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A. Gheorghita
Gr. T. Popa ' University of Medicine and Pharmacy, Iasi, Department of Biomedical Sciences, Iasi, Romania.

D. Arotăriței
Gr. T. Popa ' University of Medicine and Pharmacy, Iasi, Department of Biomedical Sciences, Iasi, Romania.

M. Turnea
Gr. T. Popa ' University of Medicine and Pharmacy, Iasi, Department of Biomedical Sciences, Iasi, Romania.

M. Ilea
Gr. T. Popa ' University of Medicine and Pharmacy, Iasi, Department of Biomedical Sciences, Iasi, Romania.

I. Duduca
Gr. T. Popa ' University of Medicine and Pharmacy, Iasi, Department of Biomedical Sciences, Iasi, Romania.

M. Rotariu
Gr. T. Popa ' University of Medicine and Pharmacy, Iasi, Department of Biomedical Sciences, Iasi, Romania.

Correspondence:
M. Ilea
Gr. T. Popa ' University of Medicine and Pharmacy, Iasi, Department of Biomedical Sciences, Iasi, Romania.

Affective modeling as a special dimension of human personality using systems of differential equations with constant coefficients

A. Gheorghita, D. Arotăriței, M. Turnea, M. Ilea, I. Duduca, M. Rotariu

Abstract
Almost a year after the beginning of the pandemic, most young people talk about strong feelings of loneliness, mental fatigue, sadness and worry. Many of them mention the need to see and embrace their friends, but also the frustrations generated by the new rules imposed at home, at school and in society. For over 400 years, a love story has been going around the world and is not going out of style at all. Love can be a very splendid thing, but it is also very complicated and seemingly very unpredictable. This article describes a mathematical approach to modeling a romantic relationship between two lovers, in particular Romeo and Juliet, using systems of differential equations with constant coefficients. The approach is quite general for shaping social interaction and can be applied to any romantic relationship of two individuals. Two mathematical models are presented as well as the mathematical methods related to them, highlighting with the help of the MATLAB program, the dynamics of the relationship between the two lovers.

Keywords: Love, Ordinary Differential systems equations, Mathematical models, MATLAB.

1. Introduction

The physical distance and the rigor of the disinfection rituals made the fight against the virus to occupy a central place in everyone's daily life. From a psychological point of view, the couple's relationships with adolescents are a very special category, being at the same time relationships of attachment, exploration of identity and training of interpersonal skills [1]. They can function as a refuge from family life or as a natural complement. The form I take for young people depends very much on the mentality of their parents and the rules imposed by them. If parents are adept at traditional family roles or rather value the intellectual development of their teenage daughters, they will certainly ignore, neglect, or violate many of the girls' needs, even if their intent is to protect them. Often, these things trigger identity conflicts in the souls of teenagers [2]. On the one hand, they feel a duty and love for their family, and on the other hand, there is frustration, fatigue, and the need for a decision that sometimes involves breaking the rules at home. This makes them vulnerable and prone to abusive relationships, rather than endeavoring to maintain a relationship and the time elapsed since the beginning of the relationship, to the detriment of the couple's security, trust and harmony [3]. As long as romantic relationships are forbidden to teenagers or are a source of reproach and ridicule in the family, they will be all the more attractive and dangerous for young people - both girls and boys. The choice of a couple partners is based on family and social relationship patterns, images of celebrity couples promoted in the media, religious or social affiliation, and many other factors. In adolescents, however, all these influences act rather unconsciously, without them being able to realize which of them are positive or negative, which their benefits are and which the risks to which they are exposed are. However, there are many things that young people feel pressured to comply with and that parents need to understand if they are to have a real chance to truly support their teens and not overwhelm them with criticism and bans. For more than a year, young people have had to move their social life, school and, in fact, almost all their activities online. If before the pandemic most relationships between young people were connected in the context of

direct interactions, face to face, after the imposition of restrictions and quarantine, most new relationships started on the net or did not start at all. And that comes at a very high cost in the emotional economy of young people, as pointed out by an army of mental health specialists around the globe [3]. Romeo and Juliet are a tragedy by William Shakespeare about the fate of two lovers who fight against destiny. It is perhaps his best-known play, being considered the most typical love story of the Renaissance. Romeo and Juliet are the story of an absolute and impossible love, which is born against the background of the war between two powerful clans. The love story between Romeo and Juliet is set in Verona, in the old days, and Shakespeare tells us that Romeo and Juliet come from the families that hate each other to death: Montague and Capulet. However, Romeo comes to a masquerade ball, organized by the Capulet family, and that's how he meets Juliet, with whom he falls in love forever. He two marries in secret, but the thread rushes: duels take place in which Romeo kills a member of the Capulet family, and Juliet's parents force her to marry another man. At the urging of a monk, Juliet avoids the wedding by drinking a liqueur that makes her look dead. With Juliet dead and buried, Romeo, who knew nothing of his girlfriend's plans, commits suicide at her grave, drinking poison. When Juliet wakes up and sees her dead lover, she stabs herself with his dagger [4].

2. Materials and methods

The general form of a system of differential equations [6] with constant coefficients is:

$$\begin{cases} \dfrac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + f_1(t) \\ \dfrac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n + f_2(t) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \dfrac{dx_n}{dt} = a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n + f_n(t) \end{cases}$$

(1)

Where:

- $x_1, x_2, \ldots x_n$ are unknown functions, and t represents the time variable
- Coefficients a_{ij} are constant.
- If the functions $f_1(t), f_2(t), .., f_n(t)$ are equal to zero, the system (1) we will call it homogeneous.
- If the functions $f_1(t), f_2(t), .., f_n(t)$ are not equal to zero, the system (1) we will call it inhomogeneous.
- We can rewrite the system (1) in the following matrix form:

$$X' = A \cdot X + F ,$$ (2)

Where:

$$X' = \begin{pmatrix} x'_1 \\ x'_2 \\ \ldots \\ x'_n \end{pmatrix}, \quad A = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{pmatrix},$$

$$F = \begin{pmatrix} f_1(t) \\ f_2(t) \\ \ldots \\ f_n(t) \end{pmatrix}, \quad X = \begin{pmatrix} x_1(t) \\ x_2(t) \\ \ldots \\ x_n(t) \end{pmatrix}$$

(3)

- The homogeneous system can be written in the form:

$$X' = A \cdot X$$ (4)

We will present a very simple model of romantic relationships [1], which assumes that the person's feelings are affected only by the other person's feelings. We will note through $x(t)$ Romeo's feelings, through $y(t)$ Juliet's feelings. These feelings will be measured on a scale of -1 to 1, with negative values representing a level of anxiety and positive values that will represent a level of affection [7]. To design an appropriate system to describe how the feelings of the two lovers change over time, proposed the following system of differential equations [1]:

$$\begin{cases} \dfrac{dx}{dt} = \alpha y(t) \\ \dfrac{dy}{dt} = \beta x(t) \\ x(0) = x_0, \ y(0) = y_0 \end{cases}$$

(5)

The constants α, β denotes "terms of relationship." A positive constant indicates that there is an affection of the other person and thus the feelings between the two lovers in-crease. A negative constant, on the other hand, indicates that the other person's affection largely rejects these feelings and thus there is uncertainty in their relationship. The best known application of systems of differential equations in the study of the dynamics of the relationship between Romeo and Juliet was proposed by the mathematician Strogatz in 1994. An obvious difficulty in any model that defines love between two individuals is what is meant by love and how we can model in a meaningful way.

The Strogatz type model of a love affair between Romeo and Juliet is of the form [2]:

$$\begin{cases} \dfrac{dx}{dt} = ax(t) + by(t) \\ \dfrac{dy}{dt} = cx(t) + dy(t) \\ x(0) = x_0, \ y(0) = y_0 \end{cases}$$

(6)

Where $x(t)$ quantify Romeo's love (or hate if $x(t) < 0$) for Juliet at the time and $y(t)$ quantify Juliet's love for

Romeo (or hate if $y(t) < 0$).

Parameters a, b, c, d quantify Romeo's romantic style and Juliet's romantic style as follows [3]:

- The parameter a would describe how Romeo is encouraged by his feelings
- The parameter b would describe how Romeo is encouraged by Juliet's feelings.
- The parameter c would describe how Juliet is encouraged by her feelings
- The parameter d would describe how Juliet is encouraged by Romeo's feelings

We have the following cases:

- $a < 0, b > 0$ - We have a cautious love of Romeo
- $a < 0, b < 0$ - We have no feelings between the two. Romeo is a lonely guy
- $a > 0, b < 0$ - Romeo is a narcissistic guy.
- $a > 0, b > 0$ - Partners are very eager to have a relationship.

- $b = 0$ - We can say that Romeo ignores Juliet's feelings.
- $b > 0, c > 0$ We have mutual love or quarrel.
- $b > 0, c < 0$ - We have an infinite cycle of love versus quarrel.
- $b < 0, c > 0$ - We have an infinite cycle of love versus quarrel
- $b < 0, c < 0$ - We have an unrequited love between our partners
- $a + d < 0$ - We have a cold relationship.
- $a + d > 0$ - We have an intensity of feelings between the two partners.

3. Results & Discussion

The model we will present has some disadvantages such as: if one of the two lovers is afraid to enter into a relationship then there can never be a love affair between the two, the love / hate between the two can last forever, hatred can become love if the other's love is strong enough.

A. Case : $\alpha \cdot \beta > 0$

$$\begin{cases} \dfrac{dx}{dt} = \alpha y(t) \\ \dfrac{dy}{dt} = \beta x(t) \end{cases} \rightarrow \begin{cases} x''(t) = \alpha y'(t) \\ y'(t) = \beta x(t) \end{cases} \rightarrow \begin{cases} x''(t) = \alpha \beta x(t) \\ y'(t) = \beta x(t) \end{cases} \rightarrow \begin{cases} x(t) = C_1 e^{\sqrt{\alpha\beta}t} + C_2 e^{-\sqrt{\alpha\beta}t} \\ y'(t) = \beta x(t) \end{cases}$$

$$\begin{cases} x(t) = C_1 e^{\sqrt{\alpha\beta}t} + C_2 e^{-\sqrt{\alpha\beta}t} \\ y(t) = \dfrac{1}{\alpha} x'(t) \end{cases} \rightarrow \begin{cases} x(t) = C_1 e^{\sqrt{\alpha\beta}t} + C_2 e^{-\sqrt{\alpha\beta}t} \\ y(t) = C_1 \dfrac{\sqrt{\alpha\beta}}{\alpha} e^{\sqrt{\alpha\beta}t} - C_2 \dfrac{\sqrt{\alpha\beta}}{\alpha} e^{-\sqrt{\alpha\beta}t} \end{cases} \qquad (7)$$

Because:
$$\begin{cases} \lim_{t\to\infty} e^{\sqrt{\alpha\beta}t} = +\infty \\ \lim_{t\to\infty} e^{-\sqrt{\alpha\beta}t} = 0 \end{cases} \qquad (8)$$

The future of the relationship between the two lovers depends on the term $: e^{\sqrt{\alpha\beta}t}$.
We can say that Romeo's feelings towards Juliet would vary as follows:

$$\begin{cases} C_1 > 0 \rightarrow x(t) \rightarrow \infty \\ C_1 < 0 \rightarrow x(t) \rightarrow -\infty \\ C_1 = 0 \rightarrow x(t) \rightarrow 0 \end{cases} \qquad (9)$$

From the initial conditions :

$$\begin{cases} x(0) = x_0 \\ y(0) = y_0 \end{cases} \rightarrow \begin{cases} C_1 + C_2 = x_0 \\ C_1 - C_2 = \dfrac{\alpha}{\sqrt{\alpha\beta}} y_0 \end{cases} \rightarrow \begin{cases} C_1 = \dfrac{x_0}{2} + \dfrac{\alpha}{2\sqrt{\alpha\beta}} y_0 \\ C_2 = \dfrac{x_0}{2} - \dfrac{\alpha}{2\sqrt{\alpha\beta}} y_0 \end{cases} \qquad (10)$$

$C_1 = 0 \rightarrow x_0 = -\dfrac{\alpha y_0}{\sqrt{\alpha\beta}}$

For , we get the condition that determines whether Romeo's feelings evolve into love or hate. We can say that Romeo's feelings towards Juliet would vary according to the initial conditions as follows:

- If: $x_0 > -\dfrac{\alpha y_0}{\sqrt{\alpha\beta}}$, then there will be an eternal love between the two partners.

- If : $x_0 < -\dfrac{\alpha y_0}{\sqrt{\alpha\beta}}$, then love will turn into eternal hatred between the two partners.

- If $x_0 = \dfrac{\alpha y_0}{\sqrt{\alpha\beta}}$, then there will be a feeling of indifference between the two partners.

B. Case $\alpha \cdot \beta < 0$

$$\begin{cases} \dfrac{dx}{dt} = \alpha y(t) \\ \dfrac{dy}{dt} = \beta x(t) \end{cases} \rightarrow \begin{cases} x''(t) = \alpha y'(t) \\ y'(t) = \beta x(t) \end{cases} \rightarrow \begin{cases} x''(t) = \alpha\beta x(t) \\ y'(t) = \beta x(t) \end{cases} \rightarrow \begin{cases} x(t) = C_1 \cos(\sqrt{\alpha\beta}t) + C_2 \sin(\sqrt{\alpha\beta}t) \\ y'(t) = \beta x(t) \end{cases}$$

$$\begin{cases} x(t) = C_1 \cos(\sqrt{\alpha\beta}t) + C_2 \sin(\sqrt{\alpha\beta}t) \\ y(t) = \dfrac{1}{\alpha} x'(t) \end{cases} \rightarrow \begin{cases} x(t) = C_1 \cos(\sqrt{\alpha\beta}t) + C_2 \sin(\sqrt{\alpha\beta}t) \\ y(t) = -C_1 \dfrac{\alpha\beta}{\alpha} \sin(\sqrt{\alpha\beta}t) + C_2 \dfrac{\sqrt{\alpha\beta}}{\alpha} \cos(\sqrt{\alpha\beta}t) \end{cases}$$

Table 1: The explanations of love.

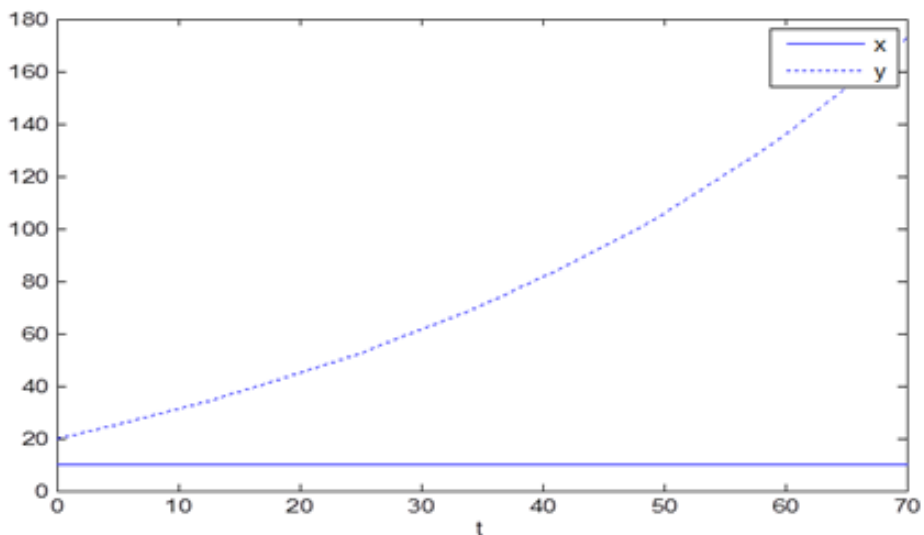| Parameters sign	Initial condition	The explanations of love
$\alpha > 0, \beta > 0$	$x_0 > 0, y_0 > 0$	The entanglement of love
$\alpha > 0, \beta > 0$	$x_0 < 0, y_0 < 0$	Embrace hate
$\alpha > 0, \beta > 0$	$x_0 > 0, y_0 < 0$	We have three cases
$\alpha > 0, \beta > 0$	1. $x_0 > -\dfrac{\alpha y_0}{\sqrt{\alpha\beta}}$	The entanglement of love
$\alpha > 0, \beta > 0$	2. $x_0 < -\dfrac{\alpha y_0}{\sqrt{\alpha\beta}}$	Embrace hate
$\alpha > 0, \beta > 0$	3. $x_0 = \dfrac{\alpha y_0}{\sqrt{\alpha\beta}}$	The indifference of the two lovers

Fig. 1: Case: $a = b = 0$. Romeo's indifference leads to an increase in Juliet's passion.
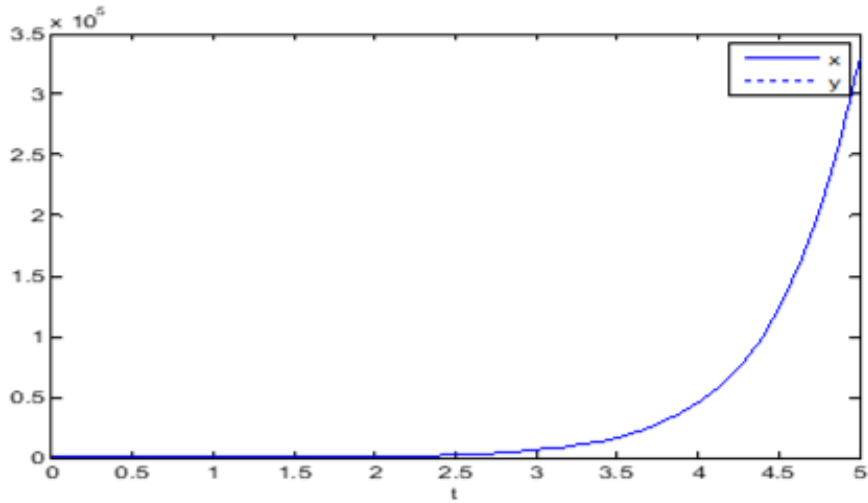
Fig. 2: Case: $a = b = c = d = 1$. In this case, all coefficients have maximum values, so love is lived to the maximum, we can even talk about a blind love
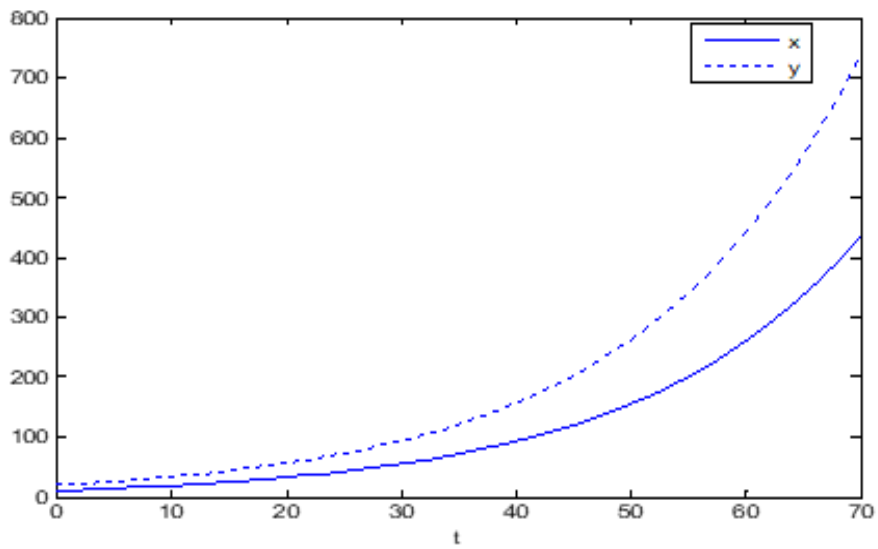

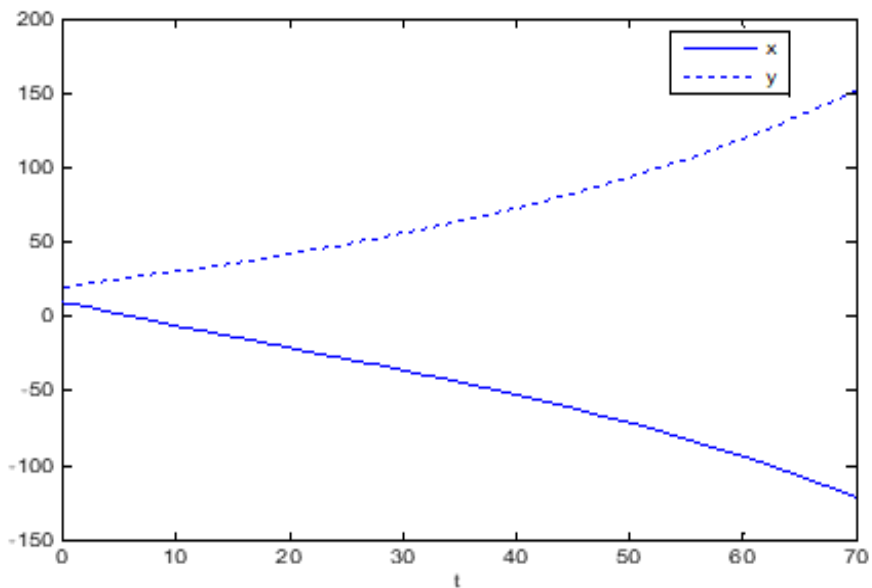Fig. 3 Case: $a < 0, b > 0$. We have a cautious love of Romeo

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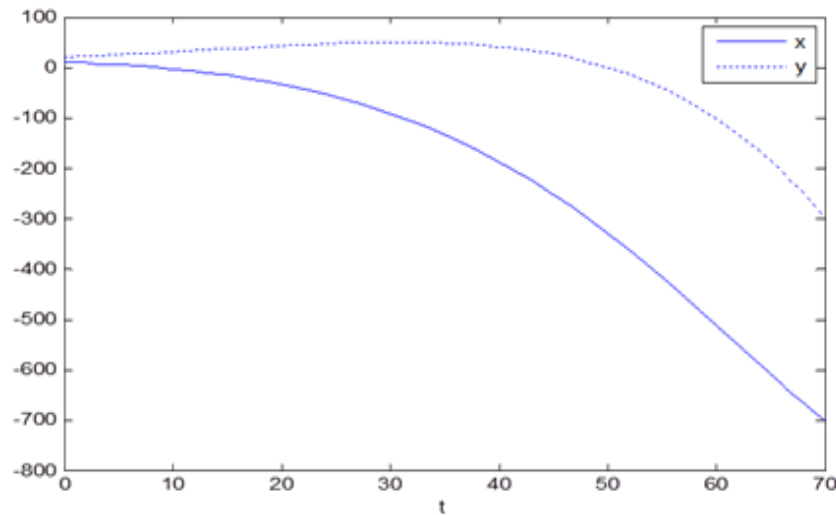Fig. 4 Case: $a < 0, b < 0$. Romeo is a lonely guy.

Fig. 5   Romeo is a narcissistic guy

8. Conclusions

In Romania, is estimated that one in three marriages ends in divorce. Other data show that about 33% of people find it difficult to maintain a relationship. In general, one of the main conflicts between two partners revolves around a topic of romantic connection between the two partners. Men generally participate less in this connection, women participate more intensely, as such, and it starts from the beginning on a plan of imbalance in terms of relationship harmony. In this article we used systems of Differential Equations with constant coefficients to analyze the stability of the relationship between two lovers, the particular case being the famous Romeo and Juliet. The choice of numerical values for all parameters was completely arbitrary. We believe that there may be other factors that define the relationship between couples, which should be taken into account such as: health, immaturity (physical, emotional, intellectual, etc.) or serious deficiencies and defects of character. Using MATLAB simulations, I noticed that the most common problem for couples has to do with the difficulty of maintaining enough energy and enthusiasm over time. Many   partners find that long-term relationships are tiring and boring. They feel that the passion is declining faster than expected. Psychologists have sought to find out how difficult it is to conquer a person. From the article, it can be seen that the role of "hard to conquer" does not necessarily create a greater desire. On the contrary, a cold attitude can discourage and drive away an aspirant. Finally, we can ask ourselves the eternal question: "What are the ideal conditions that lead to a true love between two lovers?" Here concludes your finding to with object of your studies.

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