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New Philosophical View Merging Infinity, Imaginary Number and Zero and a New Arithmetic Solving Indeterminants

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Abstract

If the expression $i = \sqrt{-1}$ has a solution that is not a real number, what might be that? So the question is a philosophical and foundational one, simply to wonder about what we are dealing with. Concern about the foundations of mathematics therefore means that the problem of imaginary numbers is in fact a *problem* and cannot be dismissed as superfluous or trivial. What is the essence of $i = \sqrt{-1}$? We discover the very nature of $i = \sqrt{-1}$ to be $i = \sqrt{-1} = -\infty$. From Euler formula $exp2\pi i = 1$, we deduce the imaginary phase of the zero, that is $0 = 2\pi i$. Thus we tame the zero. We develop new axioms to tame infinity, zero and the imaginary number. Hence we develop a new arithmetic, solves the problem of indeterminants. Eventually, many paradoxes were resolved if infinity had taken to be both even and odd simultaneously.

Keywords: Philosophy of mathematics, Foundations of Mathematics, Logic, Infinity, Imaginary numbers, Zero, Paradoxes, Indeterminants.

Introduction

Abstract simply means outside space and time, not physical. In this sense all mathematical objects are abstract, there are infinitely many numbers. So, the reasonable conclusion is that all numbers and indeed all mathematical objects are abstract. Mathematical entities can be 'seen' with 'the mind's eye [1]. The historical struggles with infinity were perhaps more strident and fear laden than those for zero. Again, the reasons were religious. For people felt that thinking about infinity was tantamount to thinking about the Creator - If one actually endeavored to manipulate infinity as a mathematical object just as we routinely manipulate numbers and variables and other constructs in mathematics—then one was showing the utmost disrespect, and exhibiting a cavalier attitude towards the Deity. One feared being guilty of heresy or sacrilege. Many prominent nineteenth-century mathematicians strictly forbade any discussion of infinity. Of course discussions of infinity were fraught with paradoxes and apparent contradictions. These suggested deep flaws in the foundations of mathematics. They only exacerbated people's fears and uncertainties. Certainly infinity was the source of much misunderstanding and confusion. It was subject best avoided. Fear of religious fallout gave a convenient rationale for pursuing such a course [2]. The distinction between potential and actual infinities was perhaps created by Aristotle in his attempt to overcome Zeno's paradoxes. Suppose I draw a line. How many points are there on it? You're likely to say, an infinite number of points. But Aristotle would say, none. However, Aristotle maintained, if I make a cut mark in the line, I create a point. Now it has one point. I can do this over and over again, creating many points. But, two things must be noted: first, at any stage in this process I have created only a finite number of points on the line, and second, no matter how many points I have created, I can always create more. Thus, says Aristotle, the points on the line are a potential infinity, not an actual infinity [1]. The conception of the theological infinite is not a conception of an infinite collection, but rather of the unbounded or unlimited, God's infinity, the absolute infinite. The idea of the theological infinite is, for Descartes, a proof for the existence of God: since the idea of God is the idea of an actually

infinite being, and I am at most potentially infinite, I could not be responsible for this idea myself. Thus, the infinite can never be part of our perceptual experience, infinity is a revelation from God and serve as an evidence of his existence [10]. Infinity, itself is a subject of philosophy. It is a mysterious and unfathomable object that we wish to discover and to identify. "The main story is about the human mind's inability to contemplate infinity. There are warnings in the Kabbalah, the Jewish book of mysticism, about peering into this aspect of mathematics. The famous mathematician Georg Cantor is credited with discovering and pioneering this area of mathematics. Mysteriously every time Cantor attempted to seriously delve into infinity theory he experienced a mental breakdown. Kurt Gödel another famous mathematician was also mentally affected by working in this area"[3]. It was the late nineteenth-century mathematician Georg Cantor (1845-1918) who finally determined the way to tame the infinity concept. He actually showed that there are many different levels or magnitudes of infinity. It is safe to say that Cantor's notions about infinity—his concept of cardinality, and his means of stratifying infinite sets of different orders or magnitudes have been among the most profound and original ideas ever to be created in mathematics. People capitalized on the perception that Cantor was a Jew, and many of their attacks were shamelessly antisemitic—even though Cantor was in fact not a Jew. It is easy to see how Cantor might have become depressed, discouraged, and unwilling to go on. Cantor spent considerable time in asylums in an effort to cope with this calumny, and to deal with his subsequent depression. Near the end of his life, Cantor became disillusioned with mathematics [4].

The Identity $i = \sqrt{-1} = -\infty$

If the expression $i = \sqrt{-1}$ has a solution that is not a real number, what might be that? So the question is a philosophical and foundational one, simply to wonder about what we are dealing with. If numbers are in origin artifacts of counting and measuring, what kind of objects or quantities can we count or measure with imaginary numbers? As noted below, what we often get in physics and engineering is that imaginary solutions signify impossibility. Concern about the foundations of mathematics therefore means that the problem of imaginary numbers is in fact a *problem* and cannot be dismissed as superfluous or trivial. A Kantian philosophy of mathematics is going to be *realistic* in relation to empirical and phenomenal reality; imaginaries can realistically exist in our mathematical representation. Leonhard Euler said: "All such expressions as $\sqrt{-1}$, $\sqrt{-2}$, etc., are consequently impossible or imaginary numbers, since they represent roots of negative quantities; and of such numbers we may truly assert that they are neither nothing, nor greater than nothing, nor less than nothing, which.

First, we have Taylor Expansion of the function

$$x_0 = 1 \text{ about } f(x) = \sqrt{x}$$

$$\sqrt{x} = 1 + \frac{(x-1)}{2.1!} - \frac{1.3(x-1)^2}{2^2.2!} + \frac{1.3.5(x-1)^3}{2^3.3!} \dots (-1)^{n-1} \frac{1.3\dots(2n-3)(x-1)^n}{2^n.n!} + \dots$$

Put $x = -1$, the series diverges

$$\sqrt{-1} = 1 - \frac{2}{2.1!} - \frac{1.3.2^2}{2^2.2!} - \frac{1.3.5.2^3}{2^3.3!} \dots (-1)^{n-1} \frac{1.3\dots(2n-3).2^n}{2^n.n!} + \dots$$

$$\sqrt{-1} = 1 - \frac{1}{1!} - \frac{1.3}{2!} - \frac{1.3.5}{3!} \dots - \frac{1.3\dots(2n-3)}{n!} + \dots$$

$$\sqrt{-1} = -\frac{1.3}{2!} - \frac{1.3.5}{3!} \dots - \frac{1.3\dots(2n-3)}{n!} + \dots$$

$$i = \sqrt{-1} = -\sum_{n=2}^{\infty} \frac{1.3.5\dots(2n-3)}{n!} = -\infty$$

Second, we have Euler formula

$$e^{ix} = \cos x + i \sin x$$

$$e^{i\pi} = -1$$

$$i\pi = \ln(-1) = \ln(-2+1)$$

We have Taylor expansion of the function

$$\ln(x+1) = \sum (-1)^{n-1} \frac{x^n}{n}$$

$$i\pi = \ln(-2+1) = \sum (-1)^{n-1} \frac{(-2)^n}{n} = -\sum \frac{2^n}{n}$$

$$i = \sqrt{-1} = \frac{-1}{\pi} \sum_{n=1}^{\infty} \frac{2^n}{n} \rightarrow -\infty$$

Since any number less than $|\infty|$ indeed is a real number that $i = \sqrt{-1}$ can never takes, and since $\pm\infty \notin (-\infty, \infty) = \mathbf{R}$, one could reasonably deduces:

$$i = \sqrt{-1} = -\infty \quad (1)$$

This is a very logical statement. **We shall adopt it as an axiom.**

Logically, $i = \sqrt{-1} = -\infty$ as follows

- 1) It is illogical that the zero has two different reciprocals $\frac{1}{0} = \infty, \text{ and } \frac{1}{0} = -\infty$ that is $\infty = -\infty$, a ridiculous result.
- 2) It is logically that the reciprocal of the largest number equals the smallest number and vice versa.
- 3) The largest number indeed is ∞ (**infinity**), while the smallest number is not **zero**, since $-\infty < \dots < 0 < \dots < \infty$. Thus the smallest number indeed is $-\infty$.
- 4) So, logically

$$\frac{1}{\infty} = -\infty, \text{ and } \frac{1}{-\infty} = \infty. \text{ Hence}$$

$$(\infty)(-\infty) = 1$$

$$\therefore (-\infty)(-\infty) = -1$$

$$\therefore \sqrt{-1} = -\infty$$

Representation of complex roots

As we know, the roots of a function $f(x) = 0$ are those values of x where the curve intercepts the horizontal axes. This is well done for the case of real roots, why it shouldn't be true for imaginary roots?

$f(x) = x^2 + 1$ Let us plot the curve of

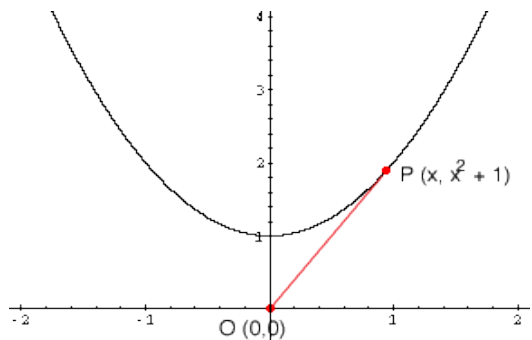


Fig. 1: the graph of $f(x) = x^2 + 1$

The curve does not intercept the horizontal axes, while it intercepts the vertical axes at $y=1$.

It is clear that in flat (Euclidean) space, the curve will eternally never intercept the horizontal x-axis. However, in non-Euclidean geometry the situation is different. In Riemann elliptical geometry, that every pair of lines will meet each other at some finitely distant point. In the hyperbolic geometry, the straight line is just drawn locally and no longer has indefinitely continued straight. The curvature of the hyperbolic line increases as distant as it is extended. At infinity where $i = -\infty$ the curve meets the infinitely extended horizontal real line where the roots located at,

$$x^2 + 1 = 0 \Rightarrow x = \mp i = \pm \infty$$

Exactly, at $i = -\infty$, the end point of the infinity real hyperbolic line is projected to the point $z = i$ on the vertical imaginary axis in the complex plane. Note that, the statements "infinity" and "imaginary" now are equivalent. Hence the real hyperbola is an imaginary (infinite) ellipse, and the real ellipse is an imaginary (infinite) hyperbola. Napier calls the square roots of negative numbers "the ghosts of real numbers" [5], following this concept the real line can be interpreted as a ground state,

while $z = i$ represents a first state and so on:
 $0 + i = 0 \text{ mod}(i) \equiv 0(\text{first state})$

$$1 + i = 1 \text{ mod}(i) \equiv 1(\text{first state})$$

This concept will help us to resolve many paradoxes reported by Euler and Penrose in the next section. Math and physics collapse at infinity where no well known mathematical tools and algebraic operations exist. Since math is well done for imaginary numbers, one can convert infinity into its imaginary form, where mathematics is well defined.

Imaginary ellipse and imaginary hyperbola.

According to our previous discussion, at infinity the hyperbola will turn to be a huge ellipse (imaginary ellipse). While the imaginary hyperbola turns to be real ellipse

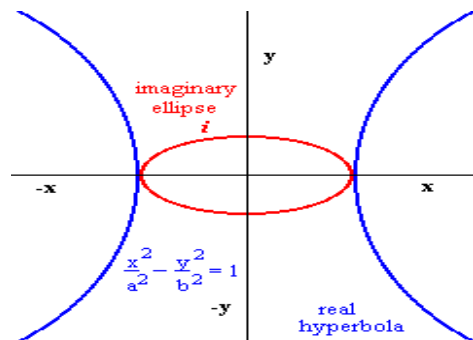


Fig. 2: the graph of the equation in blue, which is the corresponding simple equation for a hyperbola that has its center at the origin of the coordinates [5].

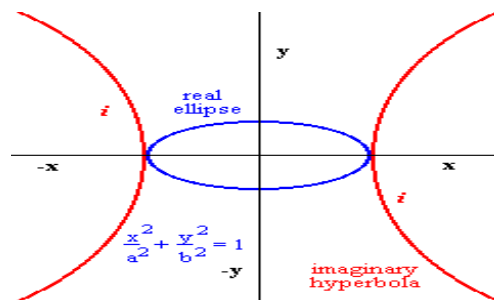


Fig. 3: The graph of the equation in blue, which is a simple equation for an ellipse that has its center at the origin of a set of rectangular coordinates [6].

Resolve the paradoxes

Historically, paradoxes and conceptual problems of mathematics have usually stemmed from the infinite. This includes, for example, Zeno's paradoxes in Greek times, infinitesimals in the seventeenth century, and the paradoxes of set theory in the late nineteenth and early twentieth centuries. In every case the problem stemmed from trying to reason with infinite quantities [1].

(a) The first paradox

Consider the following expansion

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

Put $x=2$, we get the paradox

$$-1 = 1 + 2 + 2^2 + 2^3 + \dots = \infty$$

It is obvious that $-1 \neq \infty$ in the usual sense. Let us treat this paradox from a different point of view, namely our new interpretation that discussed in the previous section. That is, -1 here is not in the ground state (level), rather it is located in the first state

$$1 + 2 + 2^2 + 2^3 + \dots = -1 + 0(\text{first state})$$

$$1 + 2 + 2^2 + 2^3 + \dots \equiv -1 + \infty$$

$$1 + 2 + 2^2 + 2^3 + \dots \equiv -1 \text{ mod}(\infty)$$

(b) The second paradox

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \dots$$

put, $x = -1$

$$\frac{1}{0} = 1 + 1 + 1 + \dots$$

$$\infty = 1 + 1 + 1 + \dots$$

accepted

put, $x = 1$

$$\frac{1}{2} = 1 - 1 + 1 - 1 \dots$$

$$\frac{1}{2} = (1 - 1) + (1 - 1) + (1 - 1) + \dots$$

$$\frac{1}{2} = 0 + 0 + 0 + \dots$$

$$\frac{1}{2} = 0$$

rejected

By our usual sense it is obvious to accept the first result

$\frac{1}{0} = \infty$, and to reject the other one $\frac{1}{2} = 0$. But math is math!

Why we do so? What is wrong? Is there any something conceptual that is missing? Again, let us treat this paradox from a different point of view. One can easily recognized that we had collected each pair together,

$$\frac{1}{2} = (1 - 1) + (1 - 1) + (1 - 1) + \dots$$

$$\frac{1}{2} = 0$$

As if we have had already assumed that, the number of terms in the infinite series is even, is infinity even? Let us try the other alternative that infinity is odd

$$\frac{1}{2} = 1 + (1 - 1) + (1 - 1) + (1 - 1) + \dots$$

$$\frac{1}{2} = 1$$

We have no reason to accept either alternative and reject the other. In order to resolve the paradox, it is reasonable to assume infinity is both even and odd. We should take into account the both alternatives simultaneously, yields

$$\frac{1}{1+x} = 1 - x + x^2 - \dots$$

$$x = 1 \Rightarrow$$

$$\therefore \frac{1}{2} = 1 - 1 + 1 - \dots = 0, \quad (\infty \quad \text{even})$$

$$\therefore \frac{1}{2} = 1 - 1 + 1 - \dots = 1, \quad (\infty \quad \text{odd})$$

Add (i) and (ii), we get

$$\frac{1}{2} + \frac{1}{2} = 0 + 1$$

$$\therefore 1 = 1$$

Eventually, the paradox is resolved if infinity is taken to be both even and odd simultaneously.

(ii) We could apply the previous method twice to resolve the paradox

$$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + \dots$$

$$x = -1 \Rightarrow$$

$$\frac{1}{(1-x)^2} = \frac{1}{4} = 1 - 2 + 3 - 4 + 5 - \dots$$

$$\frac{1}{4} = \begin{cases} (1-2) + (3-4) + (5-6) \dots; \infty & \text{even} \\ 1 + (-2+3) + (-4+5) + \dots; \infty & \text{odd} \end{cases}$$

$$\frac{1}{4} = \begin{cases} -1 - 1 - 1 - \dots; \infty & \text{even} \\ 1 + 1 + 1 + \dots; \infty & \text{odd} \end{cases}$$

$$\therefore \frac{1}{(1-x)^2} = \frac{1}{4} = \begin{cases} -1 - 1 - 1 - \dots; \infty & \text{even} \\ 1 + 1 + 1 + \dots; \infty & \text{odd} \end{cases}$$

We have no reason to accept either alternative and reject the other. In order to resolve the paradox it is reasonable to assume infinity is both even and odd. We should take into account the both alternatives simultaneously, add them together yields.

$$\frac{1}{4} + \frac{1}{4} = \frac{1}{2} = 1 - 1 + 1 - 1 + \dots$$

By the previous paradox

$$\frac{1}{2} + \frac{1}{2} = 0 + 1$$

$$\therefore 1 = 1$$

The third paradox

In his book: The Road to Reality - A Complete Guide to the Laws of the Universe, Penrose, Roger state the following paradox [7]

$$e = e^{1+2\pi i}$$

$$e = (e^{1+2\pi i})^{1+2i\pi}$$

$$e = e^{1+4\pi i - 4\pi^2}$$

$$e = e^{1-4\pi^2}$$

or

$$e^{-4\pi^2} = 1$$

Or simply:

$$e^{2i\pi} = 1$$

$$(e^{2i\pi})^{2i\pi} = (1)^{2i\pi}$$

$$\therefore e^{-4\pi^2} = 1$$

false, what is wrong ?

All previous steps satisfied a well known mathematical reasoning. How one can resolve the paradox, while still respects the previous mathematical reasoning.

New approach is needed to solve the paradox.

$$1 = e^{2\pi i} = \cos 2\pi + i \sin 2\pi$$

$$(1)^{2\pi i} = (e^{2\pi i})^{2\pi i} = (\cos 2\pi + i \sin 2\pi)^{2\pi i}$$

$$e^{\ln(1)^{2\pi i}} = e^{-4\pi^2} = (\cos 2\pi(2\pi i) + i \sin 2\pi(2\pi i))$$

$$e^{2\pi i \ln 1} = e^{-4\pi^2} = (\cos 4\pi^2 i + i \sin 4\pi^2 i)$$

R.H.S

$$e^{2\pi i \ln 1} = e^{-4\pi^2} = (\cosh 4\pi^2 + i.i \sinh 4\pi^2)$$

$$e^{2\pi i \ln 1} = e^{-4\pi^2} = (\cosh 4\pi^2 - \sinh 4\pi^2)$$

$$e^{2\pi i \ln 1} = e^{-4\pi^2} = \frac{e^{4\pi^2} + e^{-4\pi^2}}{2} + \frac{e^{4\pi^2} - e^{-4\pi^2}}{2}$$

$$e^{2\pi i \ln 1} = e^{-4\pi^2} = e^{-4\pi^2}$$

L.H.S

$$\therefore 1 = e^{2\pi i}$$

$$\therefore 0 = \ln 1 = 2\pi i$$

$$\therefore e^{2\pi i \ln 1} = e^{2\pi i (2\pi i)} = e^{-4\pi^2}$$

Note: The important point in solving the last paradox, we aware that

$$e^{ix} = \cos x + i \sin x$$

$$e^{2i\pi} = \cos 2\pi + i \sin 2\pi$$

$$e^{2i\pi} = 1$$

$$\ln e^{2i\pi} = \ln 1$$

$$\therefore 2i\pi = 0 \quad (2)$$

This is a very logical statement. We shall adopt it as an axiom.

Taming the Zero and Infinity

of course the problem which arises when one tries to consider zero and negatives as numbers is how they interact in regard to the operations of arithmetic, addition, subtraction, multiplication and division. The Indian mathematician Brahmagupta [8] tried to answer these questions. Brahmagupta attempted to give the rules for arithmetic involving zero and negative numbers in the seventh century. He explained that given a number then if you subtract it from itself you obtain zero. He gave the following rules for addition which involve zero:-*The sum of zero and a negative number is negative, the sum of a positive number and zero is positive; the sum of zero and zero is zero. A negative number subtracted from zero is positive, a positive number subtracted from zero is negative, zero subtracted from a negative number is negative, zero subtracted from a positive number is positive, zero subtracted from zero is zero.* Brahmagupta then says that any number when multiplied by zero is zero but struggles when it comes to division:-*A positive or negative number when divided by zero is a fraction with the zero as denominator. Zero divided by a negative or positive number is either zero or is expressed as a fraction with zero as numerator and the finite quantity as denominator. Zero divided by zero is zero.* Bhaskara wrote over 500 years after Brahmagupta. Despite the passage of time he is still struggling to explain division by zero. Bhaskara tried to solve the problem by writing $n/0 = \infty$. At first sight we might be tempted to believe that Bhaskara has it correct, but of course he does not. If this were true then 0 times ∞ must be equal to every number n , so all numbers are equal. The Indian mathematicians could not bring themselves to the point of admitting that one could not divide by zero. There is no single number that solves the expression "0/0.". Zero is a rich source of paradoxes. The equation $5/x=y$, where $x=0$, means that y is going to be either infinite, or

"undefined. Now, this has been criticized on the basis that $x/0$ is "undefined." "Undefined" can literally mean that it does not occur as an axiom, definition, or theorem in an axiomatic (i.e. set theoretical) number system. However, "undefined" is usually used casually to mean "meaningless." In other words, "undefined" often really means "we don't want to think about it." A more relevant or honest answer would be, "No axiomatic number system has yet been able to deal with this, so I don't know what to say [9]. Instead of the undefined phase of the zero, we use its representative form $2i\pi$, while we use i to represent the mysterious $-\infty$."

New Arithmetic Solving Indeterminants.

Consider the extended Real Numbers,

$$\square^* = [-\infty, \infty] = [i, -i]$$

Associated with additional following two axioms,

Axioms

$$i = -\infty$$

$$0 = 2i\pi$$

Similar to the definition of the zero:

- (i) $0+0=0$
- (ii) $0-0=0$
- (iii) $0 \times 0=0$
- (iv) $a - a = 0$

We state the following definition

Definition

Define $(2i\pi)$ to be the principle zero among all multiple of zero $(2ni\pi)$

We formulate definition for our principle zero $(2i\pi)$

- (i) $2i\pi + 2i\pi = 2i\pi$
- (ii) $2i\pi - 2i\pi = 2i\pi$
- (iii) $2i\pi \times 2i\pi = 2i\pi$
- (iv) $a - a = 2i\pi, \forall a$

Note, according to the above definition

- (1) $n(2i\pi) = 2i\pi + 2i\pi \dots + 2i\pi = 2i\pi$
- (2) $(2i\pi)^n = 2i\pi$

(3) Whenever you find $2i\pi + 2i\pi$ or $2i\pi - 2i\pi$

just replace each of them by $2i\pi$, that is,

$$2i\pi + 2i\pi = 2i\pi$$

$$\frac{2i\pi + 2i\pi}{2i\pi} = \frac{2i\pi}{2i\pi}$$

by (i)

$$\frac{2i\pi}{2i\pi} = \frac{2i\pi}{2i\pi}$$

Similarly

$$2i\pi - 2i\pi = 2i\pi$$

$$\frac{2i\pi - 2i\pi}{2i\pi} = \frac{2i\pi}{2i\pi}$$

by (ii)

$$\frac{2i\pi}{2i\pi} = \frac{2i\pi}{2i\pi}$$

$$(4)$$

$$2i\pi - 2i\pi = 2i\pi$$

$$(2i\pi - 2i\pi) = 2i\pi$$

$$2i\pi(1-1) = 2i\pi$$

$$2i\pi(2i\pi) = 2i\pi$$

by (iii)

Example

To illustrate the importance of the usage of the principle zero $2i\pi$ to resolve the following paradox. Consider

$$(e^{2i\pi})^i = 1^i$$

$$e^{-2\pi} = 1^i$$

$$e^{-2\pi} = e^{i \ln 1}$$

$$e^{-2\pi} = e^{i \times 0} = e^0$$

$$e^{-2\pi} = 1$$

false

Now use $0 = 2i\pi$

$$(e^{2i\pi})^i = 1^i$$

$$e^{-2\pi} = 1^i$$

$$e^{-2\pi} = e^{i \ln 1}$$

$$e^{-2\pi} = e^{i \times 0} = e^{i \times 2i\pi}$$

$$e^{-2\pi} = e^{-2\pi}$$

true

Note that if we use any other $(2ni\pi)$ zero differs than the principle zero $2i\pi$, it wouldn't solve the paradox.

For example if we use the zero $(4i\pi)$, then

$$(e^{2i\pi})^i = 1^i$$

$$e^{-2\pi} = 1^i$$

$$e^{-2\pi} = e^{i \ln 1}$$

$$e^{-2\pi} = e^{i \times 0} = e^{i \times 4i\pi}$$

$$e^{-2\pi} = e^{-4\pi}$$

false

Propositions

$$(i) \frac{x}{0} = \infty \Leftrightarrow \frac{x}{2i\pi} = -i$$

$$\therefore x = (2i\pi)(-i)$$

$$\therefore x = 2\pi$$

is the unique solution. Hence, we overcome the problem of "undefined" when dividing by zero.

$$(ii) \frac{0}{0} = \frac{2i\pi}{2i\pi} = 1$$

$$(iii) 0 \times 0 = (2i\pi)(2i\pi) = -4\pi^2$$

$$(iv) \frac{\infty}{\infty} = \frac{-i}{-i} = 1 = \frac{i}{i} = \frac{-\infty}{-\infty}$$

$$\infty \times \infty = -i \times -i = -1 = i \times i = -\infty \times -\infty$$

$$\frac{1}{\infty} = -\infty, \text{ and } \frac{1}{-\infty} = \infty$$

$$(v) 0 \times \infty = (2i\pi)(-i) = 2\pi$$

$$0 \times -\infty = (2i\pi)(i) = -2\pi$$

$$(vi) \frac{0}{\infty} = \frac{2i\pi}{-i} = -2\pi$$

$$(vii) \frac{\infty}{0} = \frac{-i}{2i\pi} = \frac{-1}{2\pi}$$

$$(iix)(0)^0 = (2i\pi)^{2i\pi} = e^{2i\pi \ln 2i\pi}$$

$$= e^{2i\pi \ln 2\pi} e^{2i\pi \ln i}$$

$$= (e^{2i\pi})^{\ln 2\pi} \left(e^{2i\pi \left(\frac{i\pi}{2}\right)} \right)$$

$$= (1)^{\ln 2\pi} e^{-\pi^2}$$

$$\therefore (0)^0 = e^{-\pi^2}$$

$$(ix)(-\infty)^{-\infty} = (i)^i = e^{i \ln i}$$

$$= e^{i \left(\frac{i\pi}{2}\right)} = e^{-\frac{\pi}{2}}$$

$$\therefore (-\infty)^{-\infty} = e^{-\frac{\pi}{2}}$$

$$(x)(\infty)^0 = (-i)^{2\pi} = e^{2\pi i \ln -i}$$

$$= e^{2\pi i (\ln -1 + \ln i)} = e^{2\pi i \left(i\pi + \frac{i\pi}{2}\right)}$$

$$\therefore (\infty)^0 = e^{-3\pi^2}$$

similarly

$$(xi)(-\infty)^0 = e^{-\pi^2}$$

Note :

$$(\infty)^0 (-\infty)^0 = e^{-3\pi^2} e^{-\pi^2} = e^{-4\pi^2}$$

$$(\infty)^0 (-\infty)^0 = (\infty \times -\infty)^0 = (-i \times i)^0$$

$$= (1)^0 = (1)^{2\pi i} = e^{2\pi i \ln 1} = e^{2\pi i (0)}$$

$$= e^{2\pi i (2\pi i)} = e^{-4\pi^2}$$

consistent

$$(xii)(0)^{-\infty} = (2i\pi)^i = e^{i \ln 2i\pi}$$

$$= e^{i (\ln 2\pi + \ln i)} = e^{i \left(\ln 2\pi + \frac{i\pi}{2}\right)}$$

$$= e^{i \ln 2\pi} e^{i \left(\frac{i\pi}{2}\right)}$$

$$\therefore (0)^{-\infty} = (2\pi)^i e^{-\frac{\pi}{2}}$$

similarly

$$(0)^\infty = (2\pi)^{-i} e^{\frac{\pi}{2}}$$

Note

$$(0)^\infty (0)^{-\infty} = \frac{(0)^\infty}{(0)^\infty} = 1$$

$$(0)^\infty (0)^{-\infty} = (2\pi)^{-i} e^{\frac{\pi}{2}} (2\pi)^i e^{\frac{-\pi}{2}} = 1$$

consistent

Results

$$(1) i = \sqrt{-1} = -\infty$$

$$(2) \frac{0}{0} = \frac{2i\pi}{2i\pi} = 1$$

$$(3) 0 \times 0 = (2i\pi)(2i\pi) = -4\pi^2$$

$$(4) \frac{\infty}{\infty} = \frac{-i}{-i} = 1 = \frac{i}{i} = \frac{-\infty}{-\infty}$$

$$(5) \infty \times \infty = -i \times -i = -1 = i \times i = -\infty \times -\infty$$

$$(6) \frac{0}{\infty} = -2\pi$$

$$(7) \frac{\infty}{0} = \frac{-1}{2\pi}$$

$$(8) (0)^0 = e^{-\pi^2}$$

$$(9) (-\infty)^{-\infty} = e^{\frac{-\pi}{2}}$$

$$(10) (\infty)^0 = e^{-3\pi^2}$$

$$(11) (0)^{-\infty} = (2\pi)^i e^{\frac{-\pi}{2}}$$

$$(12) (0)^\infty = (2\pi)^{-i} e^{\frac{\pi}{2}}$$

(13). Many paradoxes are resolved if infinity had taken to be both even and odd simultaneously.

Conclusion

Abstract, simply means outside space and time, entities can be 'seen' with 'the mind's eye'. Paradoxes and conceptual problems of mathematics have usually stemmed from the infinite. In every case the problem stemmed from trying to reason with infinite quantities. Infinity, zero and the imaginary number are the most mysterious entities and unfathomable objects in mathematics. We've dug deeply to discover the essence of these mysterious mathematical entities. We investigate the nature of the relations govern them. We have made a great effort to remove the ambiguity of these mysterious entities. We develop new axioms to tame infinity zero and the imaginary number. Hence, we tamed them to resolve many paradoxes and to solve the indeterminants.

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