World Wide Journal of Multidisciplinary Research and Development

WWJMRD 2017; 3(7): 295-309
www.wwjmrd.com
Impact Factor MJIF: 4.25
e-ISSN: 2454-6615

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About the Natural Oscillations Viscoelastic Toroidal Shell with the Flowing Fluid

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Abstract
It is given to the study of natural oscillations of a viscoelastic toroidal shell with a flowing liquid, based on the theory of shells. On the basis of a geometrically nonlinear version of the semimuscular theory of shells, the equations of motion of the toroidal shell are obtained with all inertial forces components included, including tangential forces, and also taking into account the working internal hydrostatic and dynamic pressures. Based on the developed methods, the intrinsic oscillations of the toroidal shell with a flowing liquid.

Keywords: toroidal shell, pressure, fluid oscillations, viscoelasticity.

Introduction
Studies of natural oscillations of curvilinear sections of pipelines with a constant flow of liquid within the framework of the core theory began to develop in the second half of the last century. One of the first works in this area is the article by V.S. Ushakov [1], where the equation of motion of the circular section of the pipeline was obtained and its natural oscillations were studied at a constant flow velocity and internal pressure. The flow velocity was considered to be small, which made it possible to discard some small terms of the equation and reduce the solution to the investigation of the oscillations of the circular rod. Further studies in this area began to develop quite intensively in the works of T. Anni [2], I. Hill and S. Davis [3], S.S. Zhenya [4] and M.P. Paidussis [5]. The same problem within the framework of the core theory was solved in the works: P.D. Dotsenko [6-8], VA Svetlitsky [9,10], V.F. Ovchinnikov [11], V.A. Svetlitsky [12], etc., which gives the equations of motion of a curvilinear planar or spatial pipeline, the solutions of which and their analysis are presented in the form of graphs of the dependence of the frequencies of natural oscillations on various factors (pipeline curvature, fluid flow rate, pressure, etc.). Acceptable for practical calculations. Experimental studies of curved sections of pipelines with a liquid flow are described in detail in [13]. The results of investigations of this problem within the framework of the core theory can be briefly formulated as follows:
The eigenfrequencies of the curved sections of the pipeline decrease with increasing flow velocity and increase with increasing curvature of the longitudinal axis of the pipeline. In this paper, the curvilinear pipeline with a flowing liquid is considered as a toroidal shell. The equations of motion of flexural oscillations of the toroidal shell are derived on the basis of general relations of the geometrically nonlinear theory of shells. Mushtari and K.Z. Galimov [14]. This theory considers such a bending of shells, in which the maximum deflection (in this case, the radial displacement of the points of the middle surface w) Is of the same order of magnitude as the wall thickness, or even exceeds it, but small in comparison with other linear dimensions of the shell.

Statement of the problem
We consider a curvilinear pipeline section in the form of a thin-walled large-diameter pipe, through which an ideal incompressible fluid flows at a constant speed $U = const$ and constant hydrostatic pressure $p_0 = conts$. In addition to this pressure, the hydrodynamic pressure arising from the fluid motion acts on the walls of the tube. The problem is to

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investigate the frequencies and forms of intrinsic bending vibrations in the plane of curvature of a given section of the pipeline as a thin toroidal shell, taking into account the dynamic influence of the flowing fluid, internal pressure, and deformation of the middle surface of the shell with considerable displacements.

The pipeline section in question is represented as a section of a toroidal shell with a radius R the longitudinal axis passing through the centers of gravity of its cross-sections. Cross sections are circular with a radius of the mean line of the section r, shell thickness - h. The value of the ratio $\frac{h}{r}$ considered to be small, so you can use the ratio of the theory of shells based on Kirchhoff-Love.

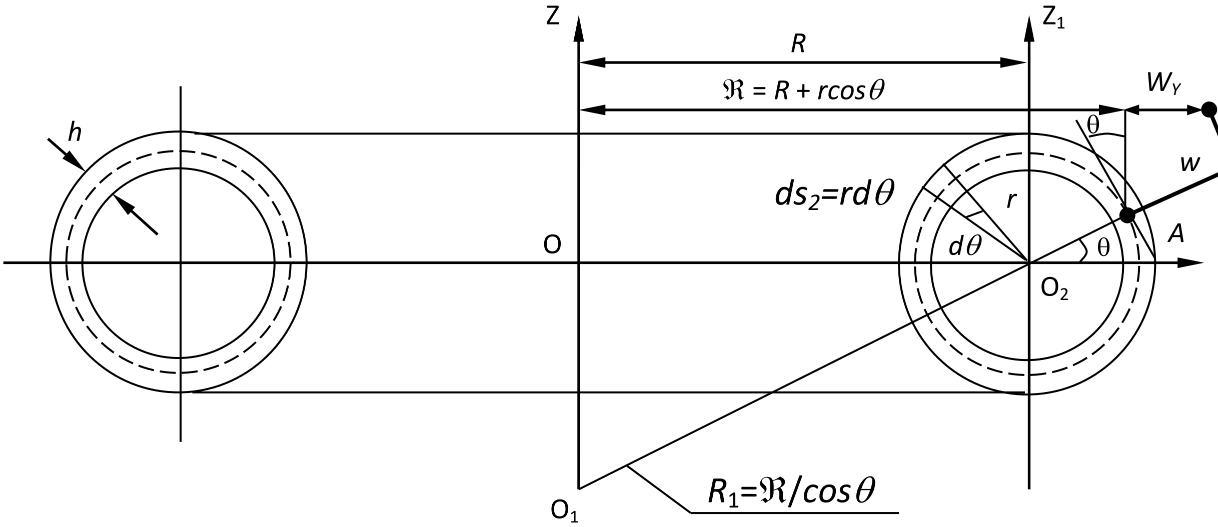

Fig.1: Curvilinear pipeline section in toroidal coordinates.

The end sections of the shell are assumed to be hinged (or rigid). Inside the shell at a speed of $U = const$ proceeds ideal incompressible fluid with a density $\rho_0 = conts$. The geometry of the curved section of the pipeline is shown in Fig. 1 as a toroidal shell with a median surface in toroidal curvilinear coordinates β, θ, where β means the central angle of the torus, and θ - angle in the cross-section of the shell $(0 \le \theta \le 2\pi)$. If the longitudinal axis of the shell is half the circumference of the radius R, as shown in Fig. 1, the angle β varies within $0 \le \beta \le \pi$. When considering the middle surface of a shell in curvilinear coordinates β, θ differentials of segments of arcs of coordinate lines ds_1 and ds_2 are connected with the differentials of the coordinates themselves through the parameters of Lame A_1 and A_2:

$$ds_1 = (R + r\cos\theta)d\beta, \qquad ds_2 = rd\theta,$$

therefore

$$A_1 = R + r\cos\theta, \qquad A_2 = r$$

Curvatures of normal sections of the middle surface of the shell in the undeformed state with $R_1 = \frac{\Re}{\cos\theta}$ and $R_2 = r$ according to Fig. 1 are defined by expressions:

$$\frac{1}{R_1} = \frac{\cos\theta}{R + r\cos\theta}, \qquad \frac{1}{R_2} = \frac{1}{r}.$$

Components of point displacement A the middle surface into position A^*, referred to the radius r (I.e. Dimensionless) and directed along the coordinates β, θ, y and along the outer normal to the middle surface, are denoted respectively u, v, W_y, w. The angle of rotation of the tangent to the midline of the contour of the cross-section is denoted by ϑ.

Differential equations and basic relations

In accordance with this theory, the equations of equilibrium of the moment forces for the toroidal shell element in the deformed state have the form (indices 1 and 2 refer to toroidal coordinates β and θ respectively) [20] :

$$\frac{\partial}{\partial\beta}(A_2T_1)+\frac{\partial}{\partial\theta}(A_1S_2)+S_1\frac{\partial A_1}{\partial\theta}-T_2\frac{\partial A_2}{\partial\beta}+A_1A_2\left(\frac{Q_1}{R_1^*}+\tau Q_2+X_1\right)=0,$$
$$\frac{\partial}{\partial\theta}(A_1T_2)+\frac{\partial}{\partial\beta}(A_2S_1)+S_2\frac{\partial A_2}{\partial\beta}-T_1\frac{\partial A_1}{\partial\theta}+A_1A_2\left(\frac{Q_2}{R_2^*}+\tau Q_1+X_2\right)=0$$
$$\frac{\partial}{\partial\beta}(A_2Q_1)+\frac{\partial}{\partial\theta}(A_1Q_2)-A_1A_2\left(\frac{T_1}{R_1^*}+\frac{T_2}{R_2^*}+S_1\tau+S_2\tau-X_3\right)=0, \quad (1)$$
$$\frac{\partial}{\partial\beta}(A_2M_1)+\frac{\partial}{\partial\theta}(A_1H_2)-H_1\frac{\partial A_1}{\partial\theta}-M_2\frac{\partial A_2}{\partial\beta}-A_1A_2Q_1=0,$$
$$\frac{\partial}{\partial\theta}(A_1M_2)+\frac{\partial}{\partial\beta}(A_2H_1)+H_2\frac{\partial A_2}{\partial\beta}-M_1\frac{\partial A_1}{\partial\theta}-A_1A_2Q_2=0,$$

Where X_1, X_2, X_3 - Components of external force vectors.

The first two equations (1) are the equations of equilibrium of forces, the last two are the equations of equilibrium of moments.

Differential equilibrium equations for the shell element (1) are nonlinear, since They contain works of effort and deformation. In addition, they are obtained for a shell in a deformed state. Therefore, these equations include radii of curvature R_1^* and R_2^* deformed middle surface of the shell. Their connection with the curvature of the initial state is expressed in accordance with [14] by the following relations:

$$\frac{1}{R_1^*}=\frac{1}{R}\left(\cos\theta-\frac{r}{R}\frac{\partial^2 w}{\partial\beta^2}\right),\ \frac{1}{R_2^*}=\frac{1}{r}\left(1-\frac{\partial\vartheta}{\partial\theta}\right). \quad (2)$$

Change in curvature of the midline of the cross section of the shell χ_2 and torsion τ are expressed in terms of the angle of rotation ϑ the following relations:

$$\chi_2=-\frac{1}{r}\frac{\partial\vartheta}{\partial\theta},\ \tau=-\frac{1}{R}\frac{\partial\vartheta}{\partial\beta}, \quad (3)$$

In accordance with the assumptions (2) - (3) of the half-shell theory of V.Vlasov's shells [15], in the first three equilibrium equations (1) we neglect the transverse force Q_1, and in the last two - torque H. As a result, in accordance with the d'Alembert principle, we obtain a system of equations for the motion of the shell in the effort:

$$\frac{r}{R}\frac{\partial T_1}{\partial\beta}+\frac{\partial S}{\partial\theta}+r\tau Q_2+rX_1^*=0,$$
$$\frac{r}{R}\frac{\partial S}{\partial\beta}+\frac{\partial T_2}{\partial\theta}+\frac{r}{R}T_1\sin\theta+\frac{r}{R_2^*}Q_2+rX_2^*=0,$$
$$\frac{r}{R_1^*}T_1+\frac{r}{R_2^*}T_2+2r\tau S-\frac{\partial Q_2}{\partial\theta}-rX_3^*=0, \quad (4)$$
$$\frac{r}{R}\frac{\partial M_1}{\partial\beta}+\frac{\partial H}{\partial\theta}-rQ_1=0,$$
$$\frac{\partial M_2}{\partial\theta}-rQ_2=0$$

where X_1, X_2, X_3 - components of inertia forces in coordinates β,θ and along the normal to the middle surface, respectively. Eliminating all the forces and moments from equations (4) except T_1 and M_2, we arrive at a single equation of motion in the effort:

$$\frac{r^2}{R^2}\frac{\partial^2 T_1}{\partial\beta^2}+\frac{r}{R}\frac{\partial}{\partial\beta}\left(\tau\frac{\partial M_2}{\partial\theta}\right)+\frac{\partial^2}{\partial\theta^2}\left(\frac{R_2^*}{R_1^*}T_1\right)-\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\left(R_2^*\frac{\partial^2 M_2}{\partial\theta^2}\right)-$$
$$-\frac{\partial}{\partial\theta}\left(\frac{r}{R}T_1\sin\theta\right)-\frac{\partial}{\partial\theta}\left(\frac{1}{R_2^*}\frac{\partial M_2}{\partial\theta}\right)+\frac{r^2}{R}\frac{\partial X_1^*}{\partial\beta}-r\frac{\partial X_2^*}{\partial\theta}-\frac{\partial^2}{\partial\theta^2}\left(R_2^*X_3^*\right)=0 \quad (5)$$

To solve the dynamic problems of the pipeline section in question, it is necessary to obtain the equation of motion of the toroidal shell in displacements. Therefore, we transform equation (5), expressing efforts T_1 and M_2 and deformation ε_1 and τ in displacements, using the relationships between the forces, deformations and displacements of the semimuscular theory of shells:

$$T_1=E_0h\frac{r}{R}\left[\left(\frac{\partial u}{\partial\beta}+W_\alpha\right)-\int_0^t R_0(t-\tau)\left(\frac{\partial u(\tau)}{\partial\beta}+W_\alpha(\tau)\right)\right]d\tau,$$
$$M_2=-\frac{E_0h^3}{12(1-\nu^2)r}\left[\frac{\partial v}{\partial\theta}-\int_0^t R_0(t-\tau)\frac{\partial v(\tau)}{\partial\theta}\right]d\tau,\ W_\alpha=w\cos\theta-\nu\sin\theta, \quad (6)$$

where $R_0(t-\tau)$ - Relaxation core material; E_0 - instantaneous modulus; h- shell thickness; ν_0 - Poisson's ratio; u, v, w - referred to the radius r dimensionless displacement components; W_y - projection onto the axis y the point A the middle surface of the shell in position A^* as a result of deformation of its contour, ν - the angle of rotation of the tangent to the midline of the section of the shell as a result of deformation of the cross section. We assume the integral terms in (6) to be small, then applying the freezing procedure [29], we note that the relations (6) are approximate of the form

$$T_1 = \bar{E}\, E_0 h \frac{r}{R}\left(\frac{\partial u}{\partial \beta} + W_y\right),$$

$$M_2 = -\frac{h^3 E_0}{12(1-\nu^2)r}\,\bar{E}\,\frac{\partial \nu}{\partial \theta},$$

$$\bar{E} = 1 - \Gamma^C(\omega_R) - i\Gamma^S(\omega_R),$$

where ω_R - Real constant; $\Gamma^C(\omega_R) = \int_0^\infty R(\tau)\cos\omega_R\tau\, d\tau$, $\Gamma^S(\omega_R) = \int_0^\infty R(\tau)\sin\omega_R\tau\, d\tau$, respectively, the cosine and sine Fourier images of the relaxation core of the material. As an example of a viscoelastic material, we take three parametric relaxation nuclei $R(t) = Ae^{-\beta t}/t^{1-\alpha}$ On the influence function $R(t-\tau)$ the usual requirements of integrability, continuity (except for $t=\tau$), sign-definiteness and monotony:

$$R > 0, \qquad \frac{dR(t)}{dt} \le 0, \quad 0 < \int_0^\infty R(t)dt < 1.$$

Substituting relations (6) into equation (5), neglecting here small nonlinear terms, we obtain the resolving equation of motion of the toroidal shell, expressed in displacements

$$\frac{\bar{E}r^2}{R^2}\frac{\partial^3 u}{\partial\beta\partial\theta^2}\cos\theta + \frac{\bar{E}r^3}{R^3}\frac{\partial^3 u}{\partial\beta^3} - \frac{\bar{E}r^2}{R^2}\frac{\partial}{\partial\theta}\left(\frac{\partial u}{\partial\beta}\sin\theta\right) + \frac{\bar{E}r^3}{R^3}\frac{\partial^2 W_\alpha}{\partial\beta^2} +$$
$$+\frac{\bar{E}r^2}{R^2}\frac{\partial}{\partial\theta}\left[\frac{\partial}{\partial\theta}(W_\alpha\cos\theta) - W_\alpha\sin\theta\right] + \frac{\bar{E}h^2}{12r^2(1-\nu^2)}\frac{\partial^3}{\partial\theta^3}\left(\frac{\partial^2\nu}{\partial\theta^2} + \nu\right) = \quad (7)$$
$$= -\frac{r^2}{E_0 hR}\frac{\partial X_1^*}{\partial\beta} + \frac{r}{E_0 h}\frac{\partial X_2^*}{\partial\theta} + \frac{r}{E_0 h}\frac{\partial}{\partial\theta}\left(\frac{\partial^2\vartheta}{\partial\theta^2}X_3^* + \frac{\partial X_3^*}{\partial\theta}\right),$$

Where X_i^* - components of inertia forces:

-tangential components in coordinates β and θ

$$X_1^* = -rh\rho\frac{\partial^2 u}{\partial t^2},\ X_2^* = -rh\rho\frac{\partial^2 \nu}{\partial t^2};$$

the normal component (along the normal to the middle surface of the shell) $X_3^* = -rh\rho\frac{\partial^2 w}{\partial t^2} + p$, where p - internal pressure, including hydrodynamic pressure, which occurs when the fluid moves, ρ - density of the shell material.

The equation of motion of the toroidal shell (7) is a differential inhomogeneous partial differential equation with four unknown quantities u, ν, w, ϑ. Adding to it three relations on the membrane theory of shells:

$$\frac{\partial\nu}{\partial\theta} + w = 0,\ \frac{r}{R}\frac{\partial\nu}{\partial\beta} + \frac{\partial u}{\partial\theta} = 0,\ \vartheta = \frac{\partial w}{\partial\theta} - \nu \qquad (8)$$

we obtain a complete system of equations with four unknowns. For a stationary fluid flow, the solution of Eqs. (7), (8) allows us to determine the frequencies and shapes of the eigenoscillations of the curved pipeline section.

Determination of hydrodynamic pressure caused by a fluid flow.

The region bounded by a toroidal cavity filled with a liquid is considered in toroidal coordinates α, β, θ, where $0 \le \alpha \le r$ - radial coordinate in the plane of the cross section of the torus (see Fig. 1), $0 \le \beta \le \beta_0$ and $-\pi \le \theta \le \pi$. Lame coefficients of the coordinate surface at $\alpha = const$ have the form [16]:

$$H_\alpha = H_\beta = \frac{c}{ch\alpha - \cos\beta},\ H_\theta = \frac{csh\alpha}{ch\alpha - \cos\beta}, \quad (9)$$

where c - scale factor.

The velocity field of an ideal incompressible fluid in the process of shell oscillation is an irrotational potential field with a potential $\varphi = \varphi(\alpha, \beta, \theta, t)$. The system of basic equations of the potential flow of an ideal incompressible fluid includes [17]:

equality of continuity (Laplace) $\nabla^2\varphi = 0$, (10)

equation of motion (Euler) $\frac{\partial\varphi}{\partial t} + Q(p) = 0,$ (11)

equation of state $p_0 = const,$ (12)

where $Q(p)$ - uniform pressure function in the fluid flow.

$$Q(p) = \frac{1}{p_0}(p - p_0), \qquad (13)$$

where p and p_0 - full and hydrostatic pressure. Here, in the expression for the normal component of inertial forces

X_3^* according to (7), the internal pressure on the pipe wall is represented as the sum

$$p = p_0 + p_{ж} = p_0 - \rho_0\, r^2\, \varphi_n \left[\frac{\partial^2 w}{\partial t^2} + \frac{U^2}{R\ r}\frac{\partial^2 w}{\partial \beta^2}\right], \quad (14)$$

where ρ_0 - fluid density, $p_0 = const$ - Constant hydrostatic pressure; $p_{ж}$ - the hydrodynamic pressure of the fluid flow in the curvilinear pipeline section, determined through Legendre functions

$$\varphi_n = -\left(\frac{1}{2} + \frac{P'_{n-\frac{1}{2}}(ch\,r)}{P_{n-\frac{1}{2}}(ch\,r)}\right)^{-1} \quad (15)$$

where $P_{n-\frac{1}{2}}(ch\,r)$ and $P'_{n-\frac{1}{2}}(ch\,r)$ - Legendre function of the first kind and its first derivative.

Considering the velocity vector of the fluid flow $\overline{U}$ in toroidal coordinates, we write down the expressions for its components by α, β, θ:

$$U_\alpha = \frac{1}{H_\alpha}\frac{\partial \varphi}{\partial \alpha}, U_\beta = \frac{1}{H_\beta}\frac{\partial \varphi}{\partial \beta}, U_\theta = \frac{1}{H_\theta}\frac{\partial \varphi}{\partial \theta}. \quad (16)$$

For the component of the velocity vector U_α, directed along the normal to the deformed surface of the shell, the smooth flow around this surface by the liquid flow must be satisfied [18]:

$$U_\alpha\Big|_{\alpha=R} = \frac{1}{H_a}\frac{\partial \varphi}{\partial \alpha}\Big|_{\alpha=R} = -r\left(\frac{\partial^2 w}{\partial t^2} + \frac{U}{H_\beta}\frac{\partial^2 w}{\partial \beta^2}\right)\Big\|_{\alpha=R}, \quad (17)$$

where w - The dimensionless component of displacement of the points of the middle surface of the shell, referred to the radius R.

Thus, the problem of determining the hydrodynamic pressure of a liquid on the pipe wall reduces to finding the potential φ, satisfying the Laplace equation (10) and conditions (15), (16) for $\alpha = R$.

Solution of the system of differential equations of motion of a toroidal shell. The differential equation of motion (7) of a curved section of a pipeline with a stationary flow of liquid, recorded in displacements u, v, w, W_y, ϑ in toroidal coordinates β, θ after the substitution of the values by the component of inertia forces X_i^* it takes the form:

$$\begin{aligned}
&\frac{r^3}{R^3}\overline{E}\frac{\partial^2 W_\alpha}{\partial \beta^2} + \frac{r^2}{R^2}\overline{E}\frac{\partial^3 u}{\partial \beta \partial \theta^2}\cos\theta + \frac{r^3}{R^3}\overline{E}\frac{\partial^3 u}{\partial \beta^3} - \overline{E}\frac{r^2}{R^2}\cos\theta\frac{\partial u}{\partial \beta} - \overline{E}\frac{r^2}{R^2}\sin\theta\frac{\partial^3 u}{\partial \theta \partial \beta} + \\
&+ \frac{r^2}{R^2}\overline{E}\frac{\partial}{\partial \theta^2}(W_\alpha \cos\theta) - \overline{E}\frac{\partial}{\partial \theta}(W_\alpha \sin\theta) + \overline{E}h_\rho^e\frac{\partial^3}{\partial \theta^2}\left(\frac{\partial^2 \vartheta}{\partial \theta^2} + \vartheta\right) = \frac{r^3}{E_0 R}\rho\frac{\partial^3 u}{\partial t^2 \partial \beta} - \\
&- r_p\frac{\partial^3 v}{\partial \theta \partial t^2} + r_h\frac{\partial^3 \vartheta}{\partial \theta^3} - r_\rho\frac{\partial^u w}{\partial \theta\, \partial t^2} + r_E\frac{\partial^2}{\partial \theta^2}(\rho_{ж})
\end{aligned} \quad (18)$$

where $u, V, w, W_\alpha, \vartheta$ - moving the shell in a toroidal coordinate;

$$h_p^2 = \frac{h^2}{12r^2(1-\nu^2)}; r_p = \frac{r^2}{E_0}\rho; r_h = \frac{r}{E_0 h}p_0; r_\rho = \frac{r^2}{E_0}\rho;\ r_E = \frac{r}{E_0 h}.$$

The last term on the right-hand side of equation (18) contains the derivative of θ of a work $\frac{\partial^2 \vartheta}{\partial \theta^2}X_3^*$. After differentiation, taking into account (18) and discarding small nonlinear terms, the following terms remain in the equation (the last three terms on the right-hand side)

$$\frac{r}{E_0 h}p_0\frac{\partial^3 \vartheta}{\partial \theta^3} - \frac{r^2}{E_0}\rho\frac{\partial^4 w}{\partial \theta^2 \partial t^2} + \frac{r}{E_0 h}\frac{\partial^2}{\partial \theta^2}(p_{ж}) \quad (19)$$

Adding to the equation of motion (18) the relation of the semimuscular theory of shells and using the formula for the hydrodynamic pressure $p_{ж}$, we obtain a complete system of equations for the problem in the displacements

$$\begin{aligned}
&\frac{r^2}{R^2}\overline{E}\frac{\partial^3 u}{\partial \beta \partial \theta^2}\cos\theta + \frac{r^3}{R^3}\overline{E}\frac{\partial^3 u}{\partial \beta^3} - \frac{r^2}{R^2}\overline{E}\frac{\partial}{\partial \theta}\left(\frac{\partial u}{\partial \beta}\sin\theta\right) + \frac{r^3}{R^3}\overline{E}\frac{\partial^2 W_\alpha}{\partial \beta^2} + \\
&+ \frac{r^2}{R^2}\overline{E}\frac{\partial}{\partial \theta}\left[\frac{\partial}{\partial \theta}(W_\alpha \cos\theta) - W_\alpha \sin\theta\right] + h_p^2\overline{E}\frac{\partial^3}{\partial \theta^3}\left(\frac{\partial^2 \vartheta}{\partial \theta^2} + \vartheta\right) = \\
&= \frac{r^3}{RE_0}\rho\frac{\partial}{\partial \beta}\left(\frac{\partial^2 u}{\partial t^2}\right) - \frac{r^2}{E_0}\rho\frac{\partial}{\partial \theta}\left(\frac{\partial^2 v}{\partial t^2}\right) + \frac{r}{E_0 h}p_{0n}\frac{\partial^3 \vartheta}{\partial \theta^3} - r_p\frac{\partial^4 w}{\partial \theta^2 \partial t^2} - \\
&- \frac{r^2}{E_0 h}\rho_0 \varphi_n\left(\frac{\partial^4 w}{\partial \theta^2 \partial t^2} + \frac{U^2}{Rr}\frac{\partial^4 w}{\partial \theta^2 \partial \beta^2}\right);
\end{aligned} \quad (20)$$

The relationship between displacements and deformations in the sex of membrane theory of shells

$$\frac{\partial v}{\partial\theta}+w=0,\qquad \frac{r}{R}\frac{\partial v}{\partial\theta}+\frac{\partial u}{\partial\theta}=0,$$
$$\vartheta=\frac{\partial w}{\partial\theta}-v,\qquad W_y=w\cos\theta-v\sin\theta \tag{21}$$

It should be noted that the displacement components u,v,w dimensional, therefore all terms of the system of equations (20), (21) are also dimensionless. To solve the system of equations (20), (21), we represent the normal component of the displacement arising during bending vibrations of the toroidal shell $w(\beta,\theta,t)$ in the form satisfying the boundary conditions at the edges of the shell (see Fig. 1):

$$w\Big|_{\substack{\beta=0\\ \beta=\pi}}=0,\qquad \frac{\partial^2 w}{\partial\beta^2}\Big|_{\substack{\beta=0\\ \beta=\pi}}=0 \tag{22}$$

And also $w(\beta,\theta,t)$ satisfying circularity conditions along the circumferential coordinate θ:

$$w(\beta,\theta,t)=f(t)\,a_m\cos m\theta\sin n\beta, \tag{23}$$

where $f(t)$ - time function $t,\ a_m=const,\ m,n$ - wave numbers that determine the shape of the shell oscillations in the circumferential and longitudinal directions, respectively. From the relations (21) between the components of displacement at a value w (23) we obtain expressions for the remaining components of the displacement and the angle of rotation:

$$u=-\frac{r}{R}\frac{n}{m^2}f(t)a_m\cos m\theta\cos n\beta,\ \ v=-\frac{1}{m}f(t)a_m\sin m\theta\sin n\beta,$$
$$\vartheta=-\frac{m^2-1}{m}f(t)a_m\sin m\theta\sin n\beta,\ \ W_\alpha=\frac{1}{2}\left(a_{m+1}\frac{m+2}{m+1}+a_{m-1}\frac{m-2}{m-1}\right)\cos m\theta\sin n\beta. \tag{24}$$

Substituting expressions (23), (24) for the displacement components and the rotation angle into the equation of motion of the shell (20) and calculating the partial derivatives with respect to β and θ, we obtain a resolving equation with respect to unknown amplitude values b_m, containing a function of time $f(t)$ and its second time derivative $f''(t)$, taking into account the relationship between displacements and deformations in the sex of membrane theory of shells:

$$f''(t)\left(\frac{r^4}{\bar{E}E_0R^2}pa_m\frac{n^2}{m^3}\sin m\theta+\frac{r^2}{\bar{E}E_0}\rho a_m\frac{1}{m}\sin m\theta+\frac{r^2}{\bar{E}E_0}\rho_0\varphi_n a_m\sin m\theta\right)=$$
$$=f(t)\Bigg\{-\frac{r^4}{R^4}\frac{n^4}{m^3}a_m\sin m\theta+\frac{r^3}{2R^3}\frac{n^2}{m}a_m\big(\sin(m-1)\theta+\sin(m+1)\theta\big)+$$
$$+\frac{r^3}{2R^3}\frac{n^2}{m}\left(a_{m+1}\frac{m+2}{m+1}+a_{m-1}\frac{m-2}{m-1}\right)\sin m\theta-\frac{r^2}{R^2}\left(a_{m+1}\frac{m+2}{m+1}+a_{m-1}\frac{m-2}{m-1}\right)\times \tag{25}$$
$$\times\left(\frac{m-2}{4}\sin(m-1)\theta+\frac{m+2}{4}\sin(m+1)\theta\right)-$$
$$-\left(\frac{h^2}{12r^2(1-\nu^2)}\left[m(m^2-1)\times(m^2-1+\frac{12r^3(1-\nu^2)}{\bar{E}E_0h^3}p_0)\right]\sin m\theta+\frac{r^3}{\bar{E}E_0h}\rho_0a_m\varphi_n\frac{U^2}{Rr}n^2m\right)\sin m\theta\Bigg\}$$

To simplify the form of equation (25) we introduce the dimensionless parameter of shell thickness h_ν:

$$h_\nu=\frac{h}{rc_\nu},\qquad c_\nu=\sqrt{12(1-\nu^2)}, \tag{26}$$

where ν - Poisson's ratio. We divide each term of equation (25) by h_ν^2. As a result, we get:

$$f(t)\left\{-\frac{r^4}{R^4h_\nu^2}\cdot\frac{n^4}{m^3}a_m\sin m\theta+\frac{r^3}{2R^3h_\nu^2}\cdot\frac{n^2}{m}\times\right.$$
$$\left[a_m(\sin(m-1)\theta+\sin(m+1)\theta)+\left(a_{m+1}\frac{m+2}{m+1}+a_{m-1}\frac{m-2}{m-1}\right)\sin m\theta\right]-$$
$$-\frac{r^2}{2R^2h_\nu^2}\left(a_{m+1}\frac{m+2}{m+1}+a_{m-1}\frac{m-2}{m-1}\right)\times((m-2)\sin(m-1)\theta+(m+2)\sin(m+1)\theta)-m(m^2-1)\cdot \quad (27)$$
$$\cdot\left(m^2-1+\frac{r}{\bar{E}E_0hh_\nu^2}p_0\right)\times a_m\sin m\theta+\frac{r}{R}\frac{r}{\bar{E}E_0hh_\nu^2}\rho_0\varphi_nU^2mn^2a_m\sin m\theta\Big\}-$$
$$-f''(t)\cdot\left[\frac{r}{\bar{E}E_0hh_\nu^2}\rho\left(\frac{r^2}{R^2}\frac{n^2}{m^3}+m+\frac{1}{m}\right)+r^2\frac{r}{\bar{E}E_0hh_\nu^2}\rho_0\varphi_nm\right]a_m\sin m\theta=0.$$

We shall further simplify the form of the resolving equation by introducing in (27) the following parameters of the density of the shell material p^*, fluid density p_0^* and internal hydrostatic pressure p_0^*:

$$\rho^*=\frac{r}{E_0hh_\nu^2}\rho,\quad \rho_0^*=\frac{r}{E_0hh_\nu^2}\rho_0,\quad p_0^*=\frac{r}{E_0hh_\nu^2}p_0. \quad (28)$$

In addition, we transform equation (27) using the curvature parameter of the toroidal shell μ, accepted in the theory of shells [14,15,19], which characterizes not only the geometry of the shell, but also its material, since it includes the Poisson's ratio:

$$\mu=\frac{r^2}{Rh}c_\nu,\qquad c_\nu^2=12(1-\nu^2) \qquad (29)$$

We assume that the intrinsic flexural oscillations of the toroidal shell occur according to a harmonic law with a circular frequency ω, i.e.

$$f(t)=d_m\sin\omega t,\qquad f''(t)=-\omega^2d_m\sin\omega t, \qquad (30)$$

Here it is taken into account that in the penultimate term of equation (27) the quantity $\frac{r^2}{R^2}$, small in comparison with wave number $m=1,2,3....$ and equating the factors for the same trigonometric functions $\sin\omega t$, ve finally get

$$\left\{-\frac{r^4}{R^4h_\nu^2}\cdot\frac{n^4}{m^3}a_m\sin m\theta+\frac{r^3}{2R^3h_\nu^2}\cdot\frac{n^2}{m}\times\right.$$
$$\left[a_m(\sin(m-1)\theta+\sin(m+1)\theta)+\left(a_{m+1}\frac{m+2}{m+1}+a_{m-1}\frac{m-2}{m-1}\right)\sin m\theta\right]-$$
$$-\frac{r^2}{2R^2h_\nu^2}\left(a_{m+1}\frac{m+2}{m+1}+a_{m-1}\frac{m-2}{m-1}\right)\times((m-2)\sin(m-1)\theta+(m+2)\sin(m+1)\theta)-m(m^2-1)\cdot \quad (31)$$
$$\cdot\left(m^2-1+\frac{r}{\bar{E}E_0hh_\nu^2}p_0\right)\times a_m\sin m\theta+\frac{r}{R}\frac{r}{\bar{E}E_0hh_\nu^2}\rho_0\varphi_nU^2mn^2a_m\sin m\theta\Big\}+$$
$$\omega^2\cdot\left[\frac{r}{\bar{E}E_0hh_\nu^2}\rho\left(\frac{r^2}{R^2}\frac{n^2}{m^3}+m+\frac{1}{m}\right)+r^2\frac{r}{\bar{E}E_0hh_\nu^2}\rho_0\varphi_nm\right]a_m\sin m\theta=0.$$

The equation of motion of a toroidal shell with a stationary fluid flow (31), obtained on the basis of a geometrically nonlinear version of the semimuscular theory of shells and the theory of the potential flow of an ideal incompressible fluid, is a homogeneous equation describing its deformation of the transverse cross sections of the envelope under bending vibrations. All terms of which are multipliers with trigonometric functions $\sin m\theta,\quad m=1,2,3...$. Thus, when $m=1$ the shell vibrations occur without deformation of the contour of the cross sections are displaced during the oscillation process as rigid ones. Therefore, the internal pressure does not affect the shape of the oscillations p_0, since the term of equation (31) containing the pressure vanishes when $m=1$. All other forms of vibration ($m=2,3,4...$), connected with the deformation of the contour of the cross section and the pressure, as can be seen from equation (31), has an effect. To solve the problem posed by the determined frequencies of the natural oscillations of the curvilinear pipeline section over all shell oscillations, we equate the coefficients for the same trigonometric functions $\sin m\theta$ at $m=1,2,3...$ from equation (31), after some transformations, we obtain an infinite system of homogeneous linear algebraic equations with respect to unknown amplitude values a_m radial component of displacement w. We consider a truncated system of homogeneous linear algebraic equations obtained from (31) with $m = 1,2,3...$ the truncation of an infinite system of linear algebraic equations does not significantly affect the accuracy of the solution of the problem, since this system is regular [20]. A study conducted in accordance with the procedure of [20] showed that the sum of the moduli of the coefficients of the minor terms of each row of the matrix A, divided by the coefficient modulus for the principal diagonal term, is less than unity for any parameter μ.

Investigation of the natural oscillations of toroidal shells with a fluid flow. Here, the results of determining the real parts (ω_{Rmn}) complex eigenfrequencies ($\omega_{mn} = \omega_{Rmn} + i\omega_{\mathrm{Im}\,n}$) bending vibrations of toroidal shells with a flowing liquid in three first shell modes $(m, n = 1,2,3)$. As the relaxation nucleus of a viscoelastic material, we take a three-parameter core $R(t) = \frac{Ae^{-\beta t}}{t^{1-\alpha}}$ Rizhanitena-Koltunova [30], which has a weak singularity, where A, α, β - parameters materials [30]. We take the following parameters: $A = 0{,}048;\ \beta = 0{,}05;\ \alpha = 0{,}1$. The complex roots of the frequency equation are determined by the Mueller method, at each iteration of the Muller method is applied by the Gauss method with the separation of the principal element [20]. Investigation of the frequencies of proper flexural vibrations of curvilinear sections of steel pipelines with a longitudinal axis in the form of a half circle $(\pi \geq \beta \geq 0)$ with a stationary flow of liquid (water) at values of its velocity u from 0 to $50\frac{м}{c}$. It was possible to estimate the influence of the flow velocity on the frequency of the first four modes ($m = 1,2,3,4$ at $n = 1,2,3$). Calculations were carried out for toroidal shells with relative magnitudes $\frac{h}{r} = \frac{1}{35}, \frac{1}{70}$ different curvature $\frac{r}{R} = \frac{1}{10}, \frac{1}{20}$, which corresponded to the parameters of curvature $\mu = 5{,}8$; 11,6 and 23,1.

Table 1: Real parts of the natural frequencies as a function of the velocity of the flowing liquid

$\frac{r}{R} = \frac{1}{20}, \frac{h}{r} = \frac{1}{60}$ $\mu = 23$		ω_{Rmn} (Гц) At the velocity of the flowing liquid in $\frac{м}{c}$		
Form of oscillation	Frequencies	$U = 0$	$U = 20$	$U = 40$
$m = 1$	ω_{R11}	26,46	21,01	17,25
	ω_{R12}	21,01	20,45	17,74
	ω_{R13}	22,92	22,72	20,55
$m = 2$	ω_{R21}	13,39	12,83	10,42
	ω_{R22}	16,67	15,82	12,51
	ω_{R23}	18,68	18,44	16,2
$m = 3$	ω_{R31}	13,02	12,29	9,28
	ω_{R32}	16,43	15,61	12,32
	ω_{R33}	18,34	18,17	15,83
$m = 4$	ω_{R41}	19,47	19,33	14,21
	ω_{R42}	20,12	20,06	12,97
	ω_{R43}	21,36	21,22	11,38

These parameters, in turn, corresponded to the following values of the curvature of the bends and bends of pipelines according to SNiP [21]: $\lambda = 0{,}57; 0{,}28$ and 0,14. The instant modulus of elasticity of the steel from which the pipes are made is assumed to be equal to $E_0 = 2 \cdot 10^5 МПП$, Poisson's ratio .
The results of the calculations are presented in Table. 1-6 and in the graphs of Fig. 2 - 5, which shows the change in real parts of the natural frequencies of bending vibrations ω_{Rmn} curvilinear sections of the steel pipeline as a function of the velocity of the flowing liquid for different values of the shell thickness. Figure 2a shows a monotonically increasing frequency dependence ω_{R21} by the shape of the oscillations at $m = 2$ the curvature parameter of the pipeline section μ and his $\frac{h}{r}$. And conversely, the more it affects the natural frequencies of oscillations. Less curvature of the tube and the thinner its walls, the lower its frequencies of natural oscillations ω_{Rmn} practically in all forms.

Table 2: Real parts of the natural frequencies as a function of the velocity of the flowing liquid

$\frac{r}{R}=\frac{1}{20}, \frac{h}{r}=\frac{1}{60}$ $\mu=11{,}5$		ω_{Rmn} (Hz) at the velocity of the flowing liquid in $\frac{м}{c}$		
Form of oscillation	Frequencies	$U=0$	$U=20$	$U=40$
$m=1$	ω_{R11}	55,34	53,47	51,13
	ω_{R12}	56,05	55,27	52,39
	ω_{R13}	60,56	59,99	57,18
$m=2$	ω_{R21}	36,26	34,09	28,57
	ω_{R22}	44,60	43,78	40,35
	ω_{R23}	51,67	50,52	47,06
$m=3$	ω_{R31}	35,02	33,01	26,51
	ω_{R32}	43,11	43,50	39,22
	ω_{R33}	50,03	49,63	46,58
$m=4$	ω_{R41}	53,01	50,31	47,44
	ω_{R42}	54,95	52,05	48,50
	ω_{R43}	55,82	53,92	49,99

Thus, the lowest oscillation frequencies from the investigated sections of the pipeline were found near the pipe with ratios $\frac{r}{R}=\frac{1}{20}$ and $\frac{h}{r}=\frac{1}{70}$ (Fig. 2,). Oscillation frequencies ω_{Rmn} for this pipe were more than 5 times less than the frequencies for the pipe with $\frac{r}{R}=\frac{1}{10}$ and $\frac{h}{r}=\frac{1}{35}$ (fig. 2b and fig. 3). Figures 2, d show the results of calculating the natural frequencies of the fluid flow velocity at h=0.05. Low frequencies of natural oscillations are considered to be the most dangerous in connection with the occurrence of resonance situations in the operation of pipelines and the possibility of loss of stability when one of the frequencies of bending vibrations $\omega_{Rmn}=0$. Pressure pipelines made of polyethylene pipes are now widely used in the transportation of gas, oil, oil products.

Table 3: Real parts of the natural frequencies as a function of the velocity of the flowing liquid

$\frac{r}{R}=\frac{1}{20}, \frac{h}{r}=\frac{1}{60}$ $\mu=11{,}5$		ω_{Rmn} (Hz) at the velocity of the flowing liquid in $\frac{м}{c}$		
Form of oscillation	Frequencies	$U=0$	$U=20$	$U=40$
$m=1$	ω_{R11}	8,61	8,01	7,42
	ω_{R12}	10,52	10,23	9,22
	ω_{R13}	12,25	12,03	11,93
$m=2$	ω_{R21}	8,30	7,83	7,12
	ω_{R22}	10,23	9,84	9,01
	ω_{R23}	11,84	11,75	11,52
$m=3$	ω_{R31}	7,13	6,70	6,32
	ω_{R32}	8,82	8,42	7,82
	ω_{R33}	10,12	9,91	9,72
$m=4$	ω_{R41}	11,13	11,05	10,11
	ω_{R42}	12,42	12,72	12,92
	ω_{R43}	14,03	13,79	13,25

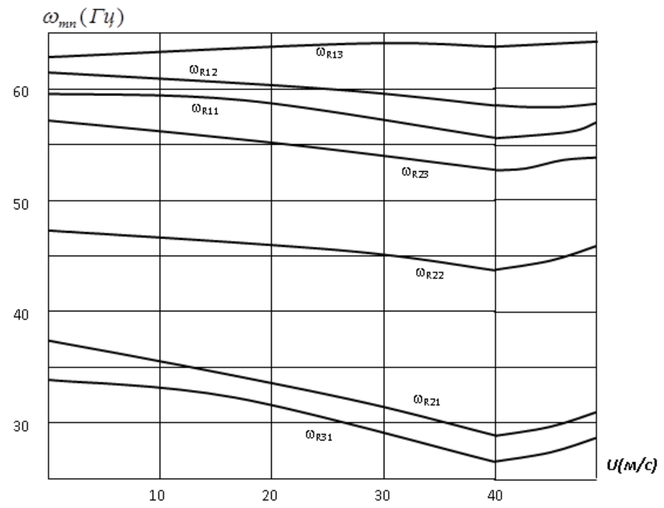

Fig.2, a. The change in real parts of the natural frequencies of bending vibrations at different velocities of a flowing liquid h=0,001.

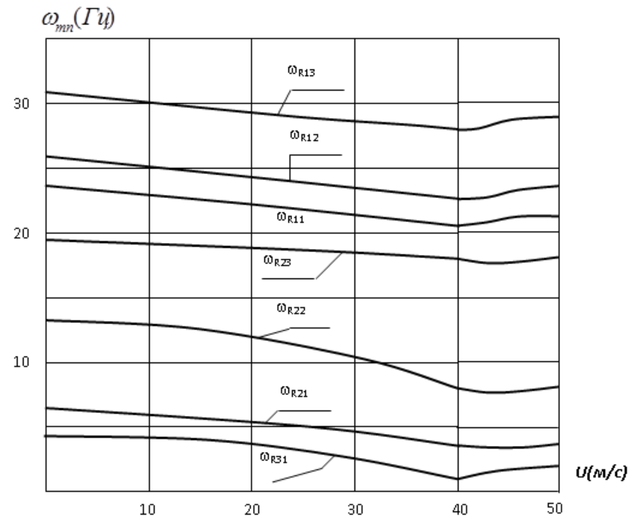

Fig.2, b. The change in real parts of the natural frequencies of bending vibrations at different velocities of a flowing liquid h=0,005.

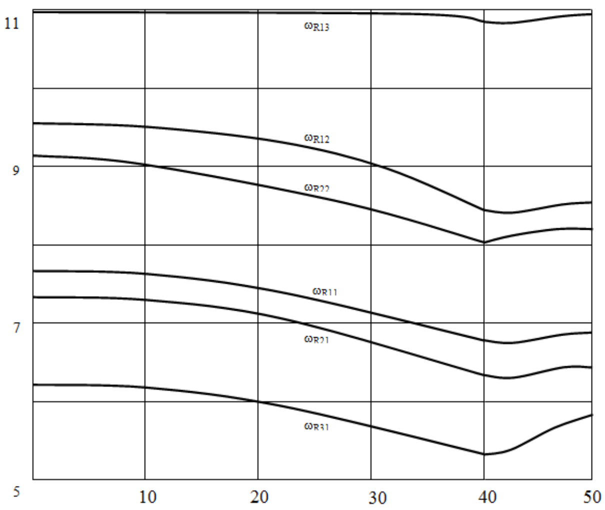

Fig. 2, c. The change in real parts of the natural frequencies of bending vibrations at different velocities of a flowing liquid h=0.001.

Literature sources contain data on the operation of such pipelines with information from (GOST 18599) "Pressure pipes from polyethylene. Technical conditions »with a range of pipes manufactured by the industry. Pipes made of polyethylene with a module of elasticity of the material $E = 500 МПа$ and Poisson's ratio $\nu = 0{,}3$, have outer diameters up to 1200 mm with a ratio of the wall thickness of the pipe to the radius of the middle surface $\frac{h}{r} = \frac{1}{8} \div \frac{1}{12}$ and are designed for internal hydrostatic pressure up to 0.8 MPa (Fig. 4). Curved sections of pipelines made of polyethylene pipes are made of sections with an external diameter of up to 630 mm and are thin-walled toroidal shells. Dynamic calculation of such areas should be carried out on the basis of shell theory. Therefore, the determination of the frequencies of natural oscillations of curvilinear pipeline sections from polyethylene pipes is carried out.

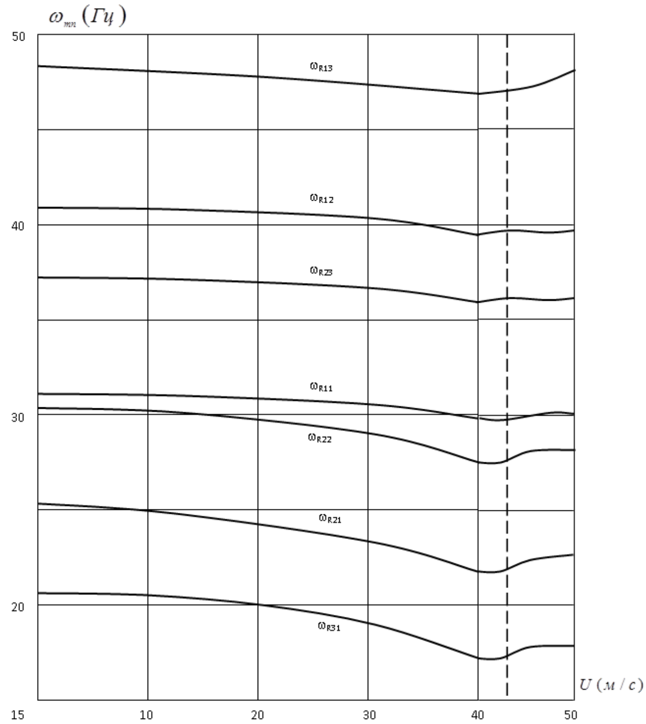

Fig. 2, d. The change in real parts of the natural frequencies of bending vibrations at different velocities of a flowing liquid h=0.05.

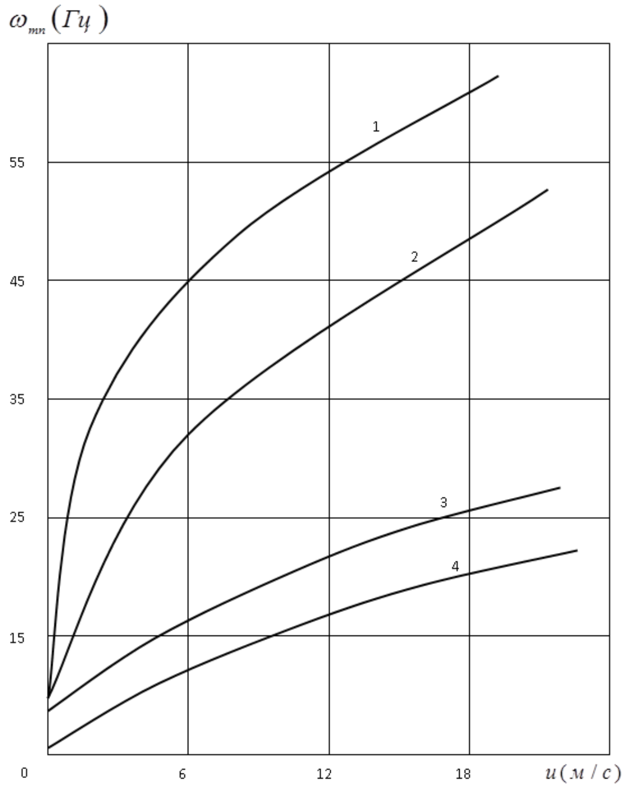

Fig.3. The change in real parts of the natural frequencies of bending vibrations from the velocity of the flowing liquid at $m = 2$.

Frequency study ω_{Rmn} by the first three forms of natural oscillations $m = 1,2,3$ curvilinear sections of polyethylene pipelines with a much smaller modulus of elasticity than in steel pipes made it possible to reveal a significant dependence of the values of the vibration frequencies on the flow velocity of the liquid. In accordance with the assortment for polyethylene pipes [22], frequency calculations were carried out for curved sections of pipelines with an external diameter of 630 mm and with a relative wall thickness $\frac{h}{r} = \frac{1}{12,4}$. It can be seen that the frequencies of tubes with a large curvature ($\frac{r}{R} = \frac{1}{10}$) much higher than the frequencies of tubes of lesser curvature ($\frac{r}{R} = \frac{1}{10}$). The main conclusion from the analysis of the results of the study of the frequencies of the own bending vibrations of polyethylene pipes with a liquid flow is that, unlike steel pipelines, the natural oscillation frequencies of these pipelines depend very much on the flow rate of the liquid. Here, the decrease in the oscillation frequencies ω_{Rmn} when the flow rate changes from 0 to $20\frac{м}{с}$ reaches 18%, which must be taken into account in the dynamic calculations of pipelines.

Table 4. Real parts of the natural frequencies as a function of the velocity of the flowing liquid

$\frac{r}{R} = \frac{1}{20}$, $\frac{h}{r} = \frac{1}{30}$, $\mu = 5,5$		ω_{Rmn} (Гц) at the velocity of the flowing liquid in $\frac{м}{с}$		
Form of oscillation	Frequencies	$U = 0$	$U = 20$	$U = 50$
$m = 1$	ω_{R11}	31,21	30,52	29,09
	ω_{R12}	40,02	39,31	38,05
	ω_{R13}	47,02	46,64	46,18
$m = 2$	ω_{R21}	24,24	53,02	20,39
	ω_{R22}	30,56	29,26	26,55
	ω_{R23}	36,44	35,91	35,27
$m = 3$	ω_{R31}	20,83	19,51	16,17
	ω_{R32}	24,91	24,01	23,34
	ω_{R33}	28,61	28,39	28,35
$m = 4$	ω_{R41}	29,02	28,92	28,98
	ω_{R42}	30,17	29,15	29,01
	ω_{R43}	31,36	30,97	30,05

As well as in pipelines, the largest natural frequencies are frequencies according to the first form of oscillations ω_{1n} at $m = 1$, where there is no deformation of the contour of the cross sections of the pipe. These frequencies correspond to the calculation of pipeline sections according to the bar theory. The lowest frequency of the self-bending oscillations of the curvilinear section is realized by shell vibration forms (at $m = 2$ and 3), corresponding to the deformed contour of cross sections and with the formation of one longitudinal half-wave of a sinusoid (at $n = 1$). The study of the natural oscillations of curvilinear sections of polyethylene pipelines has shown that, in connection with the small cured elastic modulus of polyethylene (almost 400 times less than that of steel), the oscillation frequencies for all the shell forms studied $m, n = 1,2,3$) and practically for all real geometrical sizes of the sites is much less than for the corresponding in size steel pipelines (Fig. 4).

Table 5: Real parts of natural frequencies depending on the velocity of the flowing liquid

$\frac{r}{R} = \frac{1}{20}$, $\frac{h}{r} = \frac{1}{35}$, $\mu = 4,1$		ω_{Rmn} (Гц) at the velocity of the flowing liquid in $\frac{м}{с}$		
Form of oscillation	Frequencies	$U = 0$	$U = 20$	$U = 30$
$m = 1$	ω_{R11}	6,67	6,47	5,56

	ω_{R12}	7,45	6,82	6,01
	ω_{R13}	8,74	7,91	7,51
$m=2$	ω_{R21}	4,65	6,19	2,52
	ω_{R22}	5,55	6,85	3,75
	ω_{R23}	6,02	5,51	4,69
$m=3$	ω_{R31}	2,23	1,73	0,29
	ω_{R32}	34,04	2,55	1,53
	ω_{R33}	5,54	4,91	3,25
$\frac{r}{R}=\frac{1}{20}$	ω_{R11}	1,2	0	-

In this respect, for polyethylene pipelines, a more thorough check of the condition for detuning the frequencies of external pathogens from natural frequencies is required. According to the norms of [21], the conditions for frequency detuning for the lowest oscillation frequencies have the form:

$$\frac{\omega_{R\min}}{\Omega} \geq 1{,}3 \quad \text{or} \quad \frac{\omega_{R\min}}{\Omega} \leq 0{,}7, \qquad (33)$$

where $\omega_{R\min}$ - lowest natural frequency of the pipeline; Ω - frequency of external excitation. The low frequencies of one's own flexural vibrations, in addition, can cause a loss of stability of the pipeline, when they vanish.

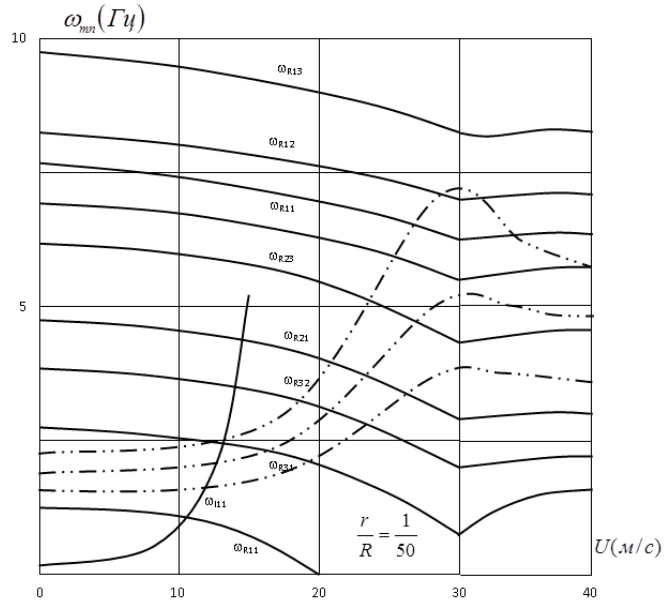

Fig.4. The change in the real parts of the natural frequencies of bending vibrations from the velocity of the flowing liquid.

Thus, when calculating the curvilinear section of a polyethylene pipeline with relative curvature $\frac{r}{R}=\frac{1}{50}$ already in the first form of oscillation ($m,n=1$) at a liquid flow rate $U=20\frac{м}{c}$ frequency $\omega_{11}=0$ (See the diagram of the dashed line in Fig. 4). This means that for such a pipeline speed $U=20\frac{м}{c}$ Is critical and it has lost stability.

With shell forms of bending vibrations ($m=2$ and 3) The contour of the cross sections is deformed (see Fig. 2.b), and the internal pressure prevents this deformation, that is, increases the rigidity of the tube and, consequently, the frequencies of the natural oscillations increase.

Table 6: Real parts of natural frequencies depending on the velocity of curvature of the shell

Relative thickness $\frac{h}{r}$	Curvature $\frac{r}{R}$	Frequencies ω_{Rm1}	ω_{Rm1} (Hz) with internal Pressure (MPa)		
			$p_0=0$	$p_0=1{,}5$	Frequency increase in%
$\frac{1}{30}$	$\frac{1}{10}$	ω_{R21}	36,23	38,81	7,5
		ω_{R31}	35,06	37,78	
	$\frac{1}{20}$	ω_{R21}	26,24	29,86	16,0

		ω_{R31}	22,86	26,04	
$\frac{1}{60}$	$\frac{1}{10}$	ω_{R21}	13,37	14,82	13,0
		ω_{R31}	13,05	14,55	
	$\frac{1}{20}$	ω_{R21}	8,39	9,97	21,0
		ω_{R31}	**7,17**	**8,49**	

To estimate the influence of the internal hydrostatic pressure on the natural oscillations, we investigated the frequencies ω_{R21} and ω_{R31} curved sections of a steel pipeline filled with a fixed liquid. The curvilinear sections of the steel pipeline have a relative curvature $\frac{r}{R}=\frac{1}{10}$ and $\frac{1}{20}$ with relative thicknesses $\frac{h}{r}=\frac{1}{70}$ and $\frac{1}{35}$ for each curvature, when the internal hydrostatic pressure changes p_0 от 0 up to 1.5 MPa, which corresponds to the actual values of the pressure in the pipelines. The results of the studies are presented in Tab. 6 and in Fig. 4. Analysis of the results showed that the internal hydrostatic pressure significantly increases the natural oscillation frequencies of pipeline sections ω_{21} and ω_{31}.

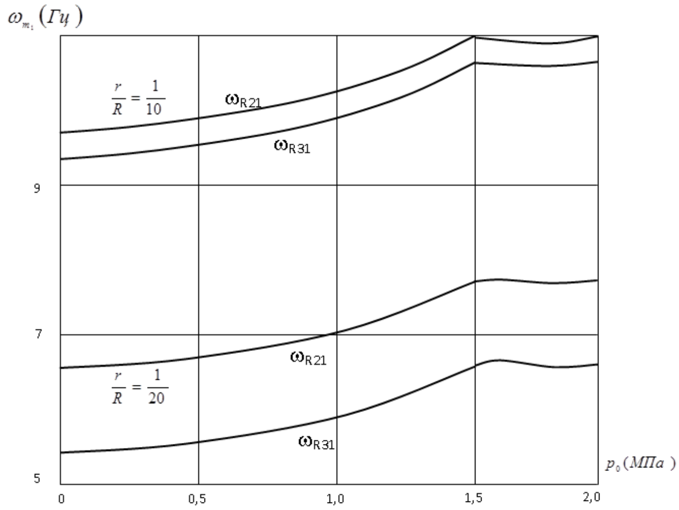

Fig.5. The change in the real parts of the natural frequencies of bending vibrations from the velocity of the flowing liquid.

It can be seen from Table 6 that an increase in pressure from 0 to 1.5 MPa increases the oscillation frequency ω_{R21} and ω_{R31} from 8 - 22 %. The greatest increase in frequencies up to 22% is obtained by the most gently sloping and most thin-walled curvilinear sections (with $\frac{r}{R}=\frac{1}{20}$ and $\frac{h}{r}=\frac{1}{60}$). This is because the internal pressure prevents the deformation of the contour of the cross sections with bending vibrations and this obstacle is the greater, the smaller the rigidity of the tube.

In less thin-walled pipes, the flexural rigidity is greater (at $\frac{h}{r}=\frac{1}{35}$), and the effect of pressure, although it takes place, but in a more moderate form, that is, the increase in frequencies reaches $8 \div 15$ %.

A study of the effect of internal hydrostatic pressure on the frequencies of natural oscillations of curvilinear sections of steel pipelines has shown that this influence is significant and should undoubtedly be taken into account in the dynamic calculations of thin-walled large-diameter pipelines. Especially strongly from the effect of internal pressure, the frequencies of natural oscillations of curvilinear polyethylene pipelines increase. The value of the modulus of elasticity of the material of these pipelines is very small (of the order of 500 MPa) as compared with the modulus of elasticity of steel ($2x10^5$ МПа), therefore, the action of internal pressure significantly increases the rigidity of the pipes, preventing the deformation of the contour of the cross section. As calculations have shown, the frequencies of oscillations along shell forms ($m=2{,}3$) with an allowable pressure in the catalog [22] equal to $p_0=0{,}8$ MPa, increase in polyethylene pipelines in $2 \div 2{,}5$ times in comparison with frequencies with the same forms, but without pressure. Also taking into account the viscous properties of the material, 15 - 10% reduces the values of natural frequencies;

Comparison of the research results with the data available in the literature. The results of the investigation of the frequencies of the curvilinear sections of the pipeline with the flowing liquid were compared with the data most widely presented in many publications in the article by S.S. Chzhenya [4], where the problem of oscillations of a curved pipe with a flowing liquid was solved on the basis of the rod theory. Therefore, we compared the frequencies of the natural oscillations in the first form ($m=1 \quad n=1$).In [4], fig. 5 shows the frequency variation graphs ω_{R11} depending on the flow rate of the liquid (water) for the steel pipe with hinged ends, with the axis in the form of a semicircle, the relative curvature $\frac{r}{R}=\frac{1}{10}$ and thin-walled $\frac{h}{r}=\frac{1}{30}$. The flow rate has changed from 0 to $50\frac{м}{c}$.

Comparison of the data obtained from the graphs shown in Fig. 5 of the paper [23] with the results of calculations using the procedure described in this paper showed that at a flow rate $U=0$ a discrepancy of 3.3%. At speed $U=50\frac{м}{c}$ the discrepancy was only 7%. The problem of natural oscillations of rectilinear pipelines with a flowing liquid, considered as closed cylindrical shells, was solved by Vol'mir [26] in cylindrical coordinates with the definition of hydrodynamic pressure using Bessel functions. Comparison of the results obtained in the monograph with the data of these studies shows that for

slightly curved tubes with a relative curvature of the order $\frac{r}{R} = \frac{1}{50} \div \frac{1}{100}$ the discrepancy is not more than 20%. In [24,25], too high results were obtained for the frequencies of natural oscillations. In connection with the fact that the hydrodynamic pressure was obtained, as for a cylindrical shell, by the Bessel functions in cylindrical coordinates, and not in toroidal ones, as in this article.
In the particular case for a dry curvilinear pipeline section, that is, a pipeline without fluid, the solutions of this work and [27] give a discrepancy of not more than 3%.
Thus, in the limiting case for the curvature parameter $\mu = 0$, that is, for a rectilinear pipeline, the solution obtained in this paper is transformed into a known solution of the problem of the natural vibrations of a cylindrical shell. Experiments on the study of the frequencies of the natural vibrations of shells are described in greater detail in the work of V.E. Breslavsky [23], where the results of testing steel closed cylindrical shells with pivotally fixed ends under pressure p_0, changed during the experiment from 0.2 to 1.2 MPa. Shells with a radius of the mean line of the section were tested $r = 15$ sm and wall thickness $h = 0{,}1$ sm. At a certain pressure p_0 flexural vibrations were excited in the envelope with the aid of an oscillator of sound frequencies. When a resonance occurs in a certain form of oscillation $m \geq 2$ excitation was turned off and oscillations of the shell were recorded on the oscilloscope. In the longitudinal direction, one half-wave of a sinusoid ($n = 1$).
The difference between experimental data [21] and calculations ω_{Rmn} at $m \geq 2$ by the formula (1) did not exceed 40%, therefore, the results of this comparison can be considered satisfactory.

Conclusions
The flow rate U flowing in the fluid pipelines (up to $20\frac{м}{c}$), which varies in the range of real velocities, has little effect on the frequencies of the natural oscillations of the curvilinear sections of the steel pipeline over all the investigated shell modes ($m = 1{,}2{,}3$,4 at $n = 1{,}2{,}3$). Oscillation frequencies ω_{Rmn} decrease with an increase in the flow rate from 0 to $20\frac{м}{c}$ not more than 7%;
for each of the sections of the pipeline considered, the largest natural frequencies are in the first form ω_{R1n} at $m = 1$. At which there is no deformation of the contour of the cross sections of the pipe, which oscillates like a beam of tubular section. These frequencies correspond to the consideration of pipeline sections according to the bar theory. Of all the frequencies ω_{1n} the frequency is the greatest ω_{R13} by the shape of the oscillations at $n = 3$, Corresponding to the formation of three half-waves of a sinusoid in the axial direction of the tube;
-for each form of oscillations of a curved pipe $(m = 1{,}2{,}3)$ frequency ω_{Rmn} at $n = 1$, which corresponds to the formation of sinusoids with oscillations of one longitudinal half-wave, is less than the others, that is, less than the frequencies for $n = 2$ and 3, corresponding to the formation of two and three half-waves in the longitudinal direction of the tube;
- the lowest frequency of bending vibrations, the most important for the dynamic calculation of the pipeline, the most important is the shell mode (with $m = 2$ and 3), corresponding to the deformed contour of the cross-section of the pipe, i.e. a cross-section with reduced bending stiffness, and with the formation of one longitudinal half-wave of a sinusoid ($n = 1$).
with an increase in the curvature of the pipeline section, i.e. the ratio $\frac{r}{R}$, at a constant relative thickness ($\frac{h}{r} = const$) frequencies ω_{Rmn} own bending vibrations increase. The same happens with an increase in the relative thickness $\frac{h}{r}$, at a constant curvature of the tube. In other words, the greater the curvature of the tube, the more rigid it becomes, and the thicker the pipe wall, the more rigid it is.
- taking into account the viscous properties of the material up to 10% reduces the values of natural frequencies.

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