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Dynamic stress - Deformed condition layer cylindrical layer from the harmonic wave

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Abstract
The paper considers an infinitely long circular cylinder consisting, in the general case, of a finite number of coaxial viscoelastic (or elastic) layers, surrounded by a deformed medium. The dynamical stressed - deformed state of a piecewise homogeneous cylindrical layer from a harmonic wave is investigated. Numerical results of stresses are obtained depending on the geometric and physicomechanical parameters of the system.

Keywords: circular cylinder, length of a wave, wave number, stress, deformation.

Introduction

The theory of diffraction of seismic waves is based on the solution of the boundary value problem of the dynamics of a continuous medium. The known number of works with success can be applied to the calculation of underground pipes. These include the work Guz A.N., Kubenko V.D. and Cherevko M.A. [1], which gives an example of calculating the diffraction of a plane harmonic P and SV wave on a rigid circular inclusion soldered into a thin elastic plate. In this case, cases are considered when the inclusion itself is either motionless or it moves together with the plate [2]. The problem of the interaction of an SV wave with a rigid circular inclusion, when on one part of its contour there is complete coherence with the surrounding medium, and on the other - sliding, contact, is solved by Parton V.Z. [3]. The fundamental work devoted to plane waves on a single inclusion is the work of Pao I.Kh. and Mao S.S. [4]. It investigates in detail the diffraction of plane waves of P and SV waves by a rigid and elastic inclusion. An analysis of the example of the calculation carried out by the same authors for a tunnel with concrete lining laid in a granite rock showed that with increasing lining, the tunnel decreases the maximum dynamic pressure; Dynamic stresses exceed the corresponding static by 10 - 15%; at $\alpha R=0,2$ (where α- wave number, R - the outer radius of the inclusion), the dynamic stresses are close to static. It was also shown in [5] that the magnitude of dynamic stresses mainly depends on the ratio of the shear moduli and the propagation velocities of the waves in the soil and in the lining, and also on the thickness of the lining. Similar problems were solved in [6, 7]. All of the above diffraction studies were solved by the method of potentials. In addition to this method, another approach to solving the diffraction problem is known, namely; Method of integral transformations (II P). A major contribution to this field has been made in the works [8,9].

Formulation of the problem.

An infinitely long circular cylinder consisting, in general, of an arbitrary number of coaxial viscoelastic layers, is surrounded by a deformable medium and is filled with liquid (gaseous) media. At a distance d from the cylinder is located a linear source of seismic (or explosive) loads (Fig. 1, a). It is assumed that the laminated package is an alternation of thin-walled and thick-walled layers of the cylinder. When describing the motion of thin-walled elements of a set, equations of the theory of such shells are used, based on the hypotheses of Krichgoff-Love. For thin-walled layers, the initial equations are the linear theory of viscoelasticity. The numbering of shell layers is made in layers - a product in the order of increasing their radii from $\kappa = 1$ and $\kappa = N$ (Fig. 1, b). The value characterizing the properties and the state of the thick-walled elements of the composite correspond to the values j =1, 2, 3,…, N, where the j-th viscoelastic layer is enclosed between the j-th and j-th shells. The parameters of the internal and external media are denoted by the indices i=1 and i=N+1. Under the assumption of a generalized plane-deformed state, the equation of motion in displacements has the form

$$(\tilde{\lambda}_j + 2\tilde{\mu}_j) grad\,div \vec{u}_j - \tilde{\mu} rot\,rot \vec{u}_j + \vec{b}_j = \rho_j \frac{\partial^2 \vec{u}_j}{\partial t^2}, (\mathrm{j} = 2,4,..\mathrm{N}+1)$$

$$\left[1+\frac{\mathrm{h}^2}{12\mathrm{R}_\kappa^2}\right]\tilde{\mathrm{E}}_\kappa \frac{\partial^2 \vartheta^{(\kappa)}}{\partial \theta^2} + \tilde{\mathrm{E}}_\kappa \frac{\partial w^{(k)}}{\partial \theta} - \frac{\tilde{\mathrm{E}}_\kappa h_k^2}{12R_\kappa^2} \frac{\partial^3 w^{(k)}}{\partial \theta^3} = \beta_1^{(\kappa)} \frac{\partial^2 \vartheta^{(\mathrm{k})}}{\partial \mathrm{t}^2} - \beta_2^{(\kappa)} q_\theta^{(k)}; \qquad (1)$$

$$-\tilde{\mathrm{E}}_\kappa \frac{d\vartheta^{(k)}}{d\vartheta} + \frac{\tilde{\mathrm{E}}_\kappa h_k^2}{12R_k^2} \frac{\partial^3 \vartheta^{(k)}}{\partial \theta^3} - \tilde{\mathrm{E}}_\kappa w^{(\mathrm{k})} - \frac{\tilde{\mathrm{E}}_\kappa h_k^2}{12R_k^2} \frac{\partial^4 w^{(k)}}{\partial \theta^4} = \beta_1^{(\kappa)} \frac{\partial^2 w^{(\mathrm{k})}}{\partial \mathrm{t}^2} + \beta_2^{(\kappa)} q_r^{(k)}, (\mathrm{k} = 1,3...\mathrm{N})$$

where $\tilde{\lambda}_j$, $\tilde{\mu}_j$ and $\tilde{\mathrm{E}}_\kappa$ operational Modules of Elasticity

$$\tilde{\lambda}_j f(t) = \lambda_{0j}\left[f(t) - \int_{-\infty}^{t} R_\lambda^{(i)}(t-\tau) f(\tau) d\tau \right],$$

$$\tilde{\mu}_j f(t) = \mu_{0j}\left[f(t) - \int_{-\infty}^{t} R_\mu^{(i)}(t-\tau) f(\tau) d\tau \right];$$

$$\tilde{E} f(t) = E_{01}\left[f(t) - \int_{-\infty}^{t} R_E(t-\tau) f(t) d\tau \right],$$

$$\beta_1^{(\kappa)} = \frac{\gamma_\kappa (1-\nu_\kappa^2) c_1^2 R_k^2}{R_1}, \; \beta_2^{(\kappa)} = \frac{R_k^2 (1-\nu_\kappa^2) c_1^2 \rho_1}{h_k R_1},$$

Here $\vec{u}_j(u_{rj}, u_{\theta j})$ - displacement vector, which depends от r, θ, t; ρ_j - density of the layer material;

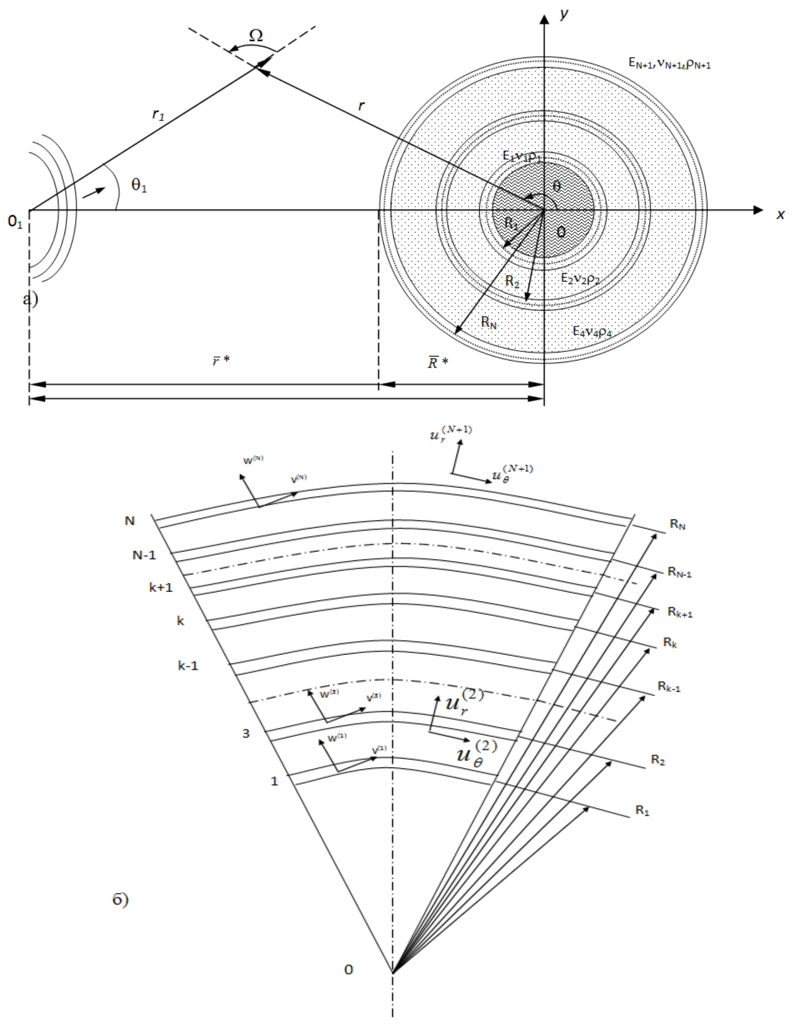

Fig.1. Calculation schemes.

$f(t)$ – some function; $R_E^{(i)}(t-\tau), R_\mu^{(i)}(t-\tau)$ and $R_\lambda^{(i)}(t-\tau)$ - relaxation core; λ_{oj}, μ_{oj} - instant elastic moduli of the viscoelastic layer, E_{01}- instantaneous elasticity moduli of the shell. At pressures up to 100 MPa, the motion of the liquid is satisfactorily described by the wave velocities for the potentials of the liquid particles [11]

$$\Delta\varphi_0 = \frac{1}{C_o^2}\frac{\partial^2\varphi_0}{\partial t^2}, \qquad (2)$$

where C_o – acoustic velocity of sound in a liquid. Potential φ_0 and the velocity vector of the liquid are related by the dependence $\vec{V} = grad\varphi_0$. Fluid pressure $r = R_0$ is determined by means of the linearized Cauchy-Lagrange integral $P = -\rho_o C_0 \frac{\partial\varphi_0}{\partial t}$ – the liquid pressure on the wall of the cylindrical layer and ρ_o – density of the liquid. Under the condition of continuous flow around a fluid, the normal component of the velocity of the liquid and the layer on the contact surface $r = R_1$ should be equal

$$\left.\frac{\partial\varphi_0}{\partial r}\right|_{r=R_1} = \left.\frac{\partial u_{r1}}{\partial t}\right|_{r=R_1}, \qquad (3)$$

where u_{r1} – moving the layer along the normal.
The problem is solved in the displacement potentials, for this purpose we represent the displacement vector in the form:

$$\vec{u}_j = grad\ \varphi_j + rot\vec{\psi}_j, \quad (j = 2,4.....N+1), \qquad (4)$$

where φ_j – longitudinal wave potential; $\vec{\psi}_j\ (\psi_{rj}, \psi_{\theta j})$ – vector potential of transverse waves

$$\left(\lambda_{oj} + 2\mu_{oj}\right)\nabla^2\varphi_j - \lambda_{oj}\int_{-\infty}^{t} R_\lambda^{(j)}(t-\tau)\nabla^2\varphi_j d\tau - 2\mu_{oj}\int_{-\infty}^{t} R_\mu^{(j)}(t-\tau)\nabla^2\varphi_j d\tau = \rho_j \frac{\partial^2\varphi_j}{\partial t^2} \qquad (5)$$

$$\mu_{oj}\nabla^2\vec{\psi}_j - \mu_{oj}\int_{-\infty}^{t} R_\mu^{(j)}(t-\tau)\nabla^2\vec{\psi}_j d\tau = \rho_j \frac{\partial^2\vec{\psi}_j}{\partial t^2}$$

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2} + \frac{\partial^2}{\partial\theta^2}$ – Differential operators in
Cylindrical coordinates and ν_j – Poisson's ratio [12]. The potential for the external region can be represented in the form

$$\varphi_{N+1} = \phi_{N+1}^{(p)} + \phi_{N+1}^{(s)}, \quad \psi_{N+1} = \psi_{N+1}^{(p)} + \psi_{N+1}^{(s)}, \qquad (6)$$

where $\phi_{N+1}^{(p)}$, $\psi_{N+1}^{(p)}$ – determine the unperturbed stress field; Field in the absence of any surface heterogeneity; $\phi_{N+1}^{(s)}$, $\psi_{N+1}^{(s)}$- the stress field due to the presence of a cylindrical surface. In [14] a generalization of the Lamb results was obtained and it was found that a non-perturbed field is determined

$$\phi_{N+1}^{(p)} = F_1\cos(\gamma+\theta_1)H_1^{(1)}(\alpha_{N+1}r_1)e^{-i\omega t}; \quad \psi_{N+1}^{(p)} = F_2\sin(\gamma-\theta_1)H_1^{(1)}(\beta_{N+1}r_1)e^{-i\omega t}, \qquad (7)$$

where

$$F_1 = ip/4\rho_{N+1}\omega c_{p(N+1)}; \qquad F_2 = ip/4\rho_{N+1}\omega c_{s2(N+1)},$$
$$\alpha_{N+1}^2 = \omega^2/c_{p(N+1)}^2; \qquad \beta_{N+1}^2 = \omega^2/c_{s(N+1)}^2,$$

and the quantity H_μ^{N} (Z) Is a Hankel function of the first kind and of order μ. Using the addition theorem for Bessole functions [15], expressions (8) can be rewritten in the form

$$\phi_{N+1}^{(p)} = \sum_{m=-\infty}^{\infty}[F_1(-1)^m J_m(\alpha_{N+1}r)H_{m+1}^{(1)}(\alpha_{N+1}Z)cos(\gamma+m\theta)]e^{-i\omega t}$$
$$\psi_{N+1}^{(p)} = \sum_{m=-\infty}^{\infty}[F_2(-1)^{m+1} J_m(\beta_{N+1}r)H_{m+1}^{(1)}(\beta_{N+1}Z)\sin(\gamma+m\theta)]e^{-i\omega t}$$
$$\phi_j^{(p)} = 0, \psi_j^{(p)} = 0\ (j = 1,2,.......N) \qquad (8)$$

where J_m- Bessel function of the first kind of order m. On the other hand, the external stress field, due to the presence of cylindrical inhomogeneities, is completely determined by solutions of equations (5), which are periodic in θ and are waves emerging from a specific point, which have amplitude decreasing in time. These decisions are as follows:

$$\phi_j^{(s)} = \sum_{m=-\infty}^{\infty}(A'_{jm}\cos(m\theta)+B'_{jm}\sin(m\theta))(A_{jm}H_m^{(1)}(\alpha_j r)+B_{jm}H_m^{(2)}(\alpha_j r))e^{-i\omega t}$$
$$\psi_j^{(s)} = \sum_{m=-\infty}^{\infty}(C'_{jm}\sin(m\theta)+D'_{jm}\cos(m\theta)(C_{jm}H_m^{(1)}(\beta_j r)e^{-i\omega t}+D_{jm}H_m^{(2)}(\beta_j r))e^{-i\omega t}$$
$$\phi_{N+1}^{(s)} = \sum_{m=-\infty}^{\infty}(A_{(N+1)m}\sin(m\theta)+B_{(N+1)m}\cos(m\theta))H_m^{(1)}(\alpha_{N+1}r)e^{-i\omega t} \quad (9)$$
$$\psi_{N+1}^{(s)} = \sum_{m=-\infty}^{\infty}(C_{(N+1)m}\sin(m\theta)+D_{(N+1)m}\cos(m\theta)H_m^{(1)}(\beta_{N+1}r)e^{-i\omega t}$$

where $A_{jm}, B_{jm}, C_{jm}, D_{jm}, A'_{jm}, B'_{jm}, C'_{jm}, D'_{jm}$ constants to be determined from the contact conditions $r=R_n$. (n=1,2,3,…N) the construction of a formal solution does not encounter fundamental difficulties, but the investigation of such a solution requires a huge amount of computation. Problems are reduced to solving non-homogeneous algebraic equations with complex coefficients

$$[c]\{g\}=\{p\} \quad (10)$$

In the case when $E_1=E_2\ldots=E_N$, $\rho_1=\rho_2\ldots=\rho_N$ and $\nu_1=\nu_2\ldots=\nu_N$, we obtain holes in an infinitely elastic medium. In this case the boundary is stress-free. The unknown constants from (11) are determined by the formulas:

$$A_{Nm}=-\sin\gamma(-1)^m\Delta_{Nm},\quad B_{Nm}=\cos\gamma(-1)^m\Delta_{Nm},$$
$$C_{Nm}=\cos\gamma(-1)^m\Delta_{Nm},\quad A_{Nm}=\sin\gamma(-1)^m\Delta_{Nm}, \quad (11)$$

where

$$\Delta_{Nm}=[F_1H_{m+1}(\alpha_N\overline{Z})\{E_{mk}K_{mk}-D_{mn}K_{mk}\}+[F_2H_{m+1}(\beta_N\overline{Z})\{H_{mk}K_{ma}-K_{ma}\overline{K}_{ma}\}]/\Delta_m;$$
$$\Delta_{Nm}=[F_2H_{m+1}(\alpha_N\overline{Z})\{D_{mk}\overline{E}_{mk}-E_{mn}D_{mk}\}+[F_2H_{m+1}(\beta_N\overline{Z})\{\overline{K}_{mk}\overline{D}_{mk}-K_{mn}\overline{E}_{mk}\}]/\Delta_m;$$
$$\overline{\Delta}_m=\overline{D}_{mk}K_{mn}-K_{mk}F_{mk};$$
$$D_{Nm}=[m^2+m-\frac{\beta_N^2R_N^{\ 2}}{2}]J_m(\alpha_NR_N)-X_NR_NJ_{m-1}(\alpha_NR_N);$$
$$\overline{D}_{Nm}=[m^2+m-\frac{\beta_N^2R_N^{\ 2}}{2}]H_m^{(1)}(\alpha_NR_N)-X_NR_NH^{(1)}{}_m(\alpha_NR_N);$$
$$E_{Nm}=m(m+1)J_m(\alpha_NR_N)-\alpha_NR_NJ_{m-1}(\alpha_NR_N).$$

is reduced to the following:

In this case, the circumferential stress on the cavity surface

$$\sigma_{\theta\theta}(R_i\theta_it)=\frac{-4}{\pi}\beta^2\mu\varphi_0\left(1-\frac{1}{R^2}\right)\sum_{n=0}^{\infty}(-1)^n\in_n H^{(1)}{}_n(\alpha r_o)\overline{T}_n\cos nDe^{-i\omega t},$$

where

$$\overline{T}_m=T_m(R_1)=[\alpha_1R_1H_{n-1}(\alpha_1R_1)-H_{n-1}(\alpha_1R_1)+Q_n(\beta_1R_1)]^{-1}$$
$$Q_n(\beta_1R_1)=\frac{(n^3-n+\frac{1}{2}\beta^2\ R_1^2)\beta_1R_1H_{n-1}(\beta_1R_1)-(n^2+n-\frac{1}{4}\beta_1^{\ 2}\ R_1^2)\beta^2{}_1R^2{}_1H^{(1)}{}_n(\beta_1R_1)}{(n^2-1)\beta_1R_1H^{(1)}{}_{n-1}(\beta_1R_1)-(n^2-n+\frac{1}{2}\beta^2R_1^2)H_n^{(1)}(\beta_1R_1)},\quad R^2=C_{p1}^2/C_{S1}^2$$

Coefficient of voltage concentration $\sigma_{\theta\theta}^*$ is defined as follows

$$\sigma_{\theta\theta}^*\Big|_{r=R_1}=\frac{\sigma_{\theta\theta}\big|_{r=R_1}}{\sigma_{\overline{r}\overline{r}}\big|_{r=R_1}} \quad (12)$$

where $\sigma_{\overline{r}\overline{r}}$ is defined as a function $\varphi^{(p)}$ as

$$\sigma_{\overline{r}\overline{r}}=i\ \pi\varphi_0\ \mu\ \alpha^2\ [\ H_2^{(1)}\pi\varphi\mu((\alpha\overline{r})\ +(1-R^2)\ H_o^{(1)}(\alpha\overline{r})\ ]\ e^{-i\omega t}$$

To solve the system of equations (11), in general, the Gauss method is used, with the separation of the principal element.

The movements are calculated as follows:

$$u_{r1} = \sum_{n=0}^{\infty}\left[h_1 H_n^{(1)}(h_1 r) A_n \frac{n}{r} + H_n^{(1)}(\iota_1 r) \right],$$

$$u_{r2} = \sum_{n=0}^{\infty}\left\{ \left[h_2 H_n^{(1)}(h_2 r) C_n + h_2 H_n^{(2)}(h_2 r) D_n \right] + \left[L_n (\frac{n}{r} H_n^{(1)}(\iota_2 r))_n + \frac{n}{r} H_n^{(2)}(\iota_2 r) M_n \right] \right\} \cos\theta,$$

$$u_{Q1} = -\sum_{n=0}^{\infty}\left\{ \left[\frac{n}{r} H_n^{(1)}(h_1 r) A_n + \iota_1 H_n^{(1)}(\iota_1 r) B_n \right] \right\} \sin\theta,$$

$$u_{Q2} = -\sum_{n=0}^{\infty}\left\{ \frac{n}{r} H_n^{(1)}(h_1 r) C_n + \frac{n}{r} H_n^{(2)}(h_2 r) D_n + \iota_1 H_n^{(1)}(\iota_2 r) L_n + \iota_2 H_n^{(2)}(\iota_2 r) m_n \right\} \sin\theta,$$

where $h_1 = p/c_{p1}$, $\iota_1 = p/c_{s1}$, $h_2 = p/c_{p2}$, $\iota_2 = p/c_{s2}$,

$$\sigma^{(1)}{}_{rr1} = \sum_{n=0}^{\infty}\left\{ \left[\lambda_1 H_n^{(1)}(h_1 r) + 2\mu h_1 H_n^{(1")}(h_1 r) \right] h_1^2 A_n + \frac{2\mu n}{r}[t_1 H_n^{(1)}(e_1 r) - \frac{1}{r} H_n^{(1)}(e, r)] B_n \right\} \cos\theta =$$

$$= \mu_2 \sum_{n=0} \left\{ \left[h_1^2 H_n^{(1)}(h_1 r) + 2\mu h_1 H_n^{(1")}(h_1 r) \right] h_1^2 A_n + \frac{2n}{r}[e_1 H_n^{(1)}(e_1 r) - \frac{1}{r} H_n^{(1)}(e, r)] B_n \right\} \cos\theta;$$

$$\sigma^{(1)}{}_{\theta\theta 1} = \sum \left\{ \left[\lambda_1 h_1^2 - 2\mu_1 \frac{n^2}{r^2}) + H_n^{(1)}(h_1 r) + \frac{2\mu}{r} h_1 H_n^{(1)}{}'(h_1 r) \right] A_n - \right.$$

$$\left. - \frac{2\mu n}{r}\left[e_1 H_n^{(1)}{}'(\iota_1 r) - \frac{1}{r} H_n^{(1)}(\iota, r) \right] B_n \right\} \cos\theta =$$

$$= C_{s1} \sum \left\{ \left[(k_1^2 - r) h_1^2 - \frac{2n^2}{r^2} \right] H_n^{(1)}(h_1 r) + \frac{2h_1}{r} H_n^{(1')}(h_1 r) A_n - \frac{2h_1}{r}\left[\iota_1 H_n^{(1)'}(\iota_1 r) - \frac{1}{r} H_n^{(1)}(\iota, r) \right] B_n \right\} \sin\theta;$$

where $\gamma = \dfrac{\mu_1}{\mu_2}$, then

$$\sigma^{(1)}{}_{rr1} = \rho_2 C_{s2}^2 \sum_{n=0}^{\infty} \left\{ h_2^2 \left[(k_2{}^2 - 2) H_n^{(1)}(h_2 r) + 2 H_n^{(1)"}(h_1 r) \right] C_n + \left[(k_2{}^2 - 2) H_n^{(2)}(h_2 r) + 2 H_n^{(2)"}(h_2 r) \right] D_n \right.$$

$$\left. + \frac{n}{r}\left[\iota_2 H_n^{(1)'}(\iota_2 r) + \frac{1}{r} H_n^{(2)"}(\iota_2 r) \right] h_n \left[\iota_2 H_n^{(1)'}(\iota_2 r) - \frac{1}{r} H_n^{(2)}(\iota_2 r) \right] M_n \right\} \cos\theta,$$

$$\sigma^{(1)}{}_{r\theta 2} = \mu_2 \sum_{n=0}^{\infty} \left\{ \frac{2n}{r}\left[\frac{1}{r} H_n^{(1)}(h_2 r) - h_2 H_n^{(1)"}(h_2 r) \right] C_n + \frac{2n}{r}\left[\frac{1}{r} H_n^{(2)}(h_2 r) - h_2 H_n^{(2)"}(h_2 r) \right] D_n \right.$$

$$+ \frac{1}{r^2}\left[e_2 r H_n^{(1)'}(e_2 r) - e^2{}_2 r^2 H_n^{(1)"}(e_2 r) - n H_n^{(1)'}(e_2 r) \right] L_n +$$

$$\left. \frac{1}{r^2}\left[e_2 r H_n^{(2)'}(e_2 r) - e^2{}_2 r^2 H_n^{(2)"}(e_2 r) - n H_n^{(1)'}(e_2 r) \right] M_n \right\} \sin\theta$$

$$C_{pi}^2 = \frac{\lambda_1 + 2\mu_i}{\rho_i}; \qquad 2C_{ji}^2 = \frac{2\mu_i}{\rho_i}, \lambda_1 = \left[\frac{\lambda_1 + 2\mu_i}{\rho_i} - \frac{2\mu_i}{\rho_i} \right] \rho_1 = (C_{pi}^2 + 2C_{s1}^2)\rho_i$$

$$\lambda_2 = (C_{p2}^2 + 2C_{s2}^2)\rho_2 \quad \text{or} \quad \lambda_i = (C_{pi}^2 + 2C_{s1}^2)\rho_i \quad C_{ji}^2 = \frac{\mu_i}{\rho_i} \qquad K_2^2 = \frac{C_{p2}^2}{C_{S2}^2} \quad (\mathrm{i} = 1{,}2)$$

Elements of the matrix [C] (10) have the following form:

$$C_{13} = -\frac{1}{b}\left[nH^{1}{}_{n}(h_1 b) - h_1 bH^{1}{}_{n+1}(h_1 b)\right];\ C_{14} = -\frac{1}{b}\left[nH^{(2)}{}_{n}(h_2 b) - h_2 bH^{(2)}{}_{n+1}(h_2 b)\right];$$

$$C_{15} = \frac{n}{b}H^{(1)}{}_{n}(\iota_2 b); C_{16} = \frac{n}{b}H^{(2)}{}_{n}(\iota_2 b); C_{21} = \frac{n}{b}H^{(1)}{}_{n}(h_2 b); C_{22} = -\frac{n}{b}\left[nH^{(1)}{}_{n}(\iota_1 b) - \iota_1 bH^{(1)}{}_{n+1}(\iota_1 b)\right];$$

$$C_{23} = -\frac{n}{b}H^{(1)}{}_{n}(h_2 b);\ C_{24} = -\frac{n}{b}H^{(2)}{}_{n}(h_2 b); C_{25} = -\frac{1}{b}\left[nH^{(1)}{}_{n}(\iota_2 b) - \iota_2 bH^{(1)}{}_{n+1}(\iota_2 b)\right];$$

$$C_{26} = -\frac{1}{b}\left[nH^{(2)}{}_{n}(\iota_2 b) - \iota_2 bH^{(2)}{}_{n+1}(\iota_2 b)\right];$$

$$C_{31} = \gamma\left\{\left[K_1^2 - 2 + \frac{2(n^2 + n + n_1^2 b^2}{b^2}\right]H^{(1)}{}_{n}(h_1 b) - \frac{2h_1}{b}H^{(1)}{}_{n-1}(h_1 b)\right\};$$

$$C_{33} = -\left\{\left[K_1^2 - 2 + \frac{2(n^2 + n + n_1^2 b^2}{b^2}\right]H^{(1)}{}_{n}(h_2 b) - \frac{2h_2}{b}H^{(1)}{}_{n-1}(h_2 b)\right\};$$

$$C_{34} = \gamma\left\{\left[K_1^2 - 2 + \frac{2(n^2 + n + n_1^2 b^2}{b^2}\right]H^{(2)}{}_{n}(h_2 b) - \frac{2h_2}{b}H^{(2)}{}_{n-1}(h_2 b)\right\}$$

$$C_{32} = \gamma_1 * \frac{2n}{b^2}\left[(n-1)H^{(1)}{}_{n}(\iota_1 b) - \iota_1 bH^{(1)}{}_{n+1}(\iota_1 b)\right]; C_{35} = -\frac{2n}{b^2}\left[(n-1)H^{(1)}{}_{n}(\iota_2 b) - \iota_2 bH^{(1)}{}_{n+1}(\iota_2 b)\right];$$

$$C_{36} = -\frac{2n}{b^2}\left[(n-1)H^{(2)}{}_{n}(\iota_2 b) - \iota_2 bH^{(2)}{}_{n+1}(\iota_2 b)\right]; C_{41} = \gamma_1 * \frac{2n}{b^2}\left[(n-1)H^{(1)}{}_{n}(h_1 b) - h_1 bH^{(1)}{}_{n+1}(h_1 b)\right];$$

$$C_{43} = \frac{2n}{b^2}\left[(k-1)H^{(1)}{}_{n}(h_2 b) - h_2 bH^{(1)}{}_{n+1}(h_2 b)\right]; C_{44} = -\frac{2m}{b}\left[(л-1)H^{(2)}{}_{n}(h_2 b) - h_2 bH^{(2)}{}_{n+1}(h_2 b)\right];$$

$$C_{42} = -\left\{\frac{e_1^2 b^2 - 2n^2}{b^2}H^{(1)}{}_{n}(e_1 b) + \frac{e_1}{b}\left[H^{(1)}{}_{n-1}(e_2 b) - H^{(1)}{}_{n+1}(e_2 b)\right]\right\};$$

$$C_{45} = -\left\{\frac{e_2^2 b^2 - 2n^2}{b^2}H^{(1)}{}_{n}(e_2 b) + \frac{e_2}{b}\left[H^{(1)}{}_{n-1}(e_2 b) - H^{(2)}{}_{n+1}(e_2 b)\right]\right\};$$

$$C_{46} = -\left\{\frac{e_2^2 b^2 - 2n^2}{b^2}H^{(2)}{}_{n}(e_2 b) + \frac{e_2}{b}\left[H^{(2)}{}_{n-1}(e_2 b) - H^{(2)}{}_{n+1}(e_2 b)\right]\right\}; C_{51} = C_{52} = 0.$$

How C_{33} at $C_{53}= -C_{33}$ with an argument $C_{53}(h_2\alpha)= -C_{33}(h_2 b)$

$$C_{53} = \left[K_2^2 - 2 + \frac{2(n^2 + n - h_2{}^2\alpha_2{}^2)}{\alpha^2}\right]H^{(1)}{}_{n}(h_2\alpha) - \frac{2h_2}{\alpha}H^{(2)}{}_{n-1}(h_2\alpha);$$

$$C_{54} = \left[K_2^2 - 2 + \frac{2(n^2 + n - h_2{}^2\alpha_2{}^2)}{\alpha^2}\right]H^{(2)}{}_{n}(h_2\alpha) - \frac{2h_2}{\alpha}H^{(2)}{}_{n-1}(h_2\alpha),$$

and in the case of $C_{55}(\iota_2\alpha) = C_{55}(\iota_2 b)$

$$C_{55}=\frac{2n}{\alpha^2}\left[(n-1)H^{(1)}{}_n(\iota_2\alpha)-\iota_2\alpha H^{(1)}{}_{n+1}(\iota_2\alpha)\right];$$
$$C_{56}=\frac{2n}{\alpha^2}\left[(n-1)H^{(2)}{}_n(\iota_2\alpha)-e_2\alpha H^{(2)}{}_{n+1}(\iota_2\alpha)\right];$$
$$C_{61}=C_{62}=0$$
$$C_{62}=(h_2\alpha)=-C_{43}(h_2b)$$
$$C_{63}=\frac{2n}{\alpha^2}\left[(n-1)H^{(1)}{}_n(h_2\alpha)-h_2\alpha H^{(1)}{}_{n+1}(h_2\alpha)\right];$$
$$C_{64}=\frac{2n}{\alpha^2}\left[(n-1)H^{(1)}{}_n(h_2\alpha)-h_2\alpha H^{(1)}{}_{n+1}(h_2\alpha)\right];$$
$$C_{65}(\iota_2\alpha)=-C_{45}(\iota_2b);$$
$$C_{65}=\frac{\iota^2{}_2\alpha^2-2n^2}{\alpha^2}H^{(1)}{}_n(\iota_2\alpha)+\frac{\iota_1}{\alpha}\left[H^{(1)}{}_{n-1}(\iota_2\alpha)-H^{(1)}{}_{n+1}(\iota_2\alpha)\right];$$
$$C_{66}=\frac{\iota^2{}_2\alpha^2-2n^2}{\alpha^2}H^{(2)}{}_n(\iota_2b)+\frac{\iota_1}{\alpha}\left[H^{(1)}{}_{n-1}(\iota_2\alpha)-H^{(1)}{}_{n+1}(\iota_2\alpha)\right];$$

Thus, we can find the elements of the matrix [C] for any order

$$\overline{E}_{mk}=m(m+1)H_m^{(1)}(\alpha_N R)-\alpha_N R_m H_{m-1}(\alpha_N R);$$
$$H_{mk}=-m(m+1)J_m(\beta_N R)-\beta RJ_{m-1}(\beta_N R);$$
$$K_{mk}=-m(m+1)H_m(\beta_n R_n)-\beta_n RH_{m-1}(\beta_n R);$$
$$\overline{K}_{mk}=-m(m^2+m-\frac{\beta^2{}_nR_n{}^2}{2})-J_n(\beta_{m/n}R_{n/n-1})+\beta_{n/n-1}RJ_{m-1}(\beta_n R_n);$$
$$\overline{K}_{mk}=-m(m^2+m-\frac{\beta^2{}_N R^2}{2})-H_m(\beta_n R)+\beta_n RH_{m-1}(\beta_n R);$$

The most obvious criterion for assessing the deterministic state is the choice of the "concentration coefficient" (stresses, deformations, etc.). The main objectives of this work are:

A) study of the redistribution of stresses due to the presence of a cavity or inclusion;

B) a study of the effect of the location of the source of excitation on this distribution. In accordance with such problems, stress concentration coefficients K_{1N}, K_{2N}, K_{3N} and K_{4N} are determined by the voltage:

$$\left[\frac{\overline{\sigma}_n}{\overline{\sigma}^{(p)}{}_{1n}},\frac{\overline{\sigma}_{2n}}{\overline{\sigma}^{(p)}{}_{2n}},\frac{\overline{\sigma}_{1n}}{\overline{\sigma}^{(p)}{}_{2n}},\frac{\overline{\sigma}_{2n}}{\overline{\sigma}^{(p)}{}_{2n}}\right]=(K_{1n},K_{2n},K_{3n},K_{4n})$$

Here $(\overline{\sigma}_{1n},\overline{\sigma}_{2n})$ и $(\sigma^{(K)}{}_{1n},\sigma^{(K)}{}_{nN})$ - the main stresses, determined by the potentials (φ,ψ) and $(\varphi^{(P)}, \psi^{(P)})$. The principal stresses are related to the components of the plane stress state by the following relations [16].

$\sigma_{1,2}=0,5\{(\sigma_{rr}+\sigma_{\theta\theta})\pm[(\sigma_{rr}-\sigma_{\theta\theta})^2+4\sigma_{r\theta}{}^2)]^{1/2}\}$

it is also possible to determine the strain energy concentration coefficient, defined by expression:

$(\psi_N/t_n{}^{(p)})_{r=Rn} = s_n / s_n{}^{(p)} = K^{(n)}\, s$

where S and $S^{(H)}$ – Functions of the deformation energy plane for the same point, related to the displacement potentials. $(\varphi_{1y}\psi)$ and $(\varphi^{(n)}\psi^{(n)})$. The energy density of the strain is expressed in terms of the principal stresses:

$E_N=[\sigma^2{}_{1n}+\sigma^2{}_{2n}-2v_n\sigma_{1n}+\sigma_{2n}]/(2E_n)$,

where E_N and ν_n respectively, Young's modulus and the Poisson's ratio.

Solutions (4) and (5) define complex stresses. Consequently, the functions of the deformation energy plane have the form:

$(T+iImT)e^{2i\omega t}=[(ReT)^2+(imT)^2]^{1/2}e^{-i(2\omega t-\alpha 1)}$

$T^{(p)}(ReF^{(p)}+iImT^{(p)})e^{2i\omega t}=[(ReT^{(p)})^2+(imT^{(p)})^2]^{1/2}e^{-i(2\omega t-\alpha)}$,

where $\alpha_{1n}=\alpha rc\ tg(ImT/ReT)$, $\alpha_2=\alpha rc\ tg(Im(T^{(p)})/(ReT^{(p)}))$.

Thus, the quantity

$$K_m=\left[\frac{(\mathrm{Re}\,\overline{S})^2+(\mathrm{Im}\,\overline{S})^2}{(\mathrm{Re}\,\overline{S}^{(p)})^2+(\mathrm{Im}\,\overline{S}^{(p)})^2}\right]^{1/2}e^{i(\alpha_{1n}-\alpha_{2m})} \quad (13)$$

Does not depend on time and is complex. As a measure of concentration, we can choose the value [16]

$$S_c=/K_m/=\left[\frac{(\mathrm{Re}\,\overline{S})^2(\mathrm{Im}\,\overline{S})^2}{(\mathrm{Re}\,\overline{S}^{(p)})^2+(\mathrm{Im}\,\overline{S}^{(p)})^2}\right]^{1/2} \quad (14)$$

Notice, that

$S_{cn}=S_c(\theta,\nu_n,\gamma_n,\alpha_n;Z,\alpha_n,R_n,\eta_n)$.

Numerical results and the limiting case of large wavelengths. To study the stress concentration on a free surface, we use the absolute values of the complex quantity and the relations (13) and (14). The value of the complex function depends on the wave number α, angle θ distances $\overline{r}$, Poisson's ratio, the ratio of Young's moduli

$E^* = E_1/E_2$, density ratios $\rho^* = \rho_1/\rho_2$, geometric parameters R_1 and R_2. If all the characteristics (Fig. 1) of the mechanical system are the same ($E_1=E_2=...E_n$; $\rho_1 = \rho_2 = ... = \rho_n$; $\nu_1 = \nu_2 = \nu_3 = ... = \nu_n$) , Then the problem of the interaction of cylindrical waves with cylindrical cavities. In the particular case, the solutions obtained for the cavity coincide with the solutions of RS. Moop and W. N. Rao [2]. The results of calculations of the stress concentration are shown in Fig. 2. From the analysis of numerical results in Fig. 2. It can be seen that α, R, and n The stress concentrations of the rim face coincide. In the particular case, we consider the interaction of waves with a rigid inclusion (Fig. 1), then on the boundary $r = R_1$ The following conditions are imposed:

$$u_r = U\cos\theta \quad u_\theta = U\sin\theta \qquad (15)$$

$$(\pi R_1^2 \rho \ddot{U}_{вкл}) = \int_0^{2\pi} (\sigma_{rr}\cos\theta - \sigma_{r\theta}\sin\theta)\Big|_{r=R_1} R_1\, d\theta,$$

where ρ_{On} – inclusion density.

It is found that the transfer and rotation of the inclusion as a rigid integer are determined by the expressions:

$U = -\dfrac{\eta}{\alpha}e^{-iwt}$ [$F_1\cos\gamma H_2(\alpha_1 Z)J_1(\alpha_1R_1) - F_2\cos\gamma H_2(\beta_1Z_1)J_1(\beta_1 R_1) - C_1H_1(\beta_1 R_1) +$
$+F_1\cos\gamma H_0(\alpha_1 Z) + J_{-1}(\alpha_1R_1) + F_2\cos\gamma H_0(\beta_1 Z)J_{-1}(\beta_1 R_1) -$
$-\beta_{N-1}H_{-1}(\alpha_1R_1) + C_{-1}H_{-1}(\alpha_1R_1)$],

$V = -\dfrac{\eta}{R_N}e^{-iwt}$ [$-F_1\sin H_2(\alpha_1 Z)J_1(\alpha_1R_1) + F_2\sin\gamma H_2(\beta_1Z_1)J_1(\beta_1R_1) - A_1H_1(\alpha_1R_1) +$
$+D_1H_{-1}(\beta_1R_1) + F_1\sin\gamma H_0(\alpha_1 Z)J_{-1}(\alpha_1R_1) + F_2\sin\gamma H_0(\beta_1 Z)J_{-1}(\beta_1R_1) +$
$+A_{-1}H_{-1}(\alpha_1R_1) + D_{-1}H_{-1}(\alpha_1R_1)$];

$\theta = -\dfrac{4\eta}{\alpha^2}e^{-iwt}$ [$F_2\sin\gamma H_1(\beta_1Z_1)\{J_0(\beta_1R_1) + \dfrac{2}{\beta_N R_N}J_{-1}(\beta_1R_1)\} + D\{H_0(\beta_1R_1) + \dfrac{2}{\beta_N R_N}H_{-1}(\beta_1R_1)\}$],

where U, V – In the directions of x and y respectively, θ-angle of rotation, $\eta = p_{cp}/p_{on}$. . The solutions of equation (15) are expressed in terms of the Bessel and Hankel functions of the first and second kind of the n-th order. Of special interest is the case of a fixed inclusion, i.e. η=0. For this inclusion, the conditions on $r = R_N$ have the form:

$U=V=\theta=0$; $\quad u_r = u_\theta = 0$.

It can be verified that from the motion the inclusion is held by the forces X and Y in the directions x and y respectively and the moment M in the x-y plane, which are defined by formulas

$\bar{X} = P\beta^2R_n^2\cos\gamma(g+h)e^{-iwt}$, $\bar{Y} = P\beta^2R_n^2\sin\gamma(-g+h)e^{-iwt}$

$\dfrac{M}{R_N} = -P\beta^2R_1^2\sin\gamma H_1(\beta_1 Z)\dfrac{\alpha_1R_1H_{-1}(\alpha_1R_1)}{V_0}e^{-iwt}$

where $\alpha = P/R_{n1}$, $\alpha_1 = (1-v_1^2)/(2\alpha_1R_1)$, $\alpha_2 = (1+v_1^2)/\beta_1R_1$.

$$g = \frac{1-\nu_1}{4\alpha a}H_2(\alpha\, z)\frac{[\beta a H_1^1(\beta a) - H_1(\beta a)]}{\bar{V}_1} + \frac{1}{2\beta a}H_1(\alpha\, z)\frac{[H_1(\alpha a) - \alpha a H_1(\alpha a)]}{\bar{V}_1},$$

$$h = \frac{1-\nu_1}{4\alpha a}H_{02}(\alpha z)\frac{[\beta a H_{-1}^1(\beta a) - H_{-1}(\beta a)]}{\bar{V}_{-1}} + \frac{1}{2\beta a}H_0(\beta\, z)\frac{[\alpha a H_{-1}^1(\alpha a) - H_{-1}(\alpha a)]}{\bar{V}_{-1}},$$

Using suitable expansions for the Hankel function, it can be shown that

$$\lim\bar{y} = \frac{P\sin\gamma}{\frac{\alpha_N^2}{\beta_N^2}+1}\begin{Bmatrix}\frac{1-\nu_N}{2}\left[H_2(\alpha_N z) + H_0(\alpha_N z)\right] - \\ -\left[H_2(\beta_N z) - H_0(\beta_N z)\right]\end{Bmatrix},$$

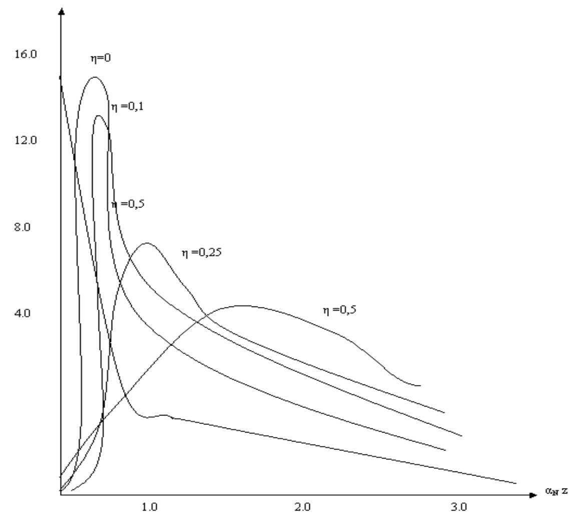

Fig. 3: Dependence of the energy concentration coefficient on the wave number α_1R_1 ($r_1/D=3,5$)

$$\lim\bar{x} = \frac{P\cos\gamma}{\frac{\alpha_N^2}{\beta_N^2}+1}\begin{Bmatrix}\frac{1-\nu_N}{2}\left[H_2(\alpha_1 z) - H_0(\alpha_1 z)\right] - \\ -\left[H_2(\beta_1 z) + H_0(\beta_1 z)\right]\end{Bmatrix},$$

$$\lim_{a\to 0}\bar{M} = 0$$

Some graphs of the nature of stress redistribution near the discontinuity surface are given. Figures 2 and 3 give graphs of the coefficient S_c as a function $\alpha_1 R_1$ for different values η and γ. These graphs show that for given η, and θ there is a value $\alpha_1R_1=\xi$, which maximizes the value S_c. The propagation of waves from a source $\bar{O}$ (Fig. 3.1) in cylindrical coordinates $\bar{\nu}$ *and* $\bar{\theta}$. The relation $|\sigma_{\bar{r}\bar{r}}/\sigma_{\bar{\theta}\bar{\theta}}|$ depending on the $\alpha\bar{r}$ at $\nu = 0,25$ (without a cylindrical body). It can be seen that radial stresses at high wave numbers are almost three times greater than $\sigma_{\theta\theta}$.

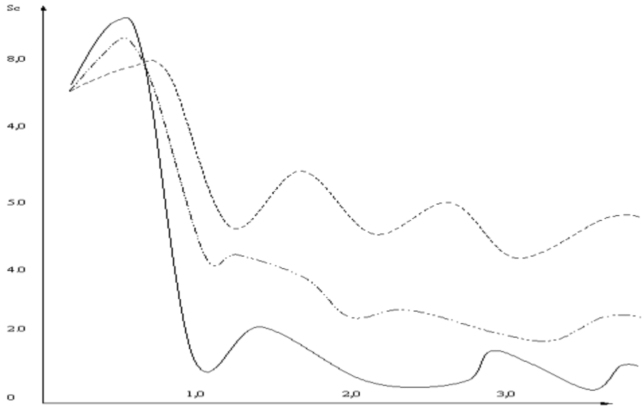

Fig. 4: Dependence of the energy concentration coefficient on the wave number (v=0.25).

In the following example, we consider the interaction of cylindrical waves with a cylindrical layer (the boundary conditions at the contact of the layer ($r = R_2$) and free surface ($\nu = R_i$) are given in (11). From the general solutions we obtain solutions for n = 1,2. The numerical results are shown in Fig. 3.4. From Fig.3.4. It is clear that the concentration of the voltage depends essentially on the location of the harmonic wave source. When $r_o / D = 2$ the dynamic concentration curve differs from static to 15%. When $\alpha_1 P_1=2$ the results of the static and dynamic stress state are radically different for close $r_o /D = 2$ source distances. Now we consider some limiting cases. Here are the results for the hole. If in equation (15) r_o Tends to infinity, then we can use the asymptotic expansions of the Hankel function for large values of the argument.

$$\lim_{r_o \to o} \sigma_{\theta\theta}\Big|_{r=R_1} \approx \frac{4}{4}\left[1-\frac{1}{k^2}\right]\sum_{m=0}^{\infty} C^{m+1} \in_{n1} S_m \cos e^{-i\omega t}$$

α - конечное .

This expression completely coincides with the expressions obtained [18] for a plane incident wave. Defining an asymptotic static solution, we obtain

$$\lim_{\alpha \to o} \sigma_{\theta\theta}\Big|_{r=R_1} \approx 4\left[1+(\frac{R_1}{r_o})^2+\frac{4}{r_0}\cos\theta\right] * \sum_{m=2}^{\infty}(-\frac{R_1}{r_o})^{m-2}(m-1)\cos m\theta$$

r_0 - final.

This solution exactly coincides with the solution of the static problem obtained by [17].

The difference between the results obtained in the present paper and the results of the ordinary wave diffraction problem justifies their consideration in many practical problems.

Table 1

R_1/r_o	3	4	5	50	60	80
Θ, Hailstones	60°	70°	90°	90°	90°	90°
$\lvert\sigma^*_{\theta\theta}\rvert$	1,541	1,536	1,525	1,414	1,416	1,416

Table 1 shows the stress concentrations as a function of R_1/r_o for different values of θ. It can be seen that the maximum stress $|\sigma^*_{\theta\theta}|$ in a cylindrical body arises when $\theta = 30^o$ ($\sigma^*_{\theta\theta} = \sigma_{\theta\theta}/ \sigma^{\overline{\kappa\kappa}}$) . At $R_1/r_o > 50$ The impact of a cylindrical source is decomposed as a plane wave, i.e. The radius of curvature of the wave can be ignored.

Conclusions

1. The problem of diffraction of harmonic waves in a cylindrical body is solved in displacement potentials. The displacement potentials are determined from the solutions of the Helmholtz equation. Arbitrary constants are determined from the boundary conditions that are placed between the bodies. As a result, the problem posed reduces to a system of inhomogeneous algebraic equations with complex coefficients that are solved by the Gauss method with the separation of the principal element.
2. Contour stresses $\sigma_{\theta\theta}$ on the free surface of cylindrical bodies reach their maximum value in

1.

$$Q=\begin{cases}\dfrac{\pi}{2}-\text{Under the action of shear waves}\\ \dfrac{\pi}{4}-\text{Under the action of longitudinal waves}\end{cases}$$

2. Contour stresses $\sigma_{\theta\theta}$ under the action of transverse harmonic waves is 15-20% greater than when exposed to longitudinal waves.
3. When the source of harmonic waves is at a distance of five radii ($\bar{V} > 5R$) from the cylindrical body, the high-frequency nature of the change in contour stresses $\sigma_{\theta\theta}$, Acting on the internal free surface, can be approximated well by the solution for a flat ($\vec{V} \to \infty$) wave. Further, all values approach the same asymptote.
4. Numerical results show that the dynamic stress concentration coefficients around cylindrical bodies depend on

A) the distance between the source and the body; B) the wave number for the sphere and the body; C) physicomechanical parameters of the sphere and body;

In the case of a cavity in an unbounded medium, the loop voltage depends on:

A) the distance between the source and the cavity; B) the wave number;

C) Poisson's ratio of the medium.

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