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Stochastic Analysis of a Deteriorating Stand by System

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Abstract
A two –unit (identical) deteriorating standby system is analyzed. When the system is operating the inspection of standby is carried out at random epochs. Using regenerative point technique in Markov Renewal theory, several reliability characteristics of interest to system designer and operation managers are obtained.

Keywords: Availability, Exponential Distribution, Markov process, Reliability

Introduction

Several papers on reliability theory study two-unit redundant system under different sets of condition. There are many electronic and other systems in which a unit fails when it is a offline. We call such a unit a deteriorating standby (DS). In the present paper we study a two –unit (identical) deteriorating standby system is analyzed. When the system is operating the inspection of standby is carried out at random epochs. Using regenerative point technique in Markov Renewal theory, several reliability characteristics of interest to system designer and operation managers are obtained. A single repair facility is available to repair the sub-unit. The repair rate of the sub-unit is constant. The repair of sub-unit is as good as new. Using regenerative point technique with the Markov-renewal process, the following reliability characteristics have been obtained:
Steady state availability of the system

Availability Analysis

A_i (t) as the probability that the system is operative at epoch t due to both the units respectively. When initially system starts from state S_i Î E. Using the similar probabilistic arguments, point wise availability are satisfied the recursive relations as follows:

$$A_0(t) = Z_0(t) + q_{01}(t) © A_1(t) + q_{02}(t) © A_2(t)$$
$$A_1(t) = Z_1(t) + q_{14}(t) © A_4(t) + q_{18}(t) © A_8(t)$$
$$A_6(t) = q_{63}(t) © A_3(t)$$
$$A_7(t) = q_{75}(t) © A_5(t) \tag{1-9}$$

Taking Laplace transform (L.T.) of the set of relations (1-9), the solution for $A_i^*(s)$ can be written in the matrix form as follows-

Taking Laplace transform of relation (1-9), and solving for, $A_0^*(s)$, we get

$$A_0^*(s) = \frac{N_2(s)}{D_2(s)}$$

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Where

$$N_2(s) = \left[q_{01}^*\left(q_{23}^*q_{14}^* + q_{32}^*q_{41}^*\right) + q_{02}^*\left(q_{23}^*q_{14}^* + q_{32}^*q_{41}^*\right)\right]Z_1^*$$
$$+q_{80}^*\left(q_{45}^*q_{32}^* + q_{54}^*q_{01}^*\right)Z_3^* \qquad (11)$$

And

$$D_2(s) = \left(1 - q_{23}^*q_{24}^*\right)\left(1 - q_{01}^*q_{10}^*\right) - q_{01}^*\left(q_{23}^* + q_{14}^*q_{53}^*\right)q_{35}^*$$
$$-q_{02}^*\left(q_{24}^*q_{36}^* + q_{63}^*q_{75}^*\right)q_{32}^* - q_{03}^*q_{36}^*\left(q_{14}^*q_{75}^* + q_{14}^*\right)q_{30}^*$$
$$-q_{02}^*\left(q_{24}^* + q_{63}^*q_{75}^*\right) - q_{24}^*\left(q_{75}^* + q_{58}^*\right)q_{45}^*. \qquad (12)$$

The steady state availability of the system due to both the units is given by

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to\infty} A_0^*(s)$$
$$= \lim_{s\to 0} S\frac{N_1(s)}{D_1(s)}.$$

Now,

$$D_2(0) = \left(1 - p_{23}p_{24}\right)\left(1 - p_{10}p_{01}\right) - p_{01}\left(p_{23} + p_{14}p_{53}\right)p_{35}$$
$$-p_{02}\left(p_{24}p_{36} + p_{63}p_{75}\right)p_{32} - p_{03}p_{36}\left(p_{14}p_{75} + p_{14}\right)$$
$$-p_{02}\left(p_{24} + p_{63}p_{75}\right) - p_{24}\left(p_{75} + p_{58}\right)p_{32}$$
$$= p_{20}\left(1 - p_{01}p_{10}\right) - p_{01}p_{23} - p_{01}p_{14}p_{53} - p_{02}p_{24}p_{36}p_{32}$$
$$-p_{03}p_{75}p_{32} - p_{03}p_{36}p_{14}p_{15} - p_{03}p_{36}p_{14} - p_{02}p_{24}$$
$$-p_{02}p_{03}p_{75} - p_{24}p_{75}p_{45} - p_{24}p_{58}p_{45} = 0.$$

Hence, by L. Hospital's rule

$$A_0 = \lim_{s\to 0}\frac{N_1(s)}{D_1'(s)} = \frac{N_1(0)}{D_1(0)}$$

Where,

$$N_1(0) = \left[p_{01}\left(p_{23}p_{24} + p_{32}p_{41}\right) + p_{02}\left(p_{23}p_{24} + p_{32}p_{41}\right)\right]\mu_1$$
$$+\left(p_{45}p_{32} + p_{54}p_{01}\right)\mu_3$$

In order to obtain $D_1'(0)$ we collect the coefficient of m_{ij} in $D_1'(s)\big|_{s=0}$ as follows -

Coefficient of $m_{01} = p_{23}p_{24} + p_{32}p_{41}$

Coefficient of $m_{02} = p_{23}p_{24} + p_{32}p_{41}$

Coefficient of $m_{23} = p_{01}p_{24} + p_{02}p_{24}$

$$= p_{24}\left(p_{01} + p_{02}\right) = p_{24}$$

Coefficient of $m_{24} = p_{01}p_{23} + p_{02}p_{23}$

$$= p_{23}\left(p_{01} + p_{02}\right) = p_{23}$$

Coefficient of $m_{32} = p_{01}p_{41} + p_{02}p_{41} + p_{54}$

$$= \left(p_{01} + p_{02}\right)p_{41} + p_{54}$$
$$= p_{41} + p_{54}$$

Coefficient of $m_{41} = p_{02}p_{32} + p_{01}p_{32}$

$$= \left(p_{01} + p_{02}\right)p_{32}$$
$$= p_{32}$$

Coefficient of $m_{45} = p_{32}$

Coefficient of $m_{54} = p_{01}.$

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