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A Mathematical and Statistical Approach for Predicting the Population Growth

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Abstract
This study is aiming to develop a mathematical approach to predict the population of Jordan until year 2100. This approach applies the simple exponential growth model and the Verhulst logistic growth equation to predict the population of Jordan utilizing predated data from 1955 to 2016. The explicit solutions for each model are exactly derived by using mathematical techniques of differentiation and integration. A Non-Linear Regression analysis was applied through Minitab. The curve-fitting tool (cftool) of MATLAB was used. Results show that the exponential model predicted the population of Jordan to be 123.169 Million in 2100, with a growth rate of 3.27% per annum. The Logistic model predicted Jordan's population to be 17.346 Million in 2100, with growth rate of 4.56 %. While, the Verhulst growth equation predicted the population of Jordan to be 12.157 Million in 2100, with a growth rate of 5.25%. A comparison between outputs of the three models was conducted to reveal the exponential model cannot be used, Logistic Based Models are more reasonable. This study will provide a deep insight into the population projection in Jordan, a country with limited resources, and in an area teeming with conflicts and wars. This study contributes in the existing literature by applying three population growth models to empirically examine the pattern of Jordan's population growth until the end of the current century.

Keywords: Exponential growth model, Logistic growth Model, Verhulst growth equation, Jordan, Non-linear Regression

Introduction

One of the most important problems in the world today is the massive growth of the population in developing countries. Population size and growth of any country are becoming of importance especially to governments. As gaining accurate information about the future population size supports planning activities [1]**.** In this work, we introduce mathematical modeling and Non-Linear Regression to estimate the populations until 2100 for one of the developing counties which is Jordan. Located in the Middle East, it is bordered by Saudi Arabia, Iraq, Syria, and Palestine [2](Appendix 1).

A mathematical model is defined as "a collection of equations based on quantitative description of a real world phenomenon, it is created in the hope that the predicted behavior will resemble the actual one" [3]. The well-known models in population growth estimation are the simple exponential growth and Verhulst Logistic growth Models. The simple exponential growth model can provide an adequate representation to population growth in ideal environment with unlimited resources. While in nature, as population size increase, population growth rate gradually decreases due to limiting factors. While, Population growth rate slows and eventually stops. This is known as Verhulst logistic growth [4] [5]. The population size at which growth stops is generally called the carrying capacity (K), which is the upper bound number of individuals of a particular population that the environment can support [4].

Statistical Software packages are used to build a statistical model for the population estimation is becoming more widely raises the value of interest **[6].** Many websites such as [6] gave predictions for the population of different countries or regions. However, in many cases these estimations are rough and can be risky to be used in planning activities especially by governments. This work proposes two different approaches to predict the population of a country or region based on current available data. A statistical Software package is used to build a statistical model for the population estimation rate. Studying population growth is becoming more widely raises the value of interest [6].

This work studies the classical models of population growth in unconstrained and constrained environments. In addition, this work proposes a statistical method to estimate the population of Jordan's population until the end of the current century; and third: to assessing the models' ability to forecast future population growth. There are enormous concerns about the consequences of human population growth for social, environment and economic development. Therefore, it is necessary to find out an influential model to predict the present and future population growth.

The construction of this paper is that in section 2, literature review is presented. In section 3 three main mathematical techniques for the prediction of population growth are examine, the explicit solutions for each model are exactly reached by using mathematical techniques of differentiation and integration. The tests of practical data and analysis for the models are done based on the explicit solutions, and these models are applied to the predictions of Jordan's population growth in section 4. Discussion of results provided in section 5, while, conclusion, implications, limitations and future research directions are in section 6.

Literature Review

Three models –exponential growth, logistic growth and the Verhulst Logistic equation - are popular in research of the population growth. The exponential growth model was proposed by Malthus in 1798 (Malthus, 1992) [7], and it is therefore called the Malthusian growth model. Verhulst proposed the logistic growth model in 1845 [8]**.** Both originated from observations of biological reproduction process. However, when it comes to human population, the constant growth rate can be observed in doubt. Human reproduce sexually and have consciousness. We cannot be sure under what condition will an individual be reproduced. Therefore, it is in doubt whether models based on constant growth rate can explain human population growth. Taking carrying capacity into regards, logistic growth model improves the preceding exponential growth model. However, whether it can describe human population growth is in dispute. We will test the exponential growth model and logistic growth model by population data of Jordan.

The use of the logistic growth model is widely established in many fields of modeling and forecasting [9]. Through mathematical modeling approach, much work has been done to further develop these models so as to predict population growth accurately **[** [10]**;** [11]**;** [5]**;** [1]]. Population growth model is a pattern of a mathematical model that is utilized to the study of population dynamics. Models allow the better perception of how processes and complex interactions work. Population growth modeling can provide a manageable way of understanding how numbers change over time or in relation to each other.

Donovan and Welden [12] indicated that the Logistic model with explicit carrying capacity is most convenient way to study population growth as the related equation contains few parameters. They also hint that the solution for basic equation of continuous-logistic model can be obtained by integrating the equation. More details of this solution are discussed later in this paper.

Mathematical Techniques for predicting Population Growth

The exponential growth model

The theory of population presented by Thomas Robert Malthus in the first edition of Principle of Population (1798) has had great influence on the progress of economic and political thoughts [7]. In 1798, Thomas R. Malthus proposed a mathematical model of population growth. He proposed by the assumption that the population grows at a rate proportional to the size of the population. An exponential growth model could represent this. The exponential growth model has the form:

$$\frac{dN}{dt} = \alpha\, N(t)\; t_0 \le t \le t_1;$$
$$N(t_0) = N_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where $N(t)$ population in time t is, N_0 is population at the beginning, α is the constant growth rate. With a little mathematical manipulation, $dN/N(t) = \alpha\, dt$, integrating both sides:

$$\int \frac{dN}{N(t)} = \int \alpha\, dt = \alpha \int dt$$

Hence, $\ln N = \alpha t + c$.The Equation (1) could be written as in one of the two forms:

$N(t) = N_0\, e^{\alpha t}$ ………… (2a) or $N(t) = e^{\alpha t + c} = N_0 e^{\alpha(t - t_0)}$ ………… (2b)

The Logistic Growth Model

In an environment that will support a limited population it is assumed that the rate of growth of population decreases as the limiting population is approached. An appropriate model is the logistic model.The logistic model has the form:

$$dN/dt = \alpha N(1 - N/K), \quad N(0) = N_0, \ldots\ldots\ldots\ldots\ldots\ldots (3a)$$

Where: $K > 0$, is the carrying capacity (the environment's maximal load), $N(t)$: is the unknown function depending on t (given the size of population at that time),α: is the constant of proportionality, K : is the size of population that the environment can long term sustain(carrying capacity). The continuous form is a differential equation and can be solved by integrating equation (3b), this will give:

$$N(t) = \frac{K}{Ae^{-\alpha t} + 1} \ldots\ldots \ldots\ldots (3b), \text{ where } A = \frac{K - N_0}{N_0}$$

The net rate of such a population will be denoted by dN/dt. This appears the speed at which population increases in size (N) as time (t) progresses. It describes a sigmoidal growth curve approaching a stable carrying capacity (K), but it is only one of the most reasonable equations that do this. Figure 1 shows the different behaviour of the two growth models, the figure shows that exponential is not bounded and the population would grow very fast, logistic growth model reaches the carrying capacity K and will stop growth afterwards.

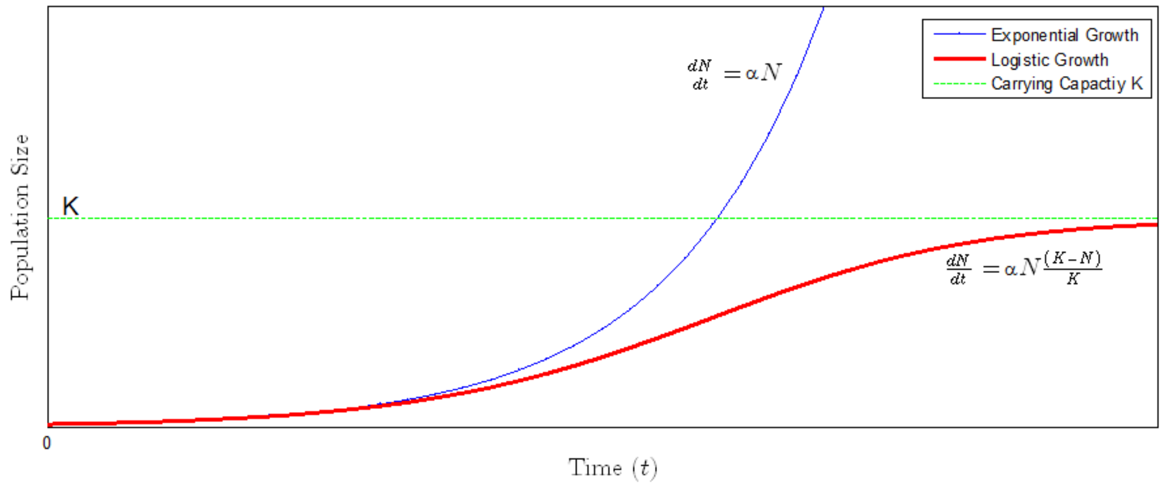

Fig. 1: The difference in the Exponential and the Logistic growth models

This analysis was introduced by Verhulst [13], who also showed that the population growth not only depends on the population size but also on how far this size is from its upper limit (K). Verhulst studied in details the exponential model proposed by Malthus, and indicated it was by far too simplistic as it only included linear terms. Verhulst hence wrote as an alternative model:

$$\frac{\alpha-\beta N(t)}{\alpha} = 1 - \frac{\beta N(t)}{\alpha} \;\ldots\ldots\ldots\;\ldots\ldots\;\ldots.\;(4)$$

Where: α and β are the vital coefficients of the population. The second term of Equation (4) describes how far the population is from its maximum limit. As the population value approaches α/β, this term becomes very small and approximates to zero, providing the correct feedback to the population growth limit. Thus, the second term models the competition for available resources, which tends to limit the population growth. Therefore, the modified equation using this new term is*:*

$$\frac{dN}{dt} = \frac{\alpha N(t)[\alpha-\beta N(t)]}{\alpha} = N(t) - \left(1 - \frac{\beta N(t)}{\alpha}\right) \ldots\ldots\ldots\ldots\ldots\;\ldots.\;\ldots\ldots.\;(5).$$

Where $t_0 \le t \le t_1; N(t_0) = N_0$

This equation is known as the Verhulst Logistic equation of population growth. Putting where N_0 represents the population at some specified time $t = 0$, equation (5) becomes:

$$\frac{dN}{dt} = \alpha N - \beta N^2 \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots..\;(6)$$

By the application of separation of variables:

$$dN = (\alpha N - \beta N^2)dt, \frac{dN}{N(\alpha-\beta N)} = dt$$

and integrating, $\int \frac{1}{\alpha}\left(\frac{1}{N} + \frac{\beta}{\alpha-\beta N}\right) dN = \int dt$ we obtain:

$$\frac{1}{\alpha}(\ln N - \ln(\alpha - \beta N) = t + c \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7).$$

At $t = 0$ and $N = N_0$, so that, $= \frac{1}{\alpha}(\ln N_0 - \ln(\alpha - \beta N_0)$, substituting C into equation (7) yields:

$$\frac{1}{\alpha}(\ln N - \ln(\alpha - \beta N) = t + \frac{1}{\alpha}(\ln N_0 - \ln(\alpha - \beta N_0)\;,$$

solving for $N(t)$ we get:

$$N(t) = \frac{\alpha/\beta}{1+(\frac{\alpha/\beta}{N_0}-1)e^{-\alpha t}} \ldots\ldots\ldots\ldots\ldots\ldots\;(8)$$

Now taking the limit as $t \to \infty$, we get:

$$\lim_{t\to\infty} N(t) = \lim_{t\to\infty}\left(\frac{\alpha/\beta}{1+(\frac{\alpha/\beta}{N_0}-1)e^{-\alpha t}}\right), \quad N(t)_{max} =$$

$$\lim_{t\to\infty} N(t) = \frac{\alpha}{\beta}\;(\alpha > 0)\;\ldots\ldots\;\ldots.\;\ldots.\;\ldots.\;\ldots..\;(9)$$

Putting $t = 1$ and $t = 2$ the values of N are N_1 and N_2 in equation (7) respectively, then we obtain the following:

$$\frac{\beta}{\alpha}(1 - e^{-\alpha}) = \frac{1}{N_1} - \frac{e^{-\alpha}}{N_0}, \quad \text{and} \quad \frac{\beta}{\alpha}(1 - e^{-2\alpha}) = \frac{1}{N_2} - \frac{e^{-2\alpha}}{N_0} \ldots\;\ldots\;\ldots\;\ldots\;\ldots\;\ldots.\;\ldots\;\ldots\;(10)$$

Dividing the second equation in relation (9) by the first equation to eliminate $^{\beta}/_{\alpha}$, we have:

$$1 + e^{-\alpha} = \left(\frac{\frac{1}{N_2} - \frac{e^{-2\alpha}}{N_0}}{\frac{1}{N_1} - \frac{e^{-\alpha}}{N_0}}\right) \ldots..\;(11)$$

So that $e^{-\alpha} = \left(\frac{\frac{1}{N_2} - \frac{e^{-2\alpha}}{N_0}}{\frac{1}{N_1} - \frac{e^{-\alpha}}{N_0}}\right) - 1$

Hence, we have:

$$e^{-\alpha} = \frac{N_0(N_2-N_1)}{N_2(N_1-N_0)} \ldots\ldots\ldots\ldots\ldots..\;(12).$$

Substituting into the first equation (9), we obtain:

$$\frac{\beta}{\alpha} = \frac{N_1^2 - N_0N_2}{N_1(N_0N_1 - 2N_0N_1 + N_1N_2)} \ldots\ldots\ldots\ldots\ldots\;\ldots\;\ldots\ldots\ldots\;(13)$$

∴ limitation value of N =

$$N(t)_{max} = \lim_{t\to\infty} N(t) = \frac{\alpha}{\beta} = \frac{N_1(N_0N_1 - 2N_0N_1 + N_1N_2)}{N_1^2 - N_0N_2} \ldots\ldots\ldots\;\ldots\;\ldots\;\ldots\;\ldots\;\ldots\;(14)$$

In this work we will denote Equation (14) as the Verhulst Equation.

Implementation and Results

In this section, the studied models will be used on actual data for Jordan. The work will try to predict the population up to year 2100.

Jordan's population estimation by exponential model:

Based on the actual of Jordan's population from year 1955 – 2016 were obtained from Worldometers Website [6] as shown below in Table 1:

Table 1: Population of Jordan from 1955 – 2016 Obtained from Worldometers [6]

Year	Actual Population	Year	Actual Population
1955	645,724	1990	3,358,453
1960	888,632	1995	4,320,158
1965	1,119,798	2000	4,767,476
1970	1,654,769	2005	5,332,982
1975	1,985,121	2010	6,517,912
1980	2,280,670	2015	7,594,547
1985	2,782,885	2016	7,747,800

The notations of time and population for the year 2017 and onwards until year 2100 are continued. To estimate the future population of Jordan, we need to determine growth rate of Jordan using the exponential growth model in Equation (2b). Using the real population of Jordan (in million) on Table 1 below with $t = 0$ corresponding to the year 1955, we have $N_0 = 645{,}742$. We can solve for the growth rate.

Now, when $N_1 = 888{,}632$ and $t = 1$

$888{,}632 = 645{,}724\ e^{5\alpha}$, It should be noticed the term 5r in the power of the exponent is because the original data are in 5 year periods.

$5\alpha = \ln\left(\frac{888{,}632}{645{,}724}\right) \Rightarrow \alpha = 0.0639$, hence the general solution of Jordan's exponential population growth model: $N(t) = 645{,}724\ e^{(0.0693)t}$ Alternatively, $N(t) = 645{,}724\ e^{0.0693(t-1955)}$

This suggests that the predicted rate of Jordan population growth is (0.0693) with the exponential growth model. Figure 2 shows the result of applying the exponential population model on Jordan data. Figure 2 shows the actual population by the bars, while the prediction is the by the line. It shows that even before reaching 2016 the errors become very high. So it cannot be used in this current form to predict the future population of Jordan. If this model should be applied it should be applied in a different approach.

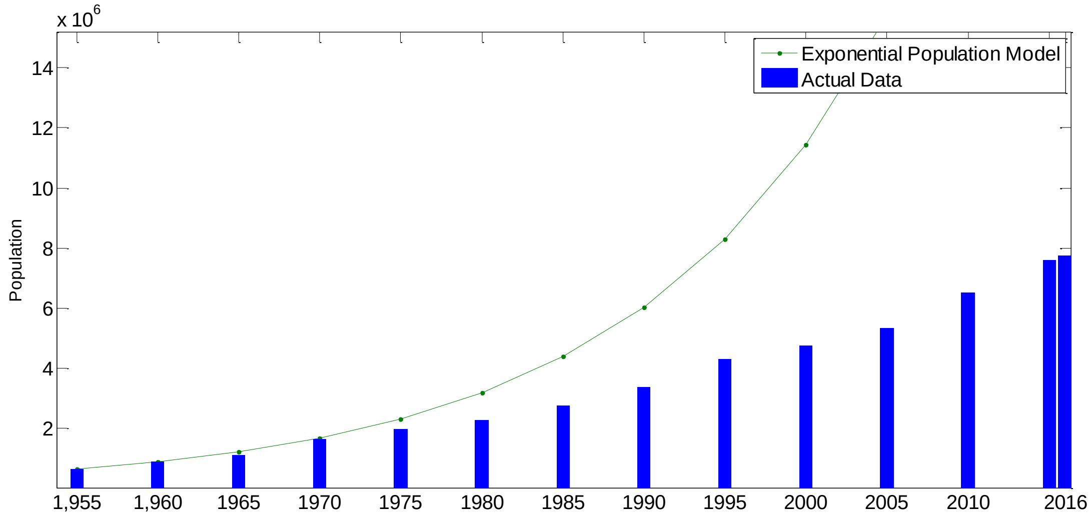

Fig. 2: The results of applying the exponential growth model for Jordan population data from year 1955- 2016

Logistic Growth model for Jordan population

Jordan population forecast in 2050 is (11,716,525) with yearly (1.2%) according to the latest United Nations Population Division estimates [14]. Thus the constant A would be equal to:

$A = \frac{K - N_0}{N_0} = \frac{\mathbf{11{,}716{,}525} - 645724}{645724}$,

This is obtained by substituting the carrying capacity and reference population that is population of Jordan for the year 1955 (from Table 1). The obtained value of term A = is 17.1447. Growth rate(r) : the term r in logistic Equation (3b) is a constant and can be determined by using the population of Jordan for the year 1955 and 1960. The estimation of r is as shown: the logistic equation is rewritten for year 1960 that is when $t = t_1$ and $N = N_1$ reapplying Equation (3b) we obtain:

$888{,}632 = \frac{11716525}{(17.1447)e^{-5\alpha}+1}$, then, $(17.1447)e^{-5\alpha} + 1 = \frac{11716525}{888632} = 13.1849$, so,

$(17.1447)e^{-5\alpha} = 12.1849 \Rightarrow \alpha = -0.0683$. For the purpose of estimation of population of Jordan from year 1965 onwards, Equation (3b) can be simplified by substituting values of terms like K, A and $e^{-\alpha}$. Hence the simplified logistic equation obtained is:

$N(t) = \frac{K}{Ae^{-\alpha t}+1} = \frac{11716525}{(17.1447)(1.0707)^{(-t+1955)}+1}$. Hence, Jordan's logistic population growth model is:

$$N(t) = \frac{11716525}{(17.1447)(1.0707)^{(-t+1955)}+1} \quad \ldots\ldots\ldots\ldots\ldots (15)$$

Population estimation from year 1970 and onwards can be obtained by substituting the values of time and population from Table 1 in Equation (15), the estimated population is the green line obtained in Figure 3, whereas the actual data are represented by the bars. Figure 3 shows that the logistic growth model can to some extent better predict Jordan's population. It also shows the predictions until year 2100. The drawback of this approach is that only two historic years were used in building the model. Moreover, the value of the limiting capacity should be known and cannot be predicted by this model.

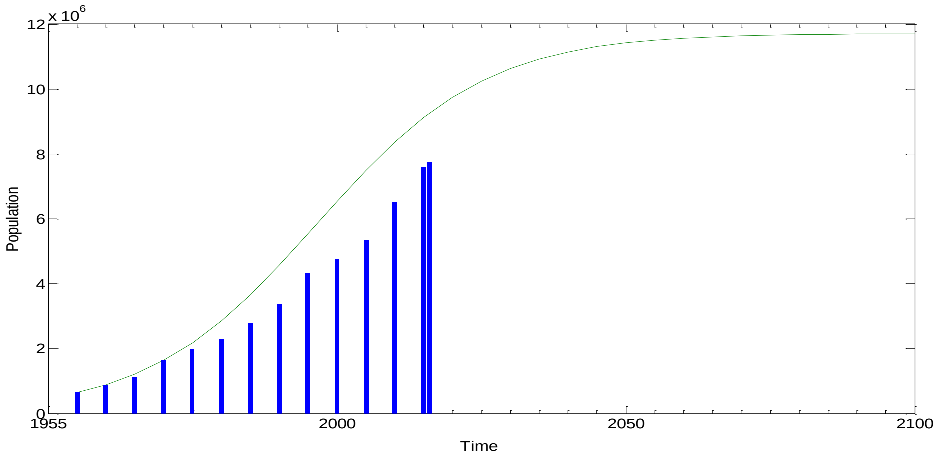

Fig 3: The population of Jordan using the Logistic Growth model

Verhulst Equation for predicting Jordan's population

Acceding to the developed model, again, based on Table 1, let t_0, t_1, and t_2 corresponding to the years 1955, 1960, and 1965, respectively. Then N_0 =645,724, N_1 =888,632, N_2 =1,119,798 also correspond. Substituting the values of N_0,N_1,N_2 in Equation (14) we obtain:

$$= \frac{888{,}632[(645{,}724)(888{,}632)-2(645{,}724)(1{,}119{,}798)+(888{,}632)(1{,}119{,}798)]}{(888{,}632)^2-[(645{,}724)(1{,}119{,}798)]}$$

= 1638012

This is the predicted carrying capacity of the population of Jordan. From Equation (11):

we obtain $e^{-\alpha} = \frac{645{,}724\,(1{,}119{,}798-888{,}632)}{1{,}119{,}798\,(888{,}632-645{,}724)} = 0.5487$

Accordingly, $\alpha = -\ln(0.5487) = 0.60020$. This also implies that the predicted rate of Jordan population growth is approximately 0.60020, With the Verhulst logistic equation. From the value of $N(t)_{max} = \frac{\alpha}{\beta}$, and, we obtained the value of $N(t)_{max} \lim_{t\to\infty} N = \frac{\alpha}{\beta} = 1638012 = \frac{0.60020}{\beta}$, $\beta = \frac{0.60020}{1638012} = 3.66 \times 10^{-7}$

Substituting the values of N_0, $e^{-\alpha}$, and $N(t)_{max} = \frac{\alpha}{\beta}$ into equation (8), we obtain:

$N(t) = \frac{1638012}{1+\left(\frac{1638012}{645{,}724}-1\right)(0.5487)^t}$, N (t) for the data of Jordan is plotted in Figure 4, where it shows that this carrying capacity is not an accurate one for Jordan. This implies that the carrying capacity should be calculated in a different approach to build accurate estimations for the population in the future.

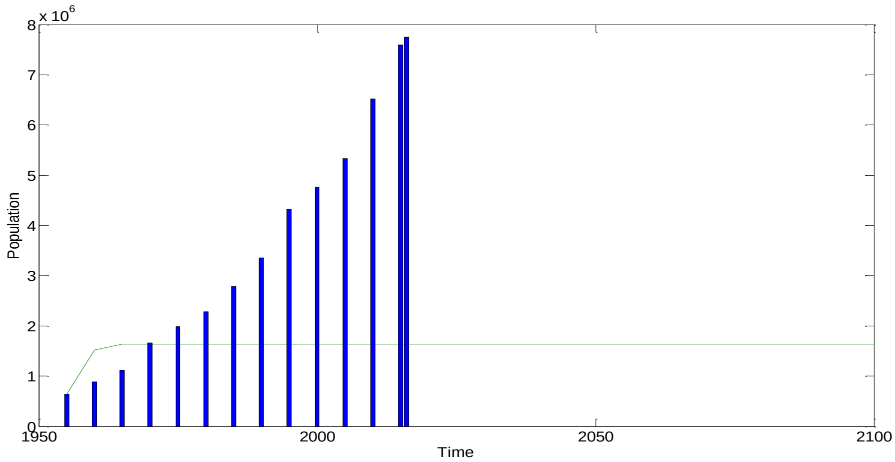

Fig 4: Verhuslt logistic equation plot on Jordan Population alongside the actual data (bars) until 2100

Non-Linear Regression Exponential Population Growth Model

In this section, we present how modern software tools applying non-Linear Regression i.e. Minitab and Matlab cftool will help predicting the population in a better way. To apply the exponential population growth model, the data of Table 1 were inputted in Minitab 17 [15]. By using a non-Linear Regression function, Equation 16 regression was obtained, which can be used to predict the population of Jordan: $N(t) = 901{,}730 \times e^{0.03278(t-1950)}$ ……. (16), where P is the population in millions, where t is the year to be predicted.

In applying the equation, Years 1955- 1975 were given a lower weight in the regression as the data is inaccurate, due to the lack of computers and national database on those years. Figure 5 shows the population growth using the exponential model. Minitab also provides 95% confidence and prediction intervals.

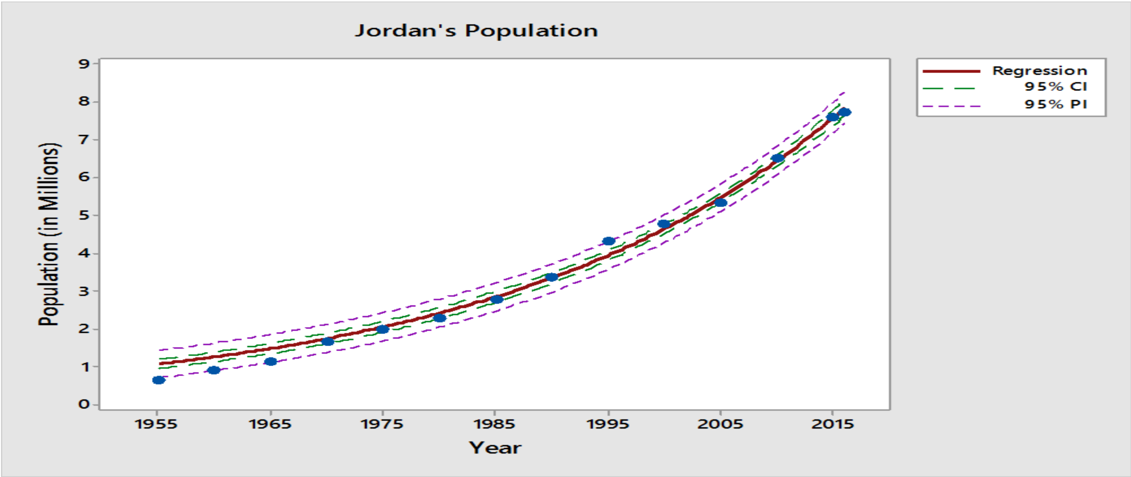

Fig 5: Non-Linear Regression Minitab 17 Output, showing Regression line, Confidence and Prediction Intervals for Jordan's actual data

Figure 6 shows both the population growth on linear and logarithmic scales until year 2100. In Figure 6, a line is placed to distinguish the original data, from the predictions. On the right side of Figure 6, a prediction interval for Jordan for years 2020-2100 is calculated. It should be noticed that the more we go into future the more, the wider the prediction interval is. It should be noticed that this model does not fit Jordan as currently the growth rate in Jordan is slowing down.

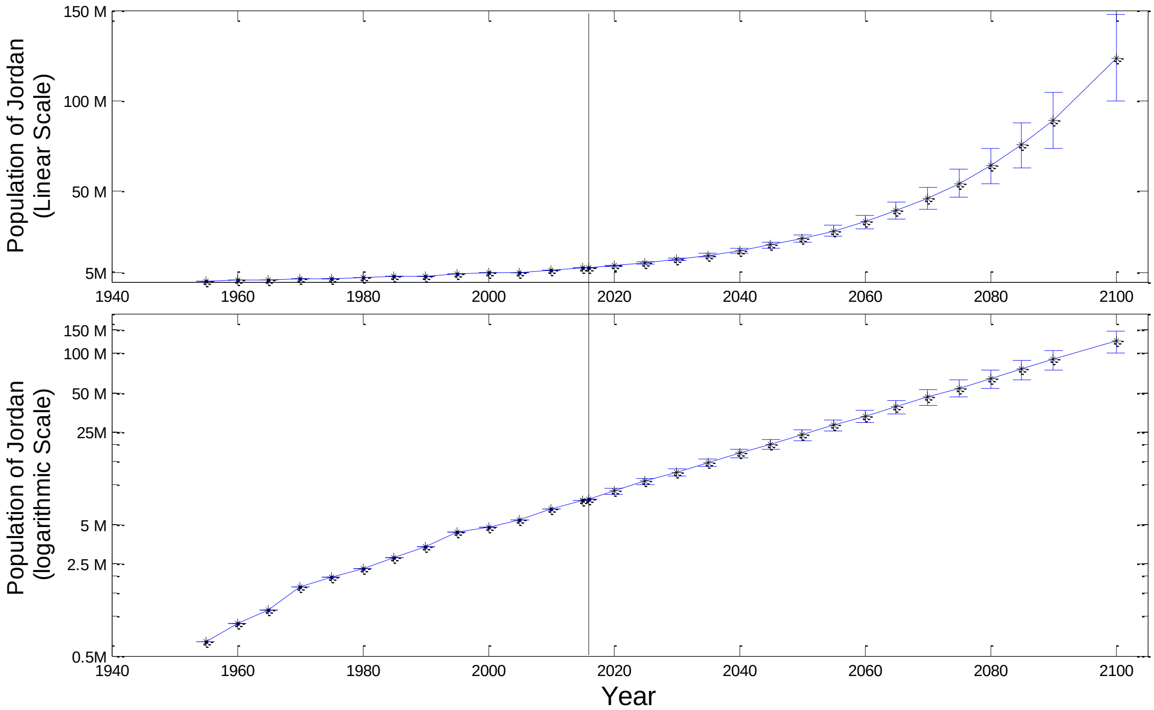

Fig. 6: Linear and Logarithmic plots of Jordan's population current (on the left) and prediction for years 2020 – 2100 (on the right)

Jordan's population estimation by Logistic model

To apply the logistic model correctly, the value of the carrying capacity (K) and the population growth rate α should be determined. The proposed in this work the MATLAB toolbox CFTOOL will be used to find the values of r and K. Figure 7 shows a screen shot from the cftool whereas Table 2 shows the final values of K and r.

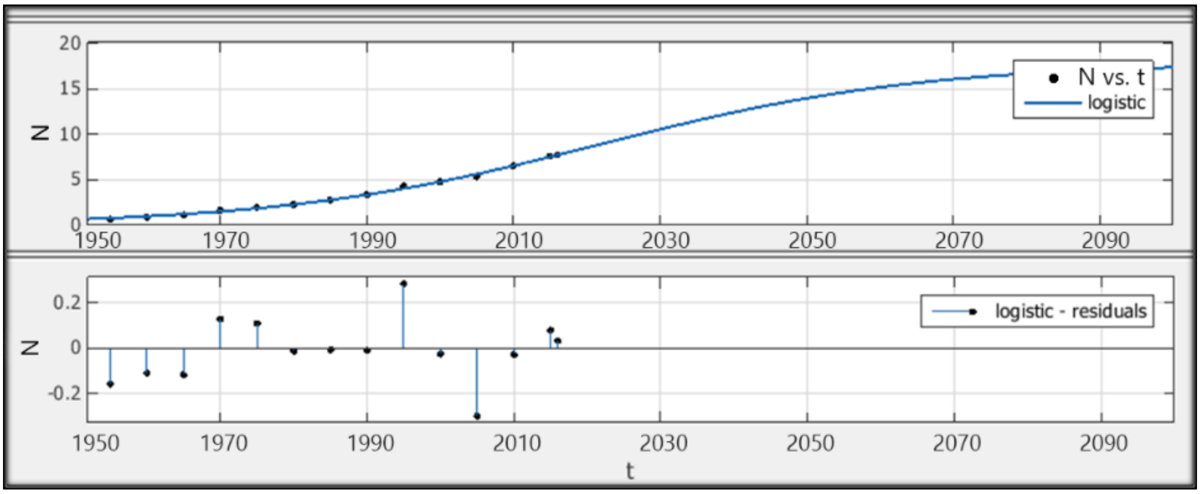

Fig. 7: Fitting the logistic growth model using MATLAB CFTOOL

Matlab Cftool provides equation 17 that fitted the Jordan's population growth data:

$$\mathrm{N(t)} = \frac{\mathrm{K}}{\left(\frac{\mathrm{K}-0.6457}{0.6457}\right)\mathrm{e}^{-\alpha(\mathrm{t}-1950)}+1} \quad \text{.........................} \quad (17)$$

The population growth rate α was found to be equal to 0.0456 with 95% confidence interval between (0.04326, 0.04795). While the carrying capacity K was equal to 17.84 with a 95% confidence interval of (13.3, 22.39), with coefficient of determination (R^2) equal to 99.67%, indicating this model fits the current data very well. It should be noticed that this approach can be used to estimate the carrying capacity based on the current data.

Jordan's population estimation by Verhulst Equation

In the final approach we are going to integrate the proposed approach in Section 4.5 using regression analysis with Verhulst Logistic equation (Equation 14). The problem found when applying this approach is that Equation 14 would give accurate estimated only if the values of $N_0, N_1,$ and N_2 are accurate. Thus population predictions of Jordan before years 1975 cannot be used. We have used the data points at t=1970, 1975 and 1980. Using Equation 14 the carrying capacity was equal to 12.255 Million. Applying this data into Matlab Cftool, the prediction model was obtained:

$$N(t) = \frac{12.255}{\left(\frac{12.255-2.28}{2.28}\right) e^{-\alpha(t-1980)}+1}$$

The population growth coefficient (α) is 0.05253 with 95% confidence interval for the population growth coefficient α of 0.05023, 0.05483. R^2 for this model is equal to 98.93%, this indicates that this model fits the data very well. Figure 8 shows the fitted equation, it should be noticed that year starts from 1980.

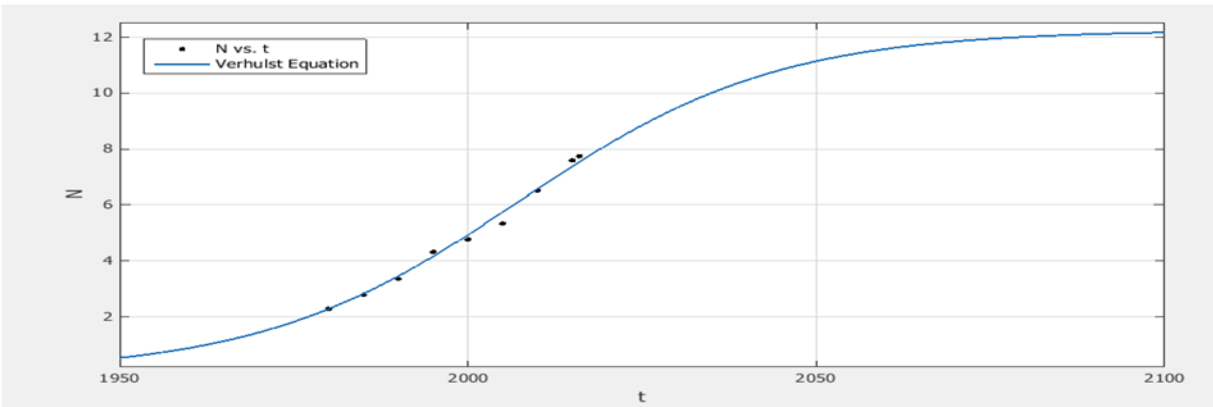

Fig. 8: the Population of Jordan with the Regression Approach applying Verhulst Equation.

Figure 9, Shows three possible charts for growth of the Jordan's population. The red curve expresses Verhulst growth pattern, the green dashed curve follows an exponential growth pattern, and the red curve is constrained so that the population is always less than or equal to some carrying capacity (K).

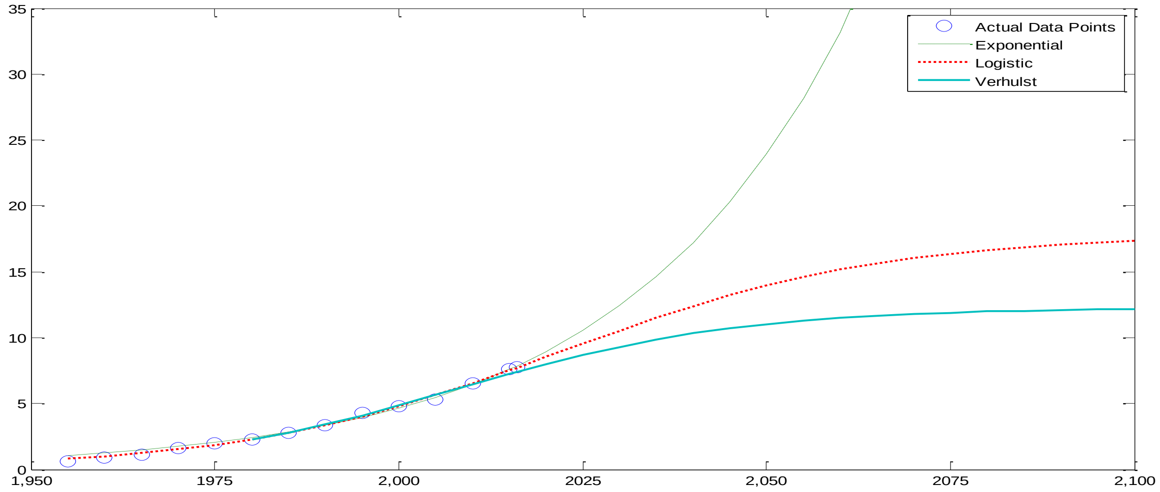

Fig. 9: Population Predictions using the Regression models of the Jordan's population Vs time

As a summary, we utilize these models to predict the population of Jordan to 2100. The predicted populations of Jordan with exponential, logistic, and Verhulst growth models that apply regression are presented in Table 2.

Table 2: Projection of Jordan's Population using exponential, logistic, and Verhulst growth models

Year	Actual Population	Projected population (in Millions)		
		Exponential model	Logistic model	Verhulst model
1955	0.646	1.062	0.804	
1960	0.889	1.252	0.998	
1965	1.120	1.474	1.236	
1970	1.655	1.737	1.525	
1975	1.985	2.046	1.875	
1980	2.281	2.411	2.293	2.281
1985	2.783	2.840	2.788	2.808
1990	3.358	3.346	3.368	3.416
1995	4.320	3.942	4.035	4.099
2000	4.767	4.644	4.791	4.843
2005	5.333	5.471	5.631	5.629
2010	6.518	6.445	6.544	6.432
2015	7.595	7.593	7.514	7.224
2016	7.748	7.846	7.713	7.379
2020		8.946	8.519	7.981
2025		10.539	9.535	8.680
2030		12.416	10.534	9.307
2035		14.627	11.494	9.854
2040		17.232	12.393	10.321
2045		20.301	13.215	10.711
2050		23.916	13.953	11.032
2055		28.176	14.601	11.292
2060		33.194	15.163	11.501
2065		39.105	15.641	11.667
2070		46.070	16.044	11.797
2075		54.275	16.381	11.900
2080		63.941	16.658	11.980
2085		75.329	16.886	12.042
2090		88.744	17.072	12.091
2095		104.549	17.224	12.128
2100		123.169	17.346	12.157

Discussion of Results

From Table 2, it can be seen that in 1990 population of Jordan was 3.358 Million and continued to increase till 1995 it was 4.320 Million. This increase was due to the first gulf war, as many Jordanians and Palestinians left Gulf countries and settled in Jordan. Also, the population begun to rise once again from 2010 – 2016, the population was 6.518 Million, and continued to increase till 2016, it was 7.748 Million. This increase was due to the situation in Syria and Iraq.

The exponential model predicted Jordan’s population to be 123.169 Million in 2100, whereas the Logistic model projected it to be 17.346 Million, and 12.157 Million by Verhulst Equation. The exponential growth model takes the time (t) only as an explanatory variable, the growth rate is constant, but the model can't explain the decrease of the population growth rate, this will cause a great problem since it only measures the time trend of a population without taking into account the other factors which affecting the population growth.

The exponential model predicts that the population would grow without bound, but this cannot possibly happen indefinitely. Most populations are constrained by limitations on resources even in the short run and none is unconstrained forever.Therefore, the conclusion cannot be applied to the long-term population growth since no weak dependent steady relationships exist in the model [16].

The Logistic growth function depends on the value of the carrying capacity (K). Its utility in real populations is limited because the dynamics of populations are complex and because it is difficult to come up with the real value for K in a given habitat. Moreover, K is not a fixed number over time; it is always changing depending on many conditions. Hence, it would be hard to make population predictions using this model. For both models nearly exponential growth at the onset, this eventually goes into saturation and converges to its carrying capacity [16]

As seen in the implementation and results, the Verhulst model is a deterministic model, meaning that it does not contain any stochastic components. Hence, this model cannot explain the fluctuations about the level of carrying capacity. It should be noticed that applying Verhulst Equation (14) to find the carrying capacity is heavily affected by the inaccuracies in the data. Thus only relaying on it will give a much deviated predictions as seen in section 4.3. An alternative approach is to only relay on recent point, which in turn is also more accurate due to computerization of governmental data collection.

According to the Table 2, Jordan's population is on growing up in the long run. However, we cannot predict population in the future accurately on a solid scientific basis because of impacts on population growth such as political and economic situations**.**We may reach a small growth rate in the future, but we do not know when, but we can't predict that based on a historical data. In this research study, we examine Malthus's model, logistic model, and Verhulst model using mathematical techniques of differentiation and integration in addition to a statistical approach to predict the population growth in Jordan until 2100. We exactly reach the explicit solutions for each model. They are greatly clear and simple for tests of practical data and analyses. Furthermore, with tables, figures, and chart, we compare predicted data and actual data of population growth for these models.

The calculations shows that the logistic model approach consists with some errors is a good tool in population estimation, this is supported by [17]. However, many valuable factors which influence population growth have been left out in these models. Therefore, determining which the better model is not an easy matter. The presented study attempt to provide an acceptable prediction for Jordan's population growth.

6. Conclusion

Population sizes and growth in any country directly affect the situation of policy, culture, economy, education, and environment... etc., as well, determine exploring and cost of natural sources. The results of this study have the following implications. First, three main popular mathematical techniques for predicting population growth was examined to predict the population sizes. Second, to contributes in the present literature by applying three population growth models to empirically examine the pattern of country's population growth in the long run. Third, Findings of this study can be useful to the government and decision makers to create economic policies accordingly.

This research, like other studies, is not without its limitations that could constitute a future research project. First, there are differences in the actual current of historical population of Jordan. Inaccuracies might be due to many reasons; some of them are even political. Second, many valuable factors such as economic, social impacts, poverty, environment, etc. has a significant influence on the estimation of the population have been left out in these models.

Last, this study serves as a basis for further research; research should obtain data from appropriate sources. Future research studies needs to address the factors that impacts population growth to get reasonable results. Studies can use other better models to estimate the population like coalition model. Future work also, should be directed into developing better methods to find the carrying capacity i.e. maximum population capacity sustained by any country.

Appendix 1: Map of Jordan

Fig. 10: Jordan Map

References

1. A. Wali, E. Kagoyire and P. Icyingeneye, "Mathematical Modeling of Uganda Population Growth," Applied Mathematical Sciences, vol. 6, no. 84, pp. 4155-4168, 2012.
2. R. W. McColl, Encyclopedia of World Geography, InfoBase Publishing, 2014.
3. G. Ledder, Differential Equations: A modeling Approach, USA: McGraw-Hill Companies Inc., 2005.
4. W. J. Edwards and C. T. Edwards, "Population Limiting Factors," Nature Education Knowledge, vol. 3, no. 10, p. 1, 2011.
5. S. S. Kulkarni, S. R. Kulkarni and S. J. Patil, "Analysis of Population Growth of India and Estimation for Future," International Journal of Innovative Research in Science, Engineering and Technology, vol. 3, no. 9,

pp. 843-850, 2014.
6. Worldometers, "Worldometers: Real World Statistics Jordan Population," Dadax, 15 Feb 2017. [Online]. Available: http://www.worldometers.info/world-population/jordan-population/. [Accessed 15 Feb 2017].
7. T. R. Malthus, An essay on the principle of population: or, A view of its past and present effects on human happiness., Reeves & Turner, 1888.
8. P. Verhulst, "Mathematical researches into the law of population growth increase[J]," Nouveaux Mémoires de l'Academie Royale des Sciences et Belles-Lettres de Bruxelles, vol. 18, pp. 1-42, 1845.
9. R. B. Banks, Growth and Diffusion Phenomena: Mathematical Frameworks and Applications, Berlin: Springer, 1994.
10. A. Taylor, "The forecasting of remote area populations: numbers aren't everything," in Demography at the Edge: Remote Human Populations in Developed Nations, D. Carson, R. Rasmussen, P. C. Ensign, L. Huskey, A. Taylor and A. Taylor, Eds., p. 39–52.
11. A. Zafar, Differential equations and their applications, 2nd ed., New Delhi: Prentice Hall, 2004.
12. T. M. Donovan and C. Welden, Spreadsheet exercises in ecology and evolution, Sinauer Associates, Incorporated, 2001.
13. P. F. Verhulst, "Notice sur la loi que la population poursuit dans son Accroissement," Correspondance math´ematique et physique, vol. 10, pp. 113-121, 1838.
14. United Nations, "World Population Prospects: The 2012 Revision," Dept. of Economic and Social Affairs, Population Division, 2013.
15. Minitab Inc., "Minitab Release 17: Statistical Software for Windows," Minitab Inc, USA, 2014.
16. H. Wei, Y. Jiang and Y. Zhang, "A Review of Two Population Growth Models and an Analysis of Factors Affecting the Chinese Population Growth," Asian Journal of Economic Modelling, vol. 3, no. 1, pp. 8-20, 2015.
17. A. J. Patel and M.B.Prajapati, "Estimation for Future Population Growth of China by using Logistic model," International Journal of scientific development and research (IJSDR), vol. 1, no. 9, pp. 52-56, 2016.
18. Central Intelligence Agency, "The World Fact book," CIA, 2017. [Online]. Available: https://www.cia.gov/library/publications/the-world-factbook/geos/jo.html. [Accessed 15 February 2017].
19. Tsoularis, "Analysis of Logistic Growth Models," Res. Lett. Math. Sci, vol. 2, pp. 23-46, 2001.
20. C. Lewis, Industrial and business forecasting methods, London: Butterworths, 1982.
21. T. Hillen, "Applications and limitations of the verhulst model for populations," Pi, no. 6, pp. 19 - 20, 2003.
22. T. Ofori, L. Ephraim and F. Nyarko, "Mathematical Model of Ghana’s Population Growth," International Journal of Modern Management Science, vol. 2, no. 2, pp. 56-66, 2013.