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Stochastic Analysis of Main Units and Subunits System

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Abstract
Introduction of redundancy is one of the well-known methods by which the reliability of a system can be improved. A standby redundant system is one in which one operating unit is followed by spare units called standbys. On failure of operating unit, a standby unit is in working mode. In the present paper we investigate the probabilistic analysis of a two-main unit and four subunit systems. The system remains operative if one main unit and two subunit are in working mode. System is failed when the main unit fails. A failed system is replaced by new one. The failed sub-unit are repairable.

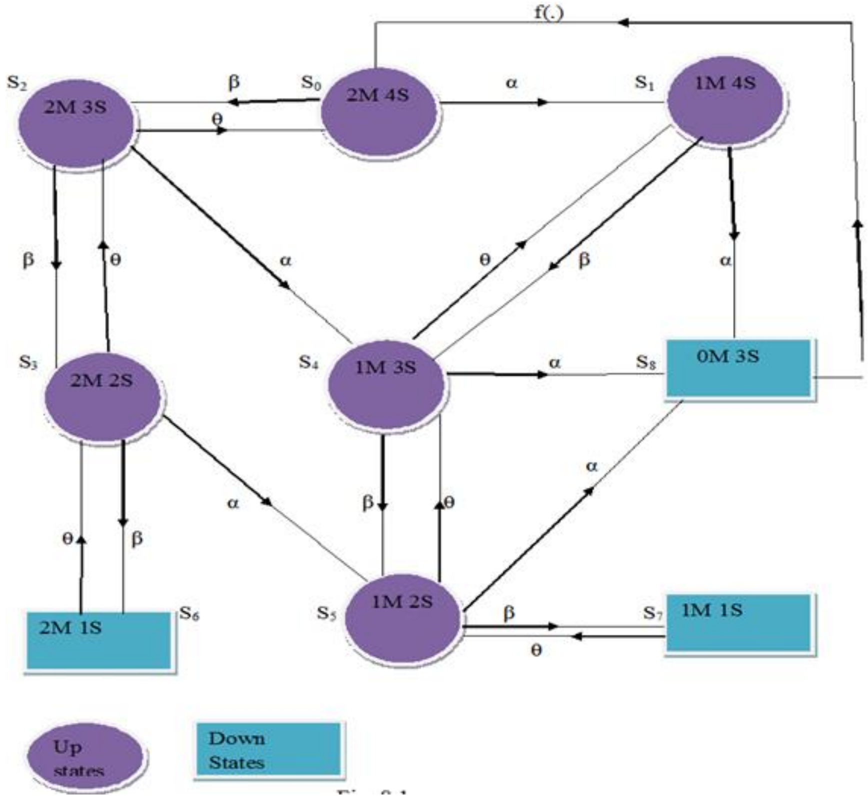
Keywords: Reliability, Availability, Mean time to system failure, busy period analysis, Markov renewal process.

Introduction
Two/three unit standby systems with two stages, that is, working or failed, have been widely discussed by a large number of researchers including (Mokaddis ET. Al. 1997, Taneja, 2005, Goel ET. Al. 2010). Introduction of redundancy is one of the well-known methods by which the reliability of a system can be improved. A standby redundant system is one in which one operating unit is followed by spare units called standbys. On failure of operating unit, a standby unit is in working mode. Ritu Mittal (2006) Analyzed stochastic analysis of a compound redundant system consisting three subsystems.

System Description and Assumption
Initially, the system starts operation with two main units and four subunits. If one main unit and two subunits work then system is in operative mode. After failure of both the main units, it is replaced by new – one. Failure time distributions of main unit and sub-units are arbitrary functions of time. A single repair facility is available to repair the sub-unit. The repair rate of the sub-unit is constant. The repair of sub-unit is as good as new. Using regenerative point technique with the Markov-renewal process, the following reliability characteristics have been obtained:
(i) Reliability of the system.
(ii) Steady state availability of the system.

Notation and States of the System
α = Constant failure rate of main unit
β = Constant failure rate of sub-unit.
θ = repair rate of sub-unit.
F (.) = replacement time pdf of main unit.
S_0 (M_2S_4) ≡ 2 main units and 4 sub-units are in normal mode.
S_1 (M_1S_4) ≡ 1 main unit and 4 sub-units are in working mode.
S_2 (M_2S_3) ≡ 2 main units and 3 sub-units are in working mode.
S_3 (M_2S_2) ≡ 2 main units and 4 sub-units are in working mode
S_4 (M_1S_3) ≡1 main unit and 3 sub-units are in working mode.
S_5 (M_1S_2) ≡ 1 main unit and 2 sub-units are in working mode.
S_6 (M_2S_1) ≡ System is in failed mode.
S_7(M_1S_1) ≡ System is in failed mode.

S_8 (M_1S_4) ≡ System is in failed mode



Analysis of Reliability and Mean Time to system Failure

Assuming that the failed states S_7 and S_8 to be the absorbing states and employing the arguments used in the theory of regenerative process, the following relation among R_i (t) be obtained:

$R_0(t) = Z_0(t) + q_{01}(t) \copyright R_1(t) + q_{02}(t) \copyright R_2(t)$

$R_1(t) = Z_1(t) + q_{14}(t) \copyright R_4(t)$

$R_2(t) = Z_2(t) + q_{20}(t) \copyright R_0(t) + q_{24}(t) \copyright R_4(t) + q_{23}(t) \copyright R_3(t)$ $R_3(t) = Z_3(t) + q_{35}(t) \copyright A_5(t) + q_{32}(t) \copyright A_2(t)$

$R_4(t) = Z_4(t) + q_{41}(t) \copyright A_1(t)$

$R_5(t) = Z_5(t) + q_{54}(t) \copyright A_4(t)$ (1-5)

Where,

$$Z_0(t) = e^{-(\alpha+\beta)t}, \qquad Z_1(t) = e^{-(\alpha+\beta)t},$$
$$Z_2(t) = e^{-(\theta+\delta+\beta)}, \qquad Z_3(t) = e^{-(\theta+\delta+\beta)},$$
$$Z_4(t) = e^{-(\theta+\alpha+\beta)}, \qquad Z_5(t) = e^{-(\theta+\alpha+\beta)}.$$

For an illustration, the equation for R_0 (t) is the sum of the following mutually exclusive contingencies:

(i) System sojourns in state S_0 up to time t. The probability of this contingency is
$e^{-(\alpha+\beta)t} = z_0(t)$, (Say).

(ii) System transits from state S_0 to S_1 during time (u, u+du); u £ t and then starting from S_1, it survives for the remaining time duration (t – u). The probability of this event is
$\int_0^t q_{01}(u)du R_1(t-u) = q_{01}(t)$ © R_1 (t).

(iii) System transits from state S_0 to S_2 during time (u, u+du); u £ t and then starting from state S_2, it survives for the remaining time duration (t – u). The probability of this event is
$\int_0^t q_{02}(u)du R_2(t-u) = q_{02}(t)$ © R_2 (t).

Taking Laplace Transform of relations (1-5) and writing the solution of resulting set of algebraic equations in the matrix form as follows

$$\begin{bmatrix} R_0^* \\ R_1^* \\ R_2^* \\ R_3^* \\ R_4^* \\ R_5^* \end{bmatrix} = \begin{bmatrix} 1 & -q_{01}^* & -q_{02}^* & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -q_{14}^* & 0 \\ -q_{20}^* & 0 & 1 & -q_{23}^* & -q_{24}^* & 0 \\ 0 & 0 & -q_{32}^* & 1 & 0 & -q_{35}^* \\ 0 & -q_{41}^* & 0 & 0 & 1 & -q_{45}^* \\ 0 & 0 & 0 & 0 & -q_{54}^* & 1 \end{bmatrix}^{-1} \begin{bmatrix} Z_0^* \\ Z_1^* \\ Z_2^* \\ Z_3^* \\ Z_4^* \\ Z_5^* \end{bmatrix}.$$
(6)

The argument‘s’ has been omitted from $R_i^*(s), q_{ij}^*(s)$ and $R_0^*(s)$ for brevity. Computing the above matri

$$R_0^*(s) = \frac{N_1(s)}{D_1(s)} \qquad (7)$$

Where

$$N_1(s) = \begin{vmatrix} Z_0^* & -q_{01}^* & -q_{02}^* & 0 & 0 & 0 \\ Z_1^* & 1 & 0 & 0 & -q_{14}^* & 0 \\ Z_2^* & 0 & 0 & -q_{23}^* & 0 & 0 \\ Z_3^* & 0 & 0 & 1 & 0 & -q_{35}^* \\ Z_4^* & -q_{41}^* & 0 & 0 & 1 & 0 \\ Z_5^* & 0 & 0 & 0 & -q_{54}^* & 1 \end{vmatrix}$$

$$D_1(s) = \begin{vmatrix} 1 & -q_{01}^* & -q_{02}^* & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -q_{14}^* & 0 \\ 0 & 0 & 0 & -q_{23}^* & 0 & 0 \\ 0 & 0 & -q_{32}^* & 1 & 0 & -q_{35}^* \\ 0 & -q_{41}^* & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -q_{54}^* & 1 \end{vmatrix}$$

Taking the inverse Laplace transforms of the expression (7), we can get the reliability of the system when system initially starts from state S_0.

To get the MTSF, we have a well-known formula

$$E(T_0) = \int R_0(t)dt = \lim_{S \to 0} R_0^*(S)$$

So that, using $q_{ij}^*(0) = p_{ij}$ and $Z_i^*(0) = \mu_i$ we get

$$E(T_0) = \frac{\mu_0 + p_{01}\mu_1 + p_{02}\mu_2 + p_{01}p_{23}\mu_3 + p_{02}p_{14}\mu_4}{1 - p_{01}p_{14}p_{24} - p_{02}p_{35}p_{32}p_{54}}. \quad (8)$$

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