World Wide Journal of Multidisciplinary Research and Development

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WWJMRD 2017; 3(7): 287-294
www.wwjmrd.com
Impact Factor MJIF: 4.25
e-ISSN: 2454-6615

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Own Waves iIn an Infinite Viscoelastic Cylindrical Panel from Variable Thickness

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Abstract
The propagation of natural waves on a viscoelastic cylindrical panel with variable thickness is considered. For the derivation of the shell equations, the principle of possible displacements (the Kirchhoff-Love hypothesis) was used. Using a variational equation and a physical equation, a system consisting of eight differential equations is obtained. After some transformations, a spectral boundary value problem is constructed with respect to complex parameters ω , For a system of eight ordinary differential equations with respect to complex form functions. The dispersion relation for a cylindrical panel.

Keywords: cylindrical shell, cylindrical panel, Hooke's law, nodal points

Introduction

Basic relations for a cylindrical shell of variable thickness. Statement of the wave problem

An infinite cylindrical shell of thickness h, density ρ, with Young's modulus E, Poisson's ratio ν, and viscosity coefficient of the material is considered. In a curvilinear orthogonal coordinate system (α_1 ; α_2 ; z) with complex Lame parameters at z = 0, the shell occupies the region

$$-\infty < \alpha_1 < +\infty; \quad 0 < \alpha_2 < l; \quad -\frac{h}{2} < z < \frac{h}{2}$$

The curvatures of the middle surface z = 0 are equal to $k_1 = 0; k_2 = \frac{1}{R}$ according to coordinates α_1 and α_2 . In the framework of the Kirchhoff-Love hypotheses, the law of variation of the displacement vector components $u_1^{(z)}$, $u_2^{(z)}$, $w^{(z)}$ the shells are defined by the following relations

$u_1^{(z)} = u - \theta_1 z$; $u_2^{(z)} = v - \theta_2 z$; $u_3^{(z)} = w$, (1)

where u, v, w – components of the displacement vector of the middle surface;
θ_1 , θ_2 - angles of rotation of the normal relative to the axes α_1 and α_2
To derive the shell equations, as before, the principle of possible displacements was used
$\delta\Pi = \delta T$ (2)
where $\delta\Pi$ – variation of the potential energy of the shell;
δT – virtual work of mass inertia forces of the shell

Novozhilov V.V. / 97 /, taking relations (1) into account, deduced the following expression for δΠ, starting from the linear theory of elasticity

$$\delta\Pi = \int_F \{T_1\delta\varepsilon_1 + T_2\delta\varepsilon_2 + S\delta\varepsilon_{12} + M_1\delta x_1 + M_2\delta x_2 + 2N\delta\tau\} d\alpha_1 d\alpha_2 \quad (3)$$

where T_1 , T_2 , S , M_1 , M_2 , N – efforts and moments; ε_1, ε_2, ε_{12}, x_1, x_2, τ – Components of deformation of the median surface F.

In the expression (5.3), terms that are of order $\frac{h}{R}$.

According to (97), the components of the tangential bending deformation of the middle surface are expressed in terms of its displacement and the angles of rotation of the normal as follows

$$\varepsilon_1 = \frac{\partial u}{\partial \alpha_1}; \varepsilon_2 = \frac{\partial \vartheta}{\partial \alpha_2} + k_2 w; \varepsilon_{12} = \frac{\partial \vartheta}{\partial \alpha_1} + \frac{\partial u}{\partial \alpha_2};$$
$$x_1 = \frac{\partial \theta_1}{\partial \alpha_1}; \quad x_2 = \frac{\partial \theta_2}{\partial \alpha_2}; \quad \tau = \frac{\partial \theta_2}{\partial \alpha_1}; \quad \theta_1 = -\frac{\partial w}{\partial \alpha_1}; \quad \theta_2 = -\frac{\partial w}{\partial \alpha_2} + k_2 \vartheta \tag{4}$$

In turn, the forces and moments are related to the deformation components by the defining relations that follow from the generalized Hooke's law:

$$T_1 = c(\varepsilon_1 + \nu\varepsilon_2) = \tilde{c}\,\varepsilon_1 + \nu T_2$$
$$M_1 = D(x_1 - \nu x_2) = \tilde{D} x_1 + \nu M_2$$
$$S = A\varepsilon_{12}; N = \mathrm{B}\tau \tag{5}$$

$(1 \leftrightarrow 2)$

where

$$\tilde{c} = \frac{\tilde{\mathrm{E}}h}{1-\nu^2}; \quad \tilde{D} = \frac{\tilde{E}h^3}{12(1-\nu^2)}; \quad A = \tilde{c}\frac{1-\nu}{2};$$
$$B = \tilde{D}(1-\nu) = \frac{\tilde{E}h^3}{12(1+\nu)}.$$

If we neglect the inertia of the rotation of the normal, then the virtual work of force

The inertia of the shell can be represented in the form:

$$\delta T = -\int_F \rho h(\ddot{u}\delta u + \ddot{\vartheta}\delta\vartheta + \ddot{w}\delta w) d\alpha_1 d\alpha_2 \tag{6}$$

After substituting expressions (3) and (6) in (2) and the standard of the integration by parts procedure, taking into account relation (4.4), we obtain the equations of motion in the form:

$$\frac{\partial T_1}{\partial \alpha_1} + \frac{\partial S}{\partial \alpha_2} = -\rho h \frac{\partial^2 u}{\partial t^2}$$
$$\frac{\partial T_2}{\partial \alpha_2} + \frac{\partial S}{\partial \alpha_1} + k_2 Q_2 = -\rho h \frac{\partial^2 \vartheta}{\partial t^2} \tag{7}$$
$$\frac{\partial Q_1}{\partial \alpha_1} + \frac{\partial Q_2}{\partial \alpha_2} - k_2 T_2 = -\rho h \frac{\partial^2 w}{\partial t^2}$$

$$Q_1 = \frac{\partial M_1}{\partial \alpha_1}$$
$$Q_2 = \frac{\partial M_2}{\partial \alpha_2} + 2\frac{\partial N}{\partial \alpha_1} \tag{8}$$

Alternative boundary conditions of a free edge, or a hard seal, with

$\alpha_2 = 0, l$ have the form:

Free edge

$$M_2 = 0; S = 0; T_2 = 0; Q_2 = 0 \tag{9}$$

hard seal

$u=0, \quad \vartheta=0, \quad w=0, \quad q_2=0 \qquad (10)$

Using relations (4), (5), (7), (8), the complete system of equations of motion can be represented in the form of eight differential equations placed relative to the first derivatives with respect to α_2:

$$\frac{\partial u}{\partial \alpha_2} = \frac{S}{A} - \frac{\partial \vartheta}{\partial \alpha_1}$$
$$\frac{\partial \vartheta}{\partial \alpha_2} = \frac{T_2}{C} - \nu\frac{\partial u}{\partial \alpha_1} - k_2 w$$
$$\frac{\partial w}{\partial \alpha_2} = -\theta_2 + k_2 \vartheta$$
$$\frac{\partial \theta_2}{\partial \alpha_2} = \frac{M_2}{D} + \nu\frac{\partial^2 w}{\partial \alpha_1^2}$$
$$\frac{\partial S}{\partial \alpha_2} = \rho h \frac{\partial^2 u}{\partial t^2} - \tilde{C}\frac{\partial^2 u}{\partial \alpha_1^2} - \nu\frac{\partial T_2}{\partial \alpha_1}$$
$$\frac{\partial T_2}{\partial \alpha_2} = \rho h \frac{\partial^2 \vartheta}{\partial t^2} - \frac{\partial S}{\partial \alpha_1} - k_2 Q_2 \tag{11}$$
$$\frac{\partial Q_2}{\partial \alpha_2} = \rho h \frac{\partial^2 \omega}{\partial t^2} + \tilde{D}\frac{\partial^4 \omega}{\partial \alpha_1^4} - \nu\frac{\partial^2 M_2}{\partial \alpha_1^2} + k_2 T_2$$
$$\frac{\partial M_2}{\partial \alpha_2} = Q_2 - 2B\frac{\partial^2 \theta_2}{\partial \alpha_1^2}$$

In the case of running along α_1 harmonic waves of the solution of the boundary value problem for system (11) with boundary conditions of type (9), (10) admit a separation of variables

$$u = z_1 \sin(k\alpha_1 - \omega t)$$
$$v = z_2 \cos(k\alpha_1 - \omega t)$$
$$w = z_3 \cos(k\alpha_1 - \omega t)$$
$$\theta_2 = z_4 \cos(k\alpha_1 - \omega t)$$
$$S = z_5 \sin(k\alpha_1 - \omega t) \tag{12}$$
$$T_2 = z_6 \cos(k\alpha_1 - \omega t)$$
$$\theta_2 = z_7 \cos(k\alpha_1 - \omega t)$$
$$M_2 = z_8 \cos(k\alpha_1 - \omega t)$$

where ω - Frequency of oscillations; K is the wave number $z_1(\alpha_2)(i = 1.8)$- function of the form of oscillations. It is further assumed that both edges of the shell $\alpha = 0$ and $\alpha_1 = l$ - Are free. After substituting relations (12) into equations (11) and boundary conditions (9), there is a spectral boundary-value problem with respect to the parameter ω for the system of eight ordinary differential equations with respect to the complex functions of the form:

$$z_1' = z_5/A + kz_2$$
$$z_2' = z_6/C + \upsilon kz_1 - k_2 z_3$$
$$z_3' = -z_4 + k_2 z_2$$
$$z_4' = z_8/D + \nu k^2 z_3 \qquad (13)$$
$$z_5' = h\left(EK^2 - \rho\omega^2\right)z_1 + \nu h^2 z_6$$
$$z_6' = -h\rho\omega^2 z_2 - kz_5 - k_2 z_7$$
$$z_7' = -h\rho\omega^2 z_3 + \overline{E}/12h^3k^4 z_3 + \nu\, k^2 z_8 + k_2 z_6$$
$$z_8' = z_7 + G/3h^3k^2 z_4\,;\; z_5 = z_6 = z_7 = z_8 = 0;$$
$$\alpha_2 = 0, l$$

When analyzing the dispersion of harmonic waves, the parameter ***k*** is assumed to be given.

Numerical analysis of the dispersion of normal waves in cylindrical panels

The relations obtained in the previous section describe the propagation of harmonic waves in infinite cylindrical panels. On the basis of the solution of the boundary-value problem (13), the Godunov orthogonal sweep method was used to perform a numerical analysis of the dispersion of these waves.

Figure 1-5 shows the real parts of the complex phase velocities of the first two modes versus the wave number for different waveguides. In all variants of calculation, the following dimensionless shell parameters

$$E = 1, \quad \rho = 1, \quad \nu = 0{,}25, \quad G = 1, \ \eta = 0{,}1.$$

The thickness h varies linearly

$$h(\alpha_2) = h_1 + (h_2 - h_1)\alpha_2 \qquad (14)$$

The solid lines in the figures correspond to the options of a panel of constant thickness ($h_1 = h_2 = 0.1$), the dashed lines characterize the panel with a wedge-shaped section. In the latter case $h_2=0.1$ and the thickness h_1 ranged from 0.001 to 0.0001. In this case, in the considered range of wave numbers from 0 to 40 with decreasing h_1 numerical convergence to the limiting solution was observed. These solutions are shown in graphs. The curvature parameter k2 is constant and takes five values from 0 to 2π . The dash-dotted lines in Figs. 1 and 2 correspond to the case of the Kirchhoff-Love plates considered earlier $\boldsymbol{\kappa}_2=0$.

In Fig. Figures 6-15 show the corresponding functions of the waveform of the deflection w for different values of the wave number.

In the long-wave range, the first mode describes the flexural oscillations of the beam type, the second mode corresponds to torsional oscillations. At small k, the velocity of the first mode tends to zero, the velocity of the second mode is always finite.

In the case of a panel of constant thickness, it is interesting to trace the influence of the transverse curvature of the panel on the velocity (C_R) wave propagation. With increasing parameter $\boldsymbol{\kappa}_2$ the tendency of increase in speed (C_R) bending mode and reducing the speed of the torsion mode. Rate of attenuation of flexural mode reduced and increased completion rates

An analogous dependence on the curvature was noted in [100], a twisting mode depending on $\boldsymbol{\kappa}_2$. It is observed in the analysis of the natural frequencies of flexural and torsional oscillations of a plane curvilinear elastic rod. In this variant of calculation this effect leads to the fact that starting from a certain value $\boldsymbol{\kappa}_2$ the dispersion curves of the first two modes, the real and imaginary parts, respectively, twice intersect each other. With further increase $\boldsymbol{\kappa}_2$ the first point of intersection is shifted to the region of small wavenumbers, and the second to the short-wave region, and then to the mid-frequency region. With increasing $\boldsymbol{\kappa}_2$ the number of nodal points of the form ***w*** also increases. The first mode has two, the second - three. In comparison with curvilinear rods, this fact also agrees with the results of [100].

Fig.1-5 (dashed lines), 11-15 - are devoted to cylindrical panels of wedge-shaped section.

Figure 1, 2 shows the qualitative difference in the behavior of the dispersion curves of the first mode corresponding to the shell and plate. If in the second case the phase velocity curve is monotonic, then in the first case a characteristic maximum in the medium-wave range is observed, which is explained by the increased flexural brutality of the shell in comparison with the plate. The velocity of the second mode, in contrast to the case of a panel of constant thickness in general, also increases with increasing curvature. Therefore, the intersections of the modes in the considered range of variation $\boldsymbol{\kappa}_2$ not visible.

With respect to all modes as well as the plate, according to Figs. 11 ± 15, the localization of the wave motion near the sharp edge is observed with increasing wave number. Simultaneously, the tangential displacements ***u*** and ***v*** tend to zero in comparison with the normal displacements ***w***. Because of this, in the short-wave range, the phase velocity of the harmonic waves in the shell also has a limiting value, which coincides with the corresponding value of the limiting phase velocity for the plate. This fact is physically obvious, since localization with increasing wave number decreases the characteristic size of the wave motion zone, with respect to which the curvature of the shell tends to zero.

At the same time, as one would expect, the greater the curvature $\boldsymbol{\kappa}_2$ The slower the transition to a site without dispersion motion $(c = const)$ with increasing wave number.

As for the localization itself, it increases with increasing curvature (for sufficiently large k, for example, for k = 10 in Fig. 11-15). Moreover, such an "increased" localization in a cylindrical panel is characteristic for both modes (real frequent complex velocities).

A feature of the oscillation modes in the wedge-shaped panel is also the dependence of the number of nodal points on the wave number. For example, as follows from Fig. 7, for the second mode with k = 1 there is one node point, for k = 3 - none, k = 5 or more - two nodal points. A detailed effect was not observed in plates of variable thickness and shells of constant thickness.

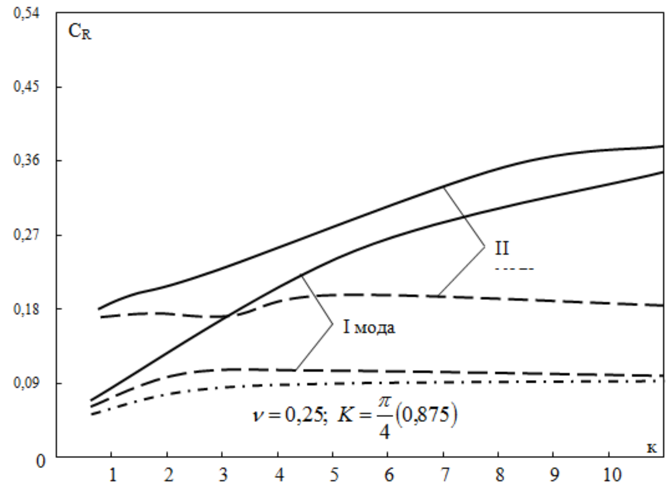

Fig.1 Change of phase I and II modes of speed by wave number.

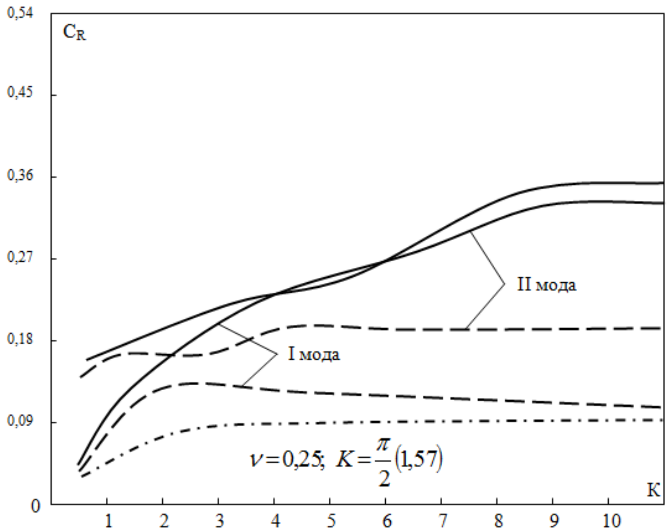

Fig.2 Change the phase I and II modes of velocity by wave number.

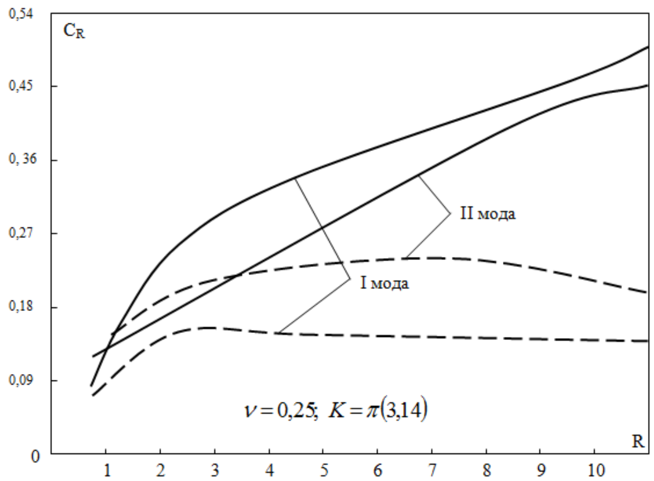

Fig.3 Change the phase I and II modes of the velocity by the wave number.

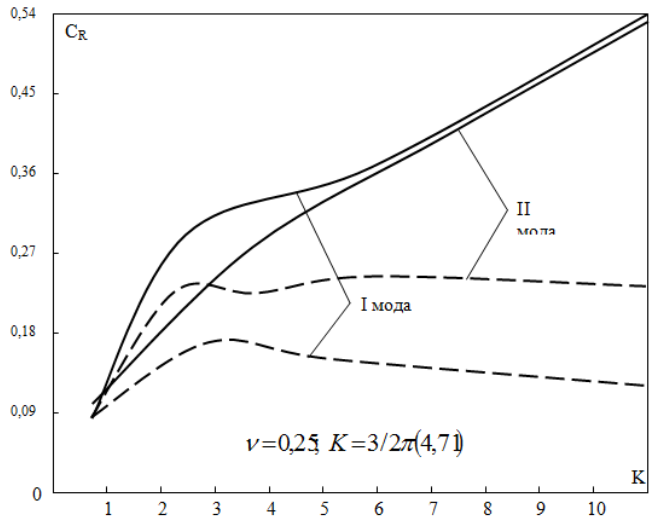

Fig.4 Change the phase I and II mode of the velocity by the wave number.

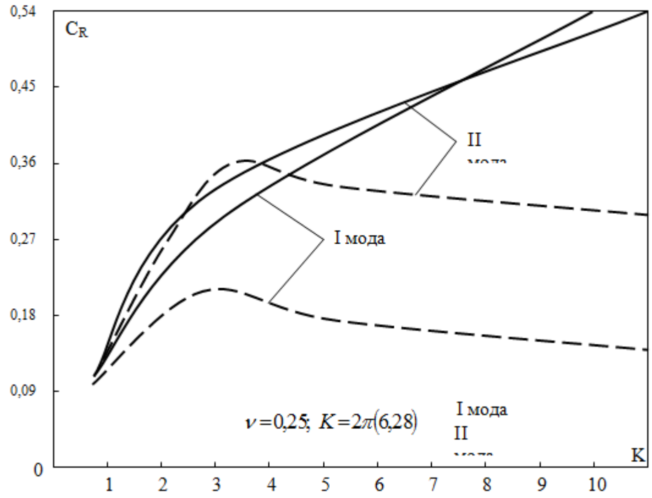

Fig.5 Change the phase I and II modes of speed by wave number.

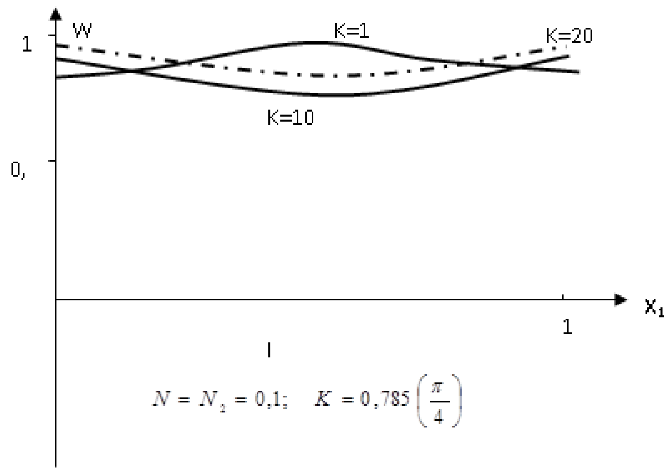

Fig.6.a. Changing the shape of the oscillations depending on the coordinate X_2.

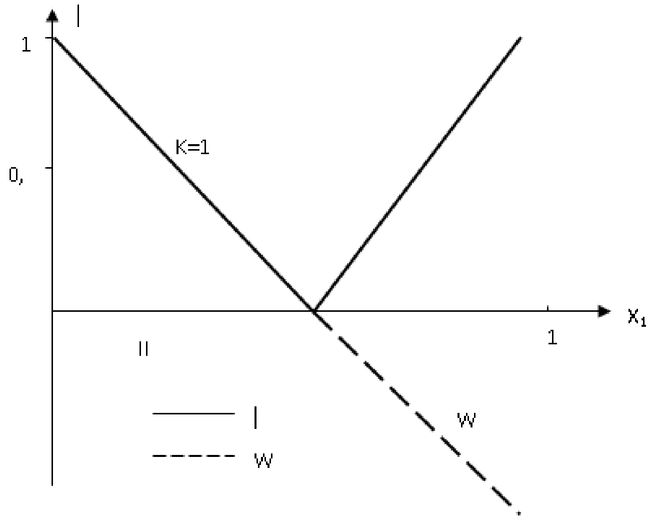

Fig.6b. Change in the shape of the oscillations depending on the coordinate X_2.

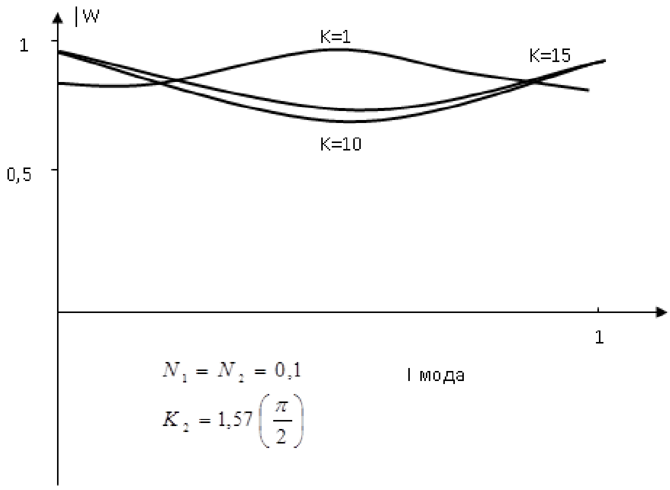

Fig.7.a. Change in the shape of the oscillations depending on the coordinate X_2.

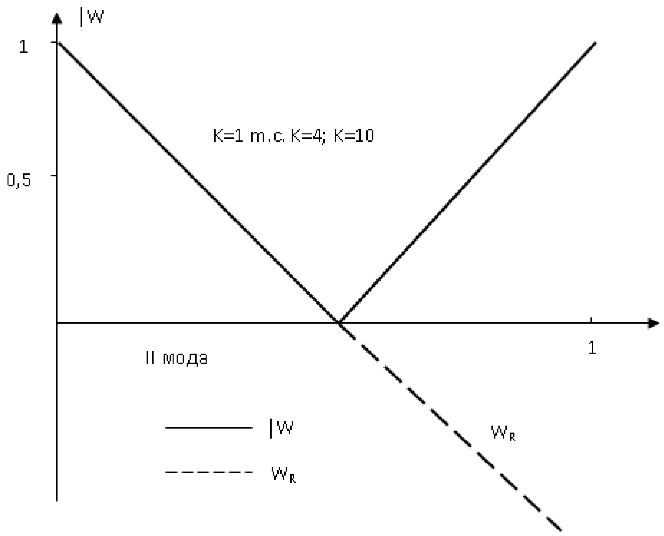

Fig.7b. Change in the shape of the oscillations depending on the coordinate X_2

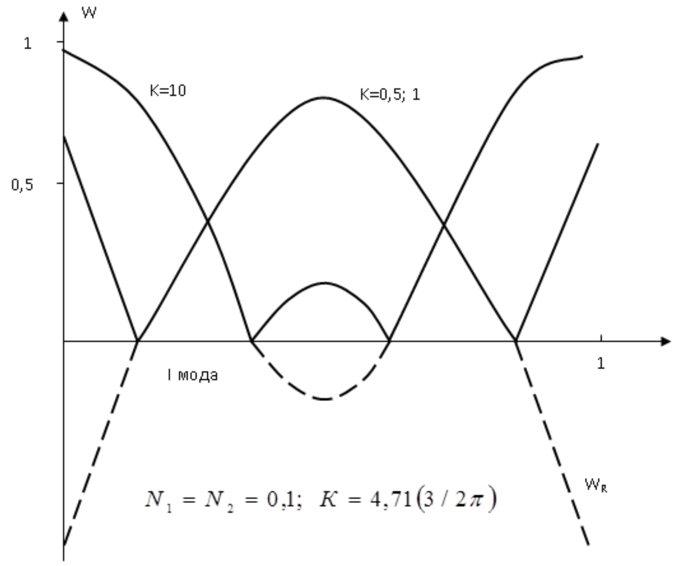

Fig.8a. Change in the shape of the oscillations depending on the coordinate X_2.

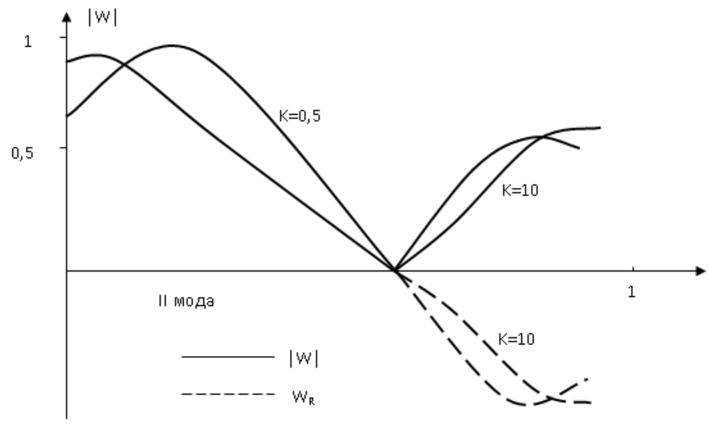

Fig.8b. Change in the shape of the oscillations depending on the coordinate X_2.

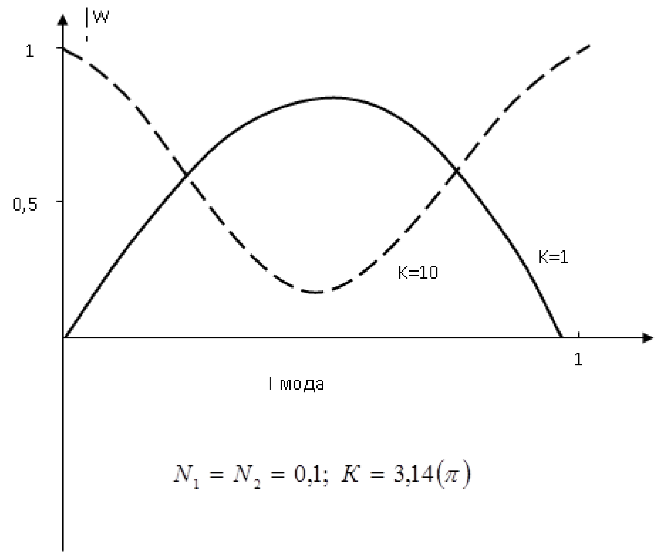

Fig. 9a. Change in the shape of the oscillations depending on the coordinate X_2.

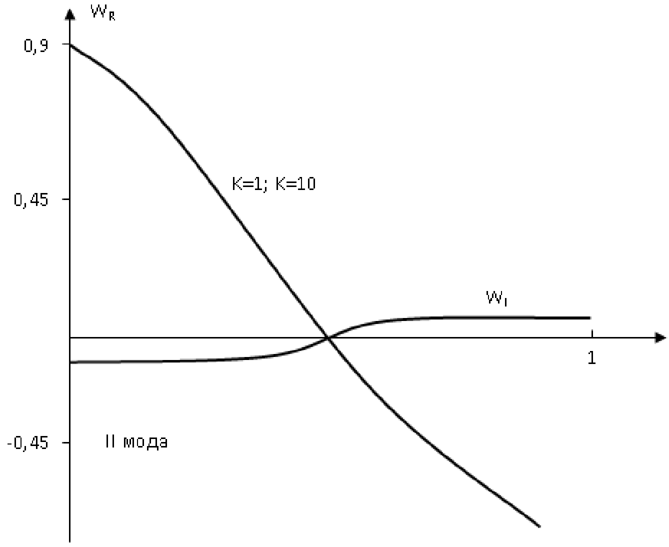

Fig.9b. Change in the shape of the oscillations depending on the coordinate X_2

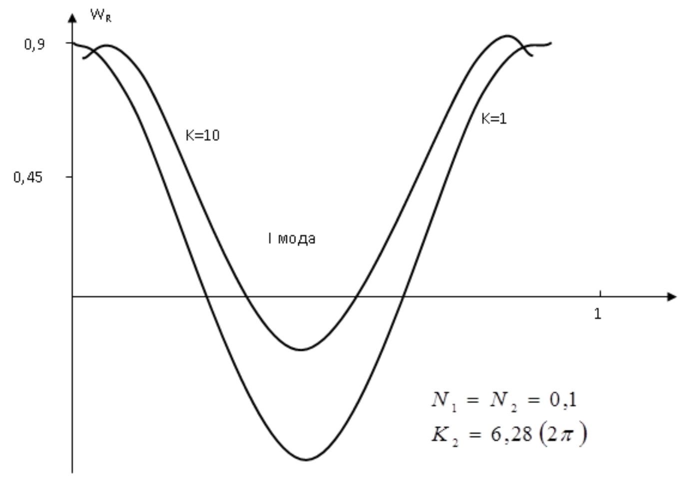

Fig.10a. Changing the shape of the oscillations depending on the coordinate X_2

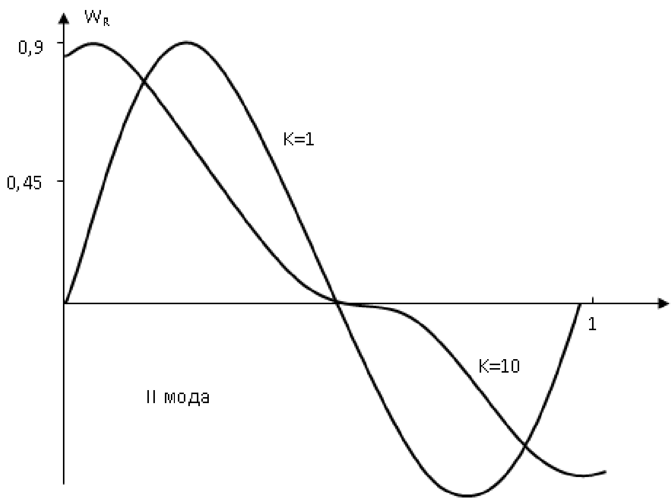

Fig.10b. Changing the shape of the oscillations depending on the coordinate $X_{2.}$

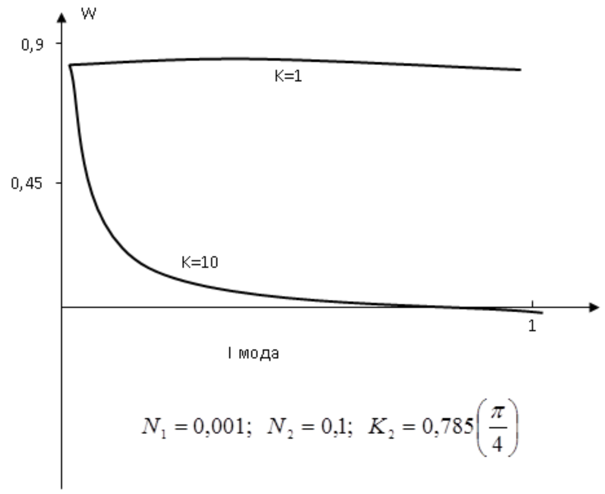

Fig.11a. Changing the shape of the oscillations depending on the coordinate $X_{2.}$

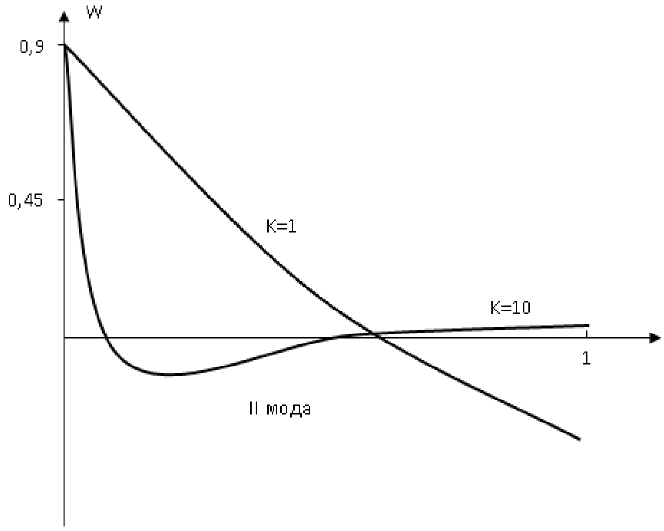

Fig.11b. Change in the shape of the oscillations depending on the coordinate $X_{2.}$

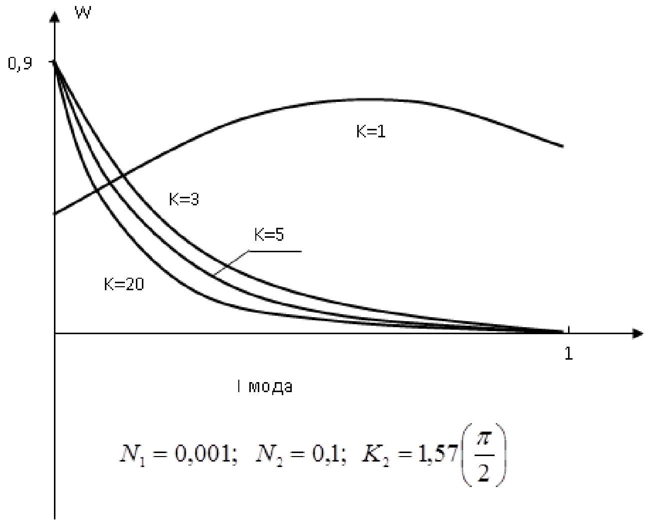

Fig. 12a. Change in the shape of the oscillations depending on the coordinate $X_{2.}$

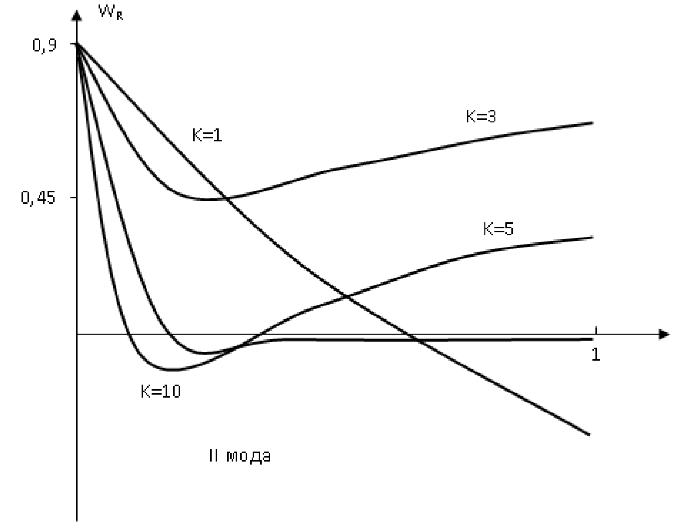

Fig.12b. Changing the shape of the oscillations depending on the coordinate X_2.

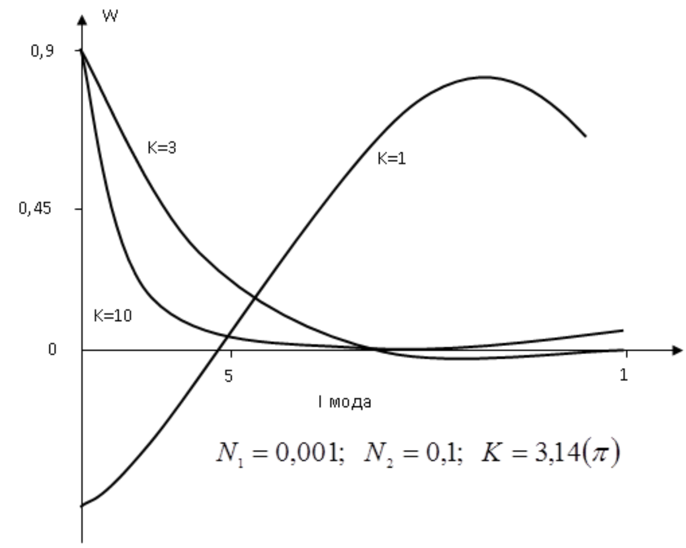

Fig. 13a. Change in the shape of the oscillations depending on the coordinate X_2.

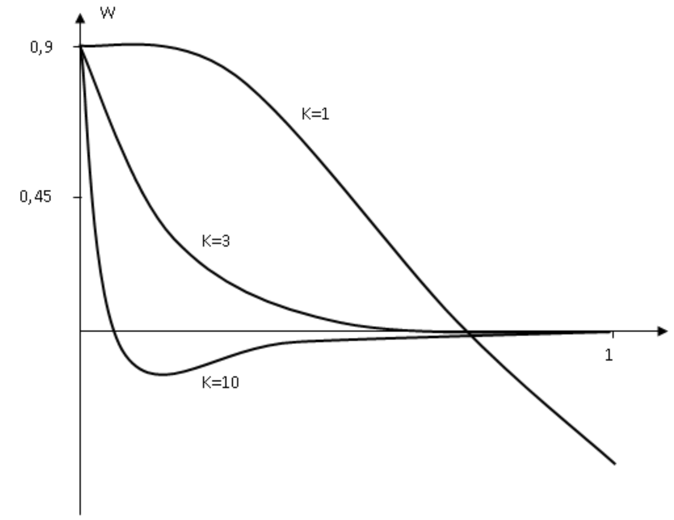

Fig.13b. Changing the shape of the oscillations depending on the coordinate X_2.

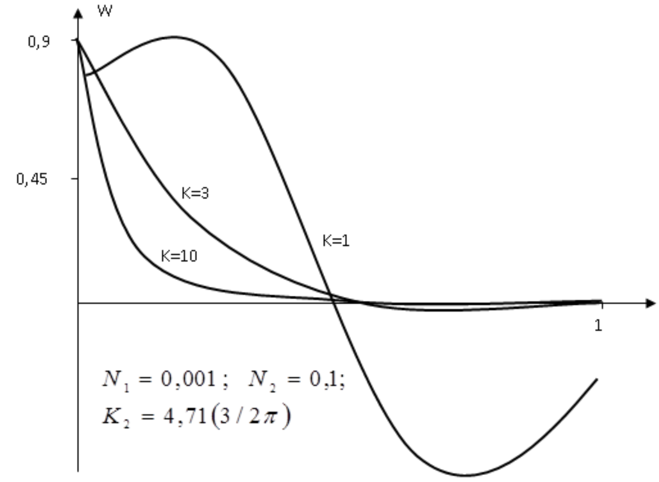

Fig.14. Change in the shape of the oscillations depending on the coordinate X_2.

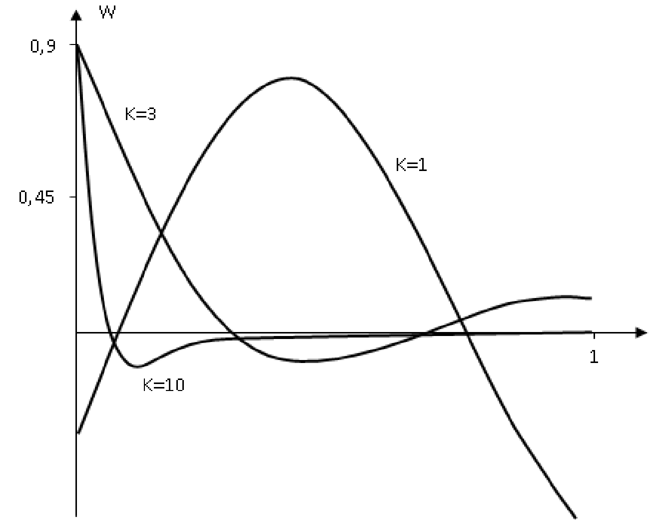

Fig.15. Change in the shape of the oscillations depending on the coordinate X_2.

Conclusions

1. With increasing curvature of a cylindrical panel of constant thickness, the propagation velocity of the first bending mode increases and the propagation velocity of the second torsional mode decreases so that, starting from a certain value of the curvature parameter, the modes intersect twice. With increasing curvature, the number of nodal points of the shape of the deflection oscillations also increases.
2. In the case of a wedge-shaped cylindrical panel, for each mode there exist limiting propagation velocities with an increase in the wave number, which coincide in magnitude with the corresponding velocities of normal waves in a wedge plate of zero curvature. In the shortwave range, the localization of motion exists and increases with increasing curvature of the panel. The number of nodal points of the shape of the oscillations of the deflection depends not only on the curvature, but also on the wave

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