WWJMRD 2017; 3(12): 42-44
www.wwjmrd.com
International Journal
Peer Reviewed Journal
Refereed Journal
Indexed Journal
UGC Approved Journal
Impact Factor MJIF: 4.25
e-ISSN: 2454-6615

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Application of Leslie Matrix Model for Projections of age-specific female Population for India

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Abstract
The idea of the future of the population is achieve with the help of the projection of the population. Leslie Matrix is an old method to project the future population. The aim of this study is to test the accuracy of the Leslie matrix, which is an age-structured version for projected future population based on fertility and survival rates. In this study, we constructed a Leslie matrix that projects the female population in India for every five-year interval between 2001 and 2051. To verify the accuracy of this technique, we compared the female data projected for 2011 with the data for the female population in India according to the 2011 census. Demonstrate the effectiveness of this method. It also summarizes the advantages and disadvantages of this method. Finally we conclude that Leslie matrix is mostly suitable for middle age group between 16-59 years. The projected total female's population is 978.93 million in 2051 year.

Keywords: Population Projection; Leslie Matrices; Fertility; Mortality.

Introduction
Population projection has become one of the most important problems in the world. The size of the population and the growth of a country directly affect the state of the economy, politics, culture, education and the environment, etc. It also helps to determine the cost of exploring natural sources. No one wants to wait until these resources are depleted because of the demographic explosion. Therefore, the study of population projection has been stared earlier. The idea of the future of the population is achieve with the help of the projection of the population. Leslie Matrix is an old method to project the future population. Leslie Matrix is a discrete age-structured model of population growth that is very popular in the field of population projections. It was invented by Patrick H. Leslie in 1945[1]. The Leslie matrix is one of the best known ways of describing the growth of populations by age, in which a population is closed to migration and where only one sex, usually the female is considered. This method requires age-specific fertility, mortality and population in the based year period. In demography, a key dimension of population dynamics is the distinction between male and female. To clarify, the population growth almost completely correlates to the fertility rates of females. Thus, just by observing female population and the growth within this subgroup, it is possible to predict population growth.

Aims & Objectives
- To check the accuracy of the Leslie matrix for projecting Indian female population.
- To project age-specific population for Indian female up to 2061.

Materials and Method
P.H. Leslie (1945) constructed a matrix to projected age-specific population. In construction a Leslie matrix there are three different set of data are required in equal age groups. First, age-specific survival rate for females Si, Second, age-specific fertility rate Fi, and third, the initial age-specific females population Pi. Now the final matrix is constructed by n X n dimension where n is number of age groups. In Leslie matrix, first row occupies by fertility data while survival rates occupy the first sub-diagonal. To project population, the t^{th} power of

the Leslie matrix is multiplied by the initial population vector x^0, where t is the number of years between the initial population and final year.

$$\text{i.e. } x_t = L^t x^0$$

$$= \begin{pmatrix} F_0 & F_1 & F_2 & \dots & F_{n-2} & F_{n-2} \\ S_0 & 0 & 0 & \dots & 0 & 0 \\ 0 & S_1 & 0 & \dots & 0 & 0 \\ 0 & 0 & S_2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & S_{n-2} & S00 \end{pmatrix} \times \begin{pmatrix} x_1^0 \\ x_2^0 \\ x_3^0 \\ x_4^0 \\ \vdots \\ x_n^0 \end{pmatrix}$$

For further information, see [2, 3, 4].

Survivorship Ratios

We have to estimate life table survival ratios, that is, proportions of birth cohorts surviving from one age interval to the next in a stationary population. I estimate ratios using L_x (average number of survivors in an age interval).

$$S_x = \frac{{}_5L_x}{{}_5L_{x-5}}$$

The estimation of the open-ended survival ratio is slightly different but still straightforward:

$$\frac{T_{85}}{T_{80}}$$

For further information, see [5, 6].

Number of Children

We used 1/(1+1.05) corresponds to a transformation of age-specific fertility rates (son and daughters) to maternity rates (only daughters), assuming that the proportion of births between males and females (SBR) is constant across mothers' ages. The number of births is also adjusted by the corresponding survival ratio from 0 to 5 years old.

Data

In this study, the age-specific Census 2001[7] and 2011[8] are used for projecting age-specific population. Where we used the 2001 census and applied the Leslie matrix to the project population during the following two consecutive 5-year intervals using the sample registration system (SRS), fertility and mortality by specific age in 2001 [9]. Check the effectiveness of the Leslie matrix by comparing the expected population for 2011 and the 2011 census. Finally we project the age-specific female population for India until 2051.

Results

Population projections for the year 2011 obtained by using the Leslie matrix constructed from initial census population 2001, ASFR and L_x in Table 1.

Table1: Input data: initial census population 2001, Lx and age-specific fertility rate (ASFR).

Age Groups	Census Population		Lx	ASFR
	2001	2011		
0-4	53460857	67297225	475632	
5-9	61735896	57399177	448996	
10-14	59362001	50060993	445384	
15-19	46391577	60763100	441921	0.0489
20-24	43551578	58255221	437080	0.2159
25-29	41969498	45272836	431264	0.1773
30-34	37004399	42401010	425533	0.0985
35-39	34621687	40856771	419830	0.0499
40-44	25924224	35939138	413283	0.0212
45-49	22597437	33419003	405246	0.0073
50-54	16777787	24637180	392765	
55-59	14105497	20783992	372725	
60-64	13965254	14729971	344826	
65-69	10360686	11492427	303677	
70-74	7198907	10102095	249438	
75-79	3296235	6355672	186288	
80-84	2313752	3598963	124702	
85+	1816284	2081255	120460	
Total	**496453556**	**585446029**	**6439049**	

Table 2 shows that the Indian female population foresaw an interval of 5 years up to 2051 years for a group of 5 years. This can be useful for developing planning for maternal health and women's education, for developing planning for older women related to health and financial security, etc. From Table 2, if we compare the total population census in 2011 with the population projected in 2011 by the Leslie matrix, we find differences only of 0.36%. This also so that the accuracy of the Leslie matrix in the projection of the female population of India.

Table 2: Projected female population by Leslie matrix for India

Age Groups	Projected female population								
	2011	2016	2021	2026	2031	2036	2041	2046	2051
0-4	67297225	74232580	76590804	76930578	80109295	85739023	91603676	96053251	99729266
5-9	57399177	63528494	70075461	72301621	72622367	75623072	80937528	86473753	90674146
10-14	50060993	56937423	63017432	69511731	71719982	72038148	75014713	80286417	85778105
15-19	60763100	49671753	56494716	62527452	68971255	71162337	71478029	74431450	79662165
20-24	58255221	60097474	49127627	55875848	61842498	68215713	70382793	70695027	73616095
25-29	45272836	57480049	59297788	48473911	55132337	61019592	67308002	69446245	69754324
30-34	42401010	44671213	56716205	58509789	47829748	54399692	60208712	66413556	68523385
35-39	40856771	41832751	44072528	55956094	57725639	47188733	53670626	59401794	65523481
40-44	35939138	40219634	41180394	43385243	55083492	56825442	46452853	52833665	58475458
45-49	33419003	35240239	39437493	40379571	42541543	54012298	55720374	45549497	51806223
50-54	24637180	32389746	34154890	38222875	39135937	41231324	52348796	54004266	44146638
55-59	20783992	23380121	30737128	32412209	36272634	37139109	39127583	49677810	51248813
60-64	14729971	19228281	21630085	28436410	29986109	33557575	34359194	36198827	45959355
65-69	11492427	12972205	16933719	19048910	25043018	26407787	29553061	30259020	31879125
70-74	10102095	9439793	10655271	13909229	15646631	20570146	21691157	24274662	24854531
75-79	6355672	7544556	7049929	7957686	10387842	11685387	15362420	16199626	18129067
80-84	3598963	4254515	5050359	4719253	5326909	6953667	7822249	10283671	10844100
85+	2081255	2790967	3461787	4182431	4373830	4766445	5758660	6672960	8331617
Total	**585446029**	**635911794**	**685683616**	**732740841**	**779751066**	**828535490**	**878800426**	**929155497**	**978935894**

In Figure 1 we verify the accuracy of the Leslie matrix by comparing the real population with the projected population for the year 2011 and it is also useful to verify the goodness of the model adaptation. Based on model adaptation, we believe that the Leslie matrix method is not reliable for the age group 0-15, but is reliable for the rest of the age groups. This statement is supported by our result in Figure 1. If we check the difference in percentage by actual and predicted population by age groups in 2011, we believe that this method is also not suitable for age groups over 60 where we find the difference more 20%.

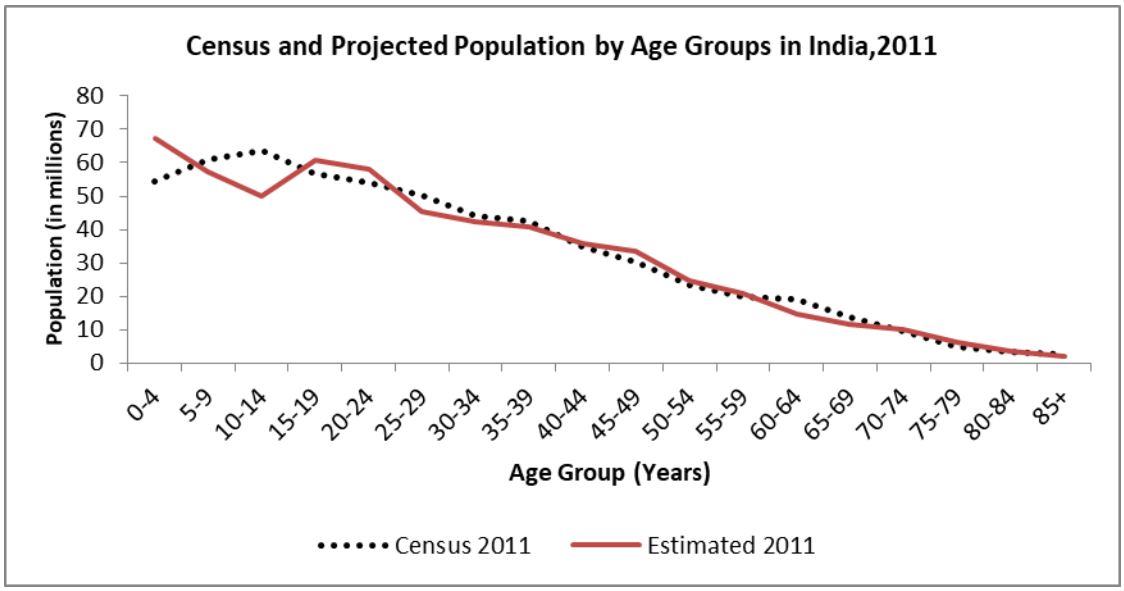

Fig 1: Comparison between census populations with projected population by age

Conclusion

Based on our results, we believe that Leslie matrix projections are not reliable for age groups of 0-15 years and age groups over 60, but are reliable for the rest of the age groups. The total percentage error to compare the 2011 total census and the population forecast for 2011 is only 0.36%. These show that the general suitability of the model. Leslie's matrix based on fertility and mortality in which the population is assumed to be closed to migration. Therefore, this method is not suitable when high in-out migration is present.

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