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Aggregate Production Planning and Marginal Concepts for Optimal Decisions Using Fuzzy Goal Programming

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Abstract
Aggregate Production Planning (APP) is a type of medium-term planning that centers on the most cost-effective way to manage production to meet changing demand and other uncertainties. In this study, the suggested technique makes reference implementation to labor levels, machine, inventory levels, overtime, warehouse capacity and backordering levels in an effort to minimize overall expenses, maximize customer satisfaction, and maximize revenue. Thus, filling the following lacuna; to pro-pose a new multi-product multi-period multi-objective APP problem through mathematical modeling; produce a FGP method to solve the multi-period, multi-product, and multi-objective APP problem; apply the proposed method in a real case study; propose a dictionary for information gathering. However, the study results were obtained using Lingo version 18 software with data gathered from Rich Pharmaceuticals Limited (RPL) based in Lagos, Nigeria. The proposed model produces a useful compromise result and general levels of DM gratification with the various fuzzy goal values. A decision maker can interactively alter the fuzzy data and associated model parameters until a satisfying answer is attained thanks to the suggested model's systematic structure for easing the decision-making process.

Keywords: Aggregate production planning, fuzzy demands, goal programing, Decision maker, linear membership function.

1. Introduction

Demand forecasting has always been crucial for the better production process. As the need for more efficient operations in the modern business environment, more effective forecasting methods are needed (Sahin et al, 2013). Forecasting is utilized in operational function to make recurring decisions regarding facility layout, capacity planning, supplier selection, and process selection. Forecasts are also necessary to understand how a company's daily operations are conducted. (Chase & Jacobs 2014). To meet the demand, footwear manufacturing companies need to find the most optimal rate of production. In each production activity, the company will always deal with costs including inventory costs, labor costs, and overtime costs, etc. (Mariyani, 2014). To optimize production costs, there are several strategies that could be done. One of them is by using aggregate planning method.

Aggregate Production Planning (APP) governs the most effective way to adjust to forecast demand in the intermediate future, often takes 6 to 24 months ahead, by adjusting regular and overtime production rates, inventory levels, labor levels, subcontracting and backordering rates, and other controllable variables (Wang R. et al., 2005). The primary inputs of APP are market demands and the manufacturing plan to meet those expectations (Leung *et al*., 2003). Production planning does this in response to changes in demand. Changing a company's production schedule on a moment’s notice can be expensive and lead to insecurity. Planning for changes in demand months in advance guarantees that the change of production schedules can occur with little effort (Hossain M. et al., 2016). APP is a general style to altering a company's production schedule to respond to changes in demand.

Saad (1982) divided all traditional decision models for resolving APP problems into six categories: (1) Linear decision rule by Holt et al. (1955), (2) Transportation method by

Bowman (1956), (3) Management Coefficient Approach by Bowman (1963), (4) linear programming method by Charnes and Cooper (1961), (5) Simulation method by Jones (1967) and (6) Search decision rule by Taubert (1968).

However, the goals and model inputs when any of these APP models are used generally are assumed to be deterministic/crisp, but the linear programming (LP) method is the most acceptable presently. Also, some involve models that are easy to formulate, while others require complicated models. As a result of certain information being insufficient or unavailable, real-world APP problems frequently have input data or related parameters that are imprecise or fuzzy, such as market demand, capacity and resource availability, as well as appropriate operating costs; Thus, the Fuzzy Aggregate Production Planning (FAPP) is enabled.

In this paper a new multi-product multi-period multi-objective APP problem is proposed. The proposed problem is modeled using multi-objective mathematical programming. Concurrently, three objective functions—minimizing total cost, enhancing customer experience, and increasing sales revenue—are taken into account. Numerous restrictions are also taken into account, including those relating to the amount of production, the amount of time available, the number of workers, the amount of inventory, the number of backorders, the capacity of the machines, the amount of warehousing space, and the available budget. Then, a Fuzzy Goal Programming (FGP) is proposed to solve the proposed model. The results of proposed approach are compared with those of the existing experimental method used in the company.

This study shall contribute to the following; (a) pro-pose a new multi-product multi-period multi-objective APP problem through mathematical modeling, (b) produce a FGP method to solve the multi-period, multi-product, and multi-objective APP problem, (c) apply the proposed method in a real case study, (d) propose a dictionary for information gathering.

2. Literature of Past Works

Based on two groups, the literature of earlier studies was reviewed. The APP and its variations are reviewed in the first group. The goal programming method is examined in the second category. The APP literature has been examined in three primary headings: 1) The traditional APP models, which take into account planning horizons, 2) APP models that take uncertainty into account; and 3) multi-objective APP models used to solve actual industrial challenges.

Classic Aggregate Production Planning

Generally speaking, APP is one of the principal production planning categories (Giannoccaro and Pontrandolfo, 2001; Mula et al., 2006). Numerous academics have extensively explored the APP problem since the classical model of linear decision rule for production and employment scheduling proposed by Holt et al. (1955, 1961). (Jain and Palekar, 2005; Leung and Wu, 2004; Wang and Liang, 2004). According to Wang and Liang (2004) APP is one of the most vital functions in production and operations management. Nam and Logendran (1992) studied APP models and clustered them in groups of optimal and near-optimal. An assessment of mathematical optimization models, including the APP, showed that the most widely accepted approach is linear programming, which has been adopted as a standard technique. Baykasoglu (2001) described APP model as medium-term capacity planning through a planning horizon of 2–18 months. Fung et al. (2003) described APP as a plan to determine production, inventory, and labor levels required to answering to all market demands. Junior and Filho (2012) reviewed the works on production planning and control for remanufacturing. Karmarkar and Rajaram (2012) discussed a rivalry version of APP model with capacity constraints. Ramezanian et al. (2012) focused on systems with multi-period, multi-product, and multi-machine with setup results. Zhang et al. (2012) presented a mixed integer linear programming model for APP problem with expansion of capacity in the production system. Jamalnia and Feili (2013) suggested a hybrid system dynamics and discrete event simulation methodology to model and simulate the APP problem. The key objective of their study was to determine the effectiveness of APP strategies regarding the Total Profit. Tonelli et al. (2013) proposed an optimization approach to face aggregate planning problems in a mixed model production environment. Furthermore, the APP models can handle the specifics of real-world issues, and they are frequently solved using effective algorithms. Numerous studies have noted that the APP cost function is convex and piecewise (Bushuev 2014). For resolving the APP problem, Bushuev (2014) developed a novel convex optimization strategy. A model for integrating process planning with production planning and control was created by Hassan Zadeh et al. (2014). They centered their thorough framework on the field of cellular manufacturing systems.

Fuzzy Aggregate Production Planning

The Fuzzy Set (FS) theory was first put forth by Zadeh (1965). It is based on an expansion of the traditional definition of set A, which states that each element x of a given universe X either belongs to set A or it does not. However, in the FS theory, an element only belongs to set A to a certain "degree of membership." There is little doubt that some fuzzy programming issues are beyond the scope of traditional mathematical programming tools. Fuzzy set theory was initially applied to traditional LP problems by Zimmermann (1976). His research focused on LP problems with fuzzy goal and constraints. The same study verified the existence of an equivalent single-goal LP problem using Bellman and Zadeh's (1970) fuzzy decision-making approach. Fuzzy mathematics programming has since evolved into a number of fuzzy optimization techniques for resolving APP problems. Currently, fuzzy techniques are often efficient in the area of decision making. Essentially every type of decision-making, including multi-objective, multi-person, and multi-stage decision-making, has used fuzzy methodologies (Tamiz,1996). Additionally, applications of fuzzy theory in management, business, and operational research are included in other studies related to fuzzy decision making (Zimmermann, 1991).

By employing integrated parametric programming, best balance, and interactive approaches, Fung et al. (2003) introduced a fuzzy multi-product aggregate production planning (FMAPP) model to cater to various situations under varied decision-making preferences. This model can also effectively improve the capability of an aggregate plan to deliver feasible disaggregate plans under varying circumstances with fuzzy demands and fuzzy capacities. In order to tackle multi-product APP choice problems in a fuzzy environment, Wang and Liang (2004a) more recently created a fuzzy multi-objective linear programming model using the piecewise linear membership function. The model can yield an effective compromise solution and the decision maker's overall levels of satisfaction. Additional research on fuzzy APP problem solving may be found in Wang and Fang (1997), Tang et al. (2000), Wang and Fang (2001), and Tang et al (2003). To optimize profit, minimize repair costs, and maximize machinery usage, Leung and Chan (2009) created a preemptive goal programming approach for the APP problem. Sakall et al. (2010) discussed a probabilistic APP model for the blending issue in a brass production. They came up with the best procedures for buying raw materials. For the APP problem with uncertainty, Li et al. (2013) suggested a brand-new hierarchical belief-rule-based inference technique.

Multi-Objective Aggregate Production Planning Zimmermann

A Fuzzy Multi-Objective Linear Programming (FMOLP) model was developed by Wang and Liang (2004) to solve the multi-product APP decision problem in a fuzzy environment. By taking into account inventory level, labor level, capacity, the time value of money, and warehouse space, the suggested model aims to reduce carrying and backordering costs, total production costs, and the changes in the rate of labor levels. The goal of the multiple-objective mathematical programming described by Tabucanon and Majumdar (1989) was to address the issue of production planning for a ship repair company. Nagarur et al. (1997) provided an example of the handling and production planning in use for pipe fitting injection molding. And more recently, Boppana and Slomp (2002) solved this issue in a corporation of machines and tools using a mathematical programming model with objectives. Wang and Fang (2003) and Dai et al. (2003) both offer a method that makes use of fuzzy linear programming (2001). The costs associated with the supply chain and demand were also taken into account by Mirzapour Al-e-Hashem et al. (2011) as uncertain parameters. For a two-level supply chain, Ghasemy Yaghin et al. (2012) suggested a fuzzy multi-objective APP model with qualitative and quantitative objectives. In an uncertain context, Mirzapour Al-e-Hashemet et al. (2012) created a multi-site, multi-period, multi-product, and multi-objective resilient APP to address conflicts between supply chain total costs, customer service standards, and worker productivity.

Fuzzy Goal Programming

When there are multiple objectives and goals that conflict, decision-makers and production managers must make difficult decisions about which objectives and goals should take precedence. The goal programming (GP) method may be suggested as a workable and useful solution to address this problem. As a productive multi-criteria and multi-objective planning technique, GP was first proposed by Charnes et al. (1955). After that, Cooper (1961) developed GP. GP is actually a development of linear programming. The GP model can help decision-makers take into account multiple objectives at once in order to come up with workable solutions, claim Chen and Tsai (2001). The following GP models are categorized depending on combinations of departures from the goals: 1) weighted GP, 2) lexicographic, and 3) min-max. Among the classes, weighted GP minimizes the weighted sum of the deviations from the goals. The weighted GP can achieve efficient and high-quality compromise solutions.

Several decision-making strategies have been expanded in fuzzy environments as a result of the introduction of fuzzy set theory in order to deal with the ambiguous nature of real-world problems. Numerous multi-objective production planning problems have been solved using fuzzy mathematical programming, particularly the fuzzy goal programming (FGP) method. For supplier selection issues with various objectives, Lee et al. (2009) and Kumar et al. (2004) presented FGP techniques. For the purpose of resolving integrated production and distribution planning problems with fuzzy multiple goals in uncertain environments, Liang (2006) suggested a FGP technique. The suggested model seeks to concurrently reduce the overall production and distribution costs, the overall quantity of returned goods, and the overall delivery time. In a multi-echelon automotive supply chain network, Torabi and Hassini (2009) suggested a multi-objective, multi-site production planning FGP model incorporating procurement and distribution plans.

A gap in earlier works has been identified, according to the literature referenced above. In this study, an APP problem with multiple objectives, multiple periods, and multiple products is suggested. The suggested solution to the problem is a FGP. Minimizing total manufacturing costs, maximizing sales revenue, and maximizing customer satisfaction are all crucial factors for the case concern in this instance. It is therefore more reasonable to describe them as three distinct objectives so that the APP model may identify a Pareto optimum that strikes a balance between these three goals. So, for the example study, the following three-objective, multi-period, multi-product FGP-APP model is developed:

3. Method and Procedure

Assumptions and Problem Definition

Following the findings of a real-world case study, the following presumptions are made for the mathematical model of the suggested APP problem.

- Production planning is done in a time horizon of T time periods ($\forall\, t = 1,2, \dots, T$).
- There is a Batch production system capable of producing all kinds of N types of products.
- Market demand can be fulfilled or backordered, however no backorder in the last t is allowed.
- There are two working shifts; Regular time production and Over time production
- A warehouse is allowed for holding final products.
- In advance, the holding cost of inventories are determined and well known.
- The workforce accommodates various skill levels ($k - levels$).
- Worker's salary is independent of unit production cost.
- At each period T, Production quantity is considered more of the safety stock for finished products.
- Hiring and firing of Manpower based on product demand is eligible and there is an allowable limit.
- In each period T, the shortage of production is recovered by overtime production in each shift.

- In each period T, the nominal and actual capacity of production machines is not the same due to unforeseen failures. So, the actual capacity of production is usually reduced by a fixed failure percentage.
- If an unforeseen failure occurs during a shift the repair process is completed in the next. This may stop, reduce, or decrease the production rate during maintenance actions
- The impreciseness and uncertainty of real-world problem and confliction of different objectives are modeled using fuzzy goals.
- Linear membership functions are defined for fuzzy goals.
- FGP used to solve the problem.

3.1 Parameters, Indices, Decision Variables and Notations

They are as stated in Tables 1 to 3

Table 1: Set of indices.

t	Number of periods in the planning horizon; $t = 1,2,\dots,T$
i	Number of product types; $i = 1,2,\dots,I$
m	Raw material type; $m = 1,2,\dots,M$
q	Types of shifts; $q \in 1,2$
w	Types of warehouses; $w = 1,2,\dots,W$
k	Skill levels of workers; $k = 1,2,\dots,K$
j	Number of objective Functions; $j = 1,2,3$

Table 2: Notation for parameters.

Parameter	Definition
CoP_{iq}	Cost of Production; for product i in shift q
DoP_{it}	Demand of product i in period t
CoB_{it}	Cost of Backordering; for product i in period t
SRe_i	Sales Revenue for product i (₦/unit)
PrT_t	Process time of product i in period t
BUL_t	The Budget upper limit in period t
AsP_{it}	Allowable shortage of product i in period t
$\overline{AMW}_t$	Available Maximum workforce in period t
$\underline{AMW}_t$	Available Minimum workforce in period t
WaO	workforce that are available for overtime (in percentage)
CoW_{kt}	Cost of workforce of level k in period t
CoH_{kt}	Cost of Hiring workforce of level k in period t
CoF_{kt}	Cost of firing workforce of level k in period t
CoR_{mtw}	Holding cost for raw material type m in period t in warehouse w
$CohP_{itw}$	Holding cost of unit of product i in period t
E_t	cumulative investment in tools and equipment in period t (currency unit)
FoW_t	fraction of the workforce variation in period t
MH_{it}	Machine hours needed to produce unit of product i in period t
MCi_t	Machine capacity that is lost due to **interruption** in period t (in percentage)
MCr_t	Machine capacity that is lost due to **repairs** in period t (in percentage)
MmC_{qt}	The maximum of machine capacity that is available in shift q in period t
MCo	The machine capacity that is available for overtime (in percentage)
ArT_{it}	Available Regular time in both shifts in period t
uMR_{im}	The units of type m raw material required to produce unit of product i
SSP_i	product i safety stock
SSR_m	Raw material type m safety stock
$\overline{MSW}_m$	The maximum available space of warehouse w
$WhCR_{wmt}$	The capacity of warehouse w for storage of raw-material type m in period t
$WhCP_{wit}$	The capacity of warehouse w for storage of finished-product i in period t
$\mathcal{D}d_i$	The Due date of product i
$\mathcal{B}_i$	Batch size of product i
DrF_i	Finished product i Defect rate

Table 3: Decision variable Notation.

Decision variable	Definition
X_{iqt}	Number of product i produced in shift q of period t
$X\beta_{iqt}$	Number batches of product *i* produced in shift *q* of period *t*
B_{it}	Backorder level of product *i* in period *t*
XW_{kt}	Number of available workers of level k in period t
XH_{kt}	Number of hired workers of level k in period t
XF_{kt}	Number of fired workers of level k in period t

XR_{mtw}	Inventory level of raw material type m at the end of period t in warehouse w
XP_{itw}	Inventory level of finished-product i in period t in warehouse w

3.2 Model Formulation

Minimize Total Cost

$$Min\ Z_1 = \sum_{i=1}^{I} \sum_{q\in\{1,2\}} \sum_{t=1}^{T} CoP_{iq} X_{iqt} + \sum_{k=1}^{K}\sum_{t=1}^{T} CoW_{kt} XW_{kt} + \sum_{k=1}^{K}\sum_{t=1}^{T} CoH_{kt} XH_{kt} + \sum_{k=1}^{K}\sum_{t=1}^{T} CoF_{kt} XF_{kt} + \sum_{i=1}^{I}\sum_{w=1}^{W}\sum_{t=1}^{T} CohP_{iwt} XP_{iwt} + \sum_{m=1}^{M}\sum_{w=1}^{W}\sum_{t=1}^{T} CoR_{mwt} XR_{mwt} + \sum_{i=1}^{I}\sum_{t=1}^{T} CoB_{lt} B_{lt} \quad (\mathbf{1})$$

The above Minimization of Total Cost function (TFC) involves the following seven terms; the per unit Production Cost, Cost of salary of the workforce, Cost of hiring, Cost of firing, Cost of holding of products, Cost of holding of raw materials, and Cost of Backordering.

Maximize Customer Satisfaction Level

$$Min\ Z_2 = \sum_{i=1}^{I} \left| \sum_{q\in\{1,2\}} \sum_{t=1}^{T} PtP_{it} X_{iqt} - Dd_i \right| \quad (2)$$

Function (2) is for achieving the customer satisfaction, this is by minimizing the difference between the delivery date (PtP_{it}) of all products and the due date (Dd_i) of all products, this in turn maximizes the customer satisfaction level. Worthy of note that delivering the product earlier to Dd_i is not to the benefit of the producer and delivering later to Dd_i is also not to the benefit of the customer, thus (2) minimizes the imbalance concurrently.

Maximize Sales Revenue

This last objective function is to realize the highest possible return from the quantities produced by regular production and overtime production including inventories and back orders.

$$Max\ Z_3 = \sum_{i=1}^{I} \sum_{q\in\{1,2\}} \sum_{t=1}^{T} SRe \times \left(XP_{iwt-1} - B_{lt-1} + X_{iqt} - XP_{iwt} + B_{lt}\right) \quad (3)$$

Constraints

The Labor-force Constraints are considered as follows:

$$\sum_{k=1}^{K} XW_{kt} \le \overline{AMW}_t\,, \qquad \forall t \quad (4)$$

$$\sum_{k=1}^{K} XW_{kt} \ge \underline{AMW_t}\,, \qquad \forall t \quad (5)$$

$$XW_{kt} = XW_{k(t-1)} + XH_{kt} - XF_{kt}\,, \quad \forall k, \forall t, t > 1 \quad (6)$$

$$XW_{kt} - XW_{k(t-1)} \le FoW_t * XW_{kt}\,, \quad \forall k, \forall t, t > 1 \quad (7)$$

Constraints (4) attests that the total labor utilized during period t does not exceed the total workforce that is available. In a similar vein, (5) guarantees that in period t, the employed workforce exceeds the available minimum workforce. Set of Constraints (6) is a workforce level balance equation that assures that the workforce with skill level k available during a given period is equal to the workforce with the same skill level k during the previous period plus the change in workforce level during the current period. The change in workforce level in each planning period cannot be greater than a benchmark number of workers in the present period, according to constraint number seven.

Time Constraints

$$\sum_{i=1}^{I} PrT_{it} * X_{iqt} \le \sum_{k=1}^{K} ArT_{qt} * XW_{Kt}\,, \qquad \forall t,\ q = 1 \quad (8)$$

$$\sum_{i=1}^{I} PrT_{it} * X_{iqt} \le \sum_{k=1}^{K} ArT_{qt} * WaO * XW_{Kt}\,, \qquad \forall t,\ q = 2 \quad (9)$$

The relationships mentioned above make sure that each working shift's necessary production time is less than or equal to the available regular production time and overtime.

Inventory Constraints

$$XP_{iwt} = XP_{iw(t-1)} + \sum_{q\in\{1,2\}} X_{iqt} - B_{it} - DoP_{it}\,, \quad \forall i, \forall w, \quad t > 1 \tag{10}$$

$$XR_{mwt} = XR_{mw(t-1)} + \sum_{q\in\{1,2\}} X_{iq(t-1)} - uRM_{im}, \quad \forall i, \forall w, \quad t > 1 \tag{11}$$

$$SSR_m \leq \sum_{w\in W} XR_{mwt}\,, \quad \forall m, \forall t, \tag{12}$$

Constraints (10) ensures that the amount of finished product type I in period t in warehouse w is equal to the amount of finished product type I in period $t-1$ in warehouse w plus the quantity of produced finished goods type I in period t in both working shifts, less the amount of product type I in period t that is on backorder and the quantity of produced finished goods type I in period t in both working shifts. A set of limitations (11) assures that there is a balance between raw materials, and (12) guarantees that the safety stock of raw materials in warehouses is satisfied.

Production Constraint

$$SSP_i \leq \sum_{q\in\{1,2\}} X_{iqt}\,, \quad \forall i, \forall t, \tag{13}$$

$$DoP_{it} \leq \left(1 - \frac{DrF_i}{\beta_i}\right) * \sum_{q\in\{1,2\}} X_{iqt} + XP_{i(t-1)}, \quad \forall i, \forall t, \quad t > 1 \tag{14}$$

Set of constraints (13), which is written for all product types and all periods of planning, guarantee the satisfaction of safety stock of finished-products in working shifts. Set of constraints (14) represents the total production of non-defected final products plus the inventory of finished-product in previous period should be greater than or equal to demand of the finished-product in current period.

Machine capacity Constraints

$$\sum_{i=1}^{I} MH_{it} * X_{iqt} \leq \overline{M}mC_{qt} - MCi_t * \overline{M}mC_{qt}, \qquad \forall t, \;\; q = 1 \tag{15}$$

$$\sum_{i=1}^{I} MH_{it} * X_{iqt} \leq MCo * \overline{M}mC_{qt} - MCr_t * MCo * \overline{M}mC_{qt}, \qquad \forall t, \;\; q = 2 \tag{16}$$

Constraints (15) and (16) pledge that in regular time and overtime, the machine capacity is assured.

Warehouse Capacity Constraint

$$\sum_{w=1}^{W} XP_{iwt} \leq \sum_{w=1}^{W} WhcP_{wit}\,, \;\; \forall i, \forall t, \tag{17}$$

$$\sum_{m=1.}^{M} \sum_{w=1}^{W} XR_{mwt} \leq \sum_{w=1}^{W} \sum_{m=1}^{M} WhcR_{mwt}\,, \quad \forall t, \tag{18}$$

$$\sum_{w=1}^{W} WhcP_{wit} + \sum_{w=1}^{W} WhcR_{mwt} \leq \overline{M}SWh_m, \;\; \forall i, \forall t, \tag{19}$$

The first two constraints (17) and (18) gives the restrictions of actual inventories of finished products and raw materials. While (19) guarantees that each warehouse at each period will not be able to allow storage capacity of products an raw materials beyond its maximum warehouse available space.

Backorder, Budget limit and Non-negativity Constraints

There is backorder obeying the following;

$$\sum_{w=1}^{W} B_{it} \leq \sum_{w=1}^{W} AsP_{it} * DoP_{it} \quad \forall i, \quad t \neq T \tag{20}$$

$$B_{iT} = 0, \qquad \forall i \tag{21}$$

$$ToCo \;\leq \sum_{t=1}^{T} BUL_t \tag{22}$$

$$X_{iqt}, X\beta_{iqt}, B_{it}, XR_{mtw}, XP_{iwt} \geq 0, \qquad \forall i, \forall q, \forall t, \forall m, \forall w \tag{23}$$

$$XL_{kt}, XH_{kt}, XF_{kt} \;\; \geq 0, \qquad \forall t, \forall k, \forall l \tag{24}$$

Constraints (20) represent the backorder level at the end of period t cannot exceed the certain percent-age of the demand which determines the upper limit of shortage. While (21) assure that there is no possibility for backordering at the end of time horizon or last period.

A restriction on the available budget for each planning

period is shown using (22), which ensures that the Total Cost (i.e., Eq. (1)) cannot go beyond the predetermined budget for the time horizon.
(23) and (24) both present non-negativity requirements on decision variables.

3.3 Fuzzy Multi-objective Goal Programing Development

In classic models of GP, the decision maker has to specify a precise aspiration level (goal) for each of the objectives. In general, especially in large-scale problems, this is a very difficult task, and the use of the Fuzzy Set theory in GP models can overcome such problem, allowing decision makers to work with imprecise aspiration levels (Yaghoobi and Tamiz, 2007). In multiobjective programming, In fuzzifying the inequality signs; " $=$ " " $\leq$ " and " $\geq$ ", Zimmermann (1978) used the symbol "~", they are to be understood as "essentially greater than or equal to" and "essentially less than or equal to". if an imprecise aspiration level is introduced to each of the objective functions then these fuzzy objectives are termed as fuzzy goals. Let g_k be the aspiration level assigned to the kth objective $Z_k(x)$. Then the fuzzy goals are:

$Z_k(x) \gtrapprox g_k$ [for maximizing $Z_k(x)$] and
$Z_k(x) \lessapprox g_k$ [for minimizing $Z_k(x)$]

In solving the problem, a general form of FGP model is considered:

$$\begin{aligned} &\text{find} \quad x \\ &\text{to satisfy;} \\ &\quad Z_k(x) \gtrapprox g_k \qquad k = 1 \dots n \\ &\quad Z_k(x) \lessapprox g_k \qquad k = n+1 \dots J \qquad (25) \\ &\text{subjet to} \\ &\quad AX \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b \\ &\quad X \geq 0 \end{aligned}$$

For this paper, a FGP is employed in solving the APP system (1) –(24). Being able to use FGP approach with fuzzy goals, the aspiration levels should be calculated. Payoff table is used when the decision maker has no enough view point to determine the aspiration levels. Zimmermann (1978) used a Payoff table to develop an upper and lower limit that was used to formulate the membership functions of the fuzzy goals.
In the general form (25), the purpose of FGP is to find compromise solution X such that all fuzzy goals are satisfied. g_k is the aspiration level for kth goal, $AX \leq b$ are system constraints in vector notation. $Z_k(x) \lessapprox g_k$ Means that the kth fuzzy goal is approximately less than or equal to the aspiration level g_k, and $Z_k(x) \gtrapprox g_k$ Means that the k-th fuzzy goal is approximately greater than or equal to the aspiration level g_k (Hannan, 1981).
The fuzzy decision-making concept of Bellman and Zadeh (1970) can be used to solve the planned multi-objective APP problem (1)–(24). Linear membership functions as proposed by Zimmermann (1978) are used to represent the fuzzy goals of decision makers.

Now, the membership function μ_k for the kth fuzzy goal $Z_k(x) \lessapprox g_k$ can be expressed as follows:

$$\mu(Z_k(x)) = \begin{cases} 1 & Z_k(x) \leq g_k \\ \dfrac{u_k - Z_k(x)}{u_k - g_k} & g_k \leq Z_k(x) \leq u_k \\ 0 & Z_k(x) \geq u_k \end{cases} \qquad (26)$$

where u_k is the upper tolerance limit for the kth fuzzy goal and $u_k - g_k$ is the tolerance p_k which is subjectively chosen and the function is as depicted in Figure 1a.

Again, the membership function μ_k for the kth fuzzy goal $Z_k(x) \gtrapprox g_k$ can be expressed as follows:

$$\mu(Z_k(x)) = \begin{cases} 1 & Z_k(x) \geq g_k \\ \dfrac{Z_k(x) - l_k}{g_k - l_k} & l_k \leq Z_k(x) \leq g_k \\ 0 & Z_k(x) \leq l_k \end{cases} \qquad (27)$$

where l_k is the lower tolerance limit for the kth fuzzy goal and $g_k - l_k$ is the tolerance p_k which is subjectively chosen and the function is as depicted in Figure 1b.

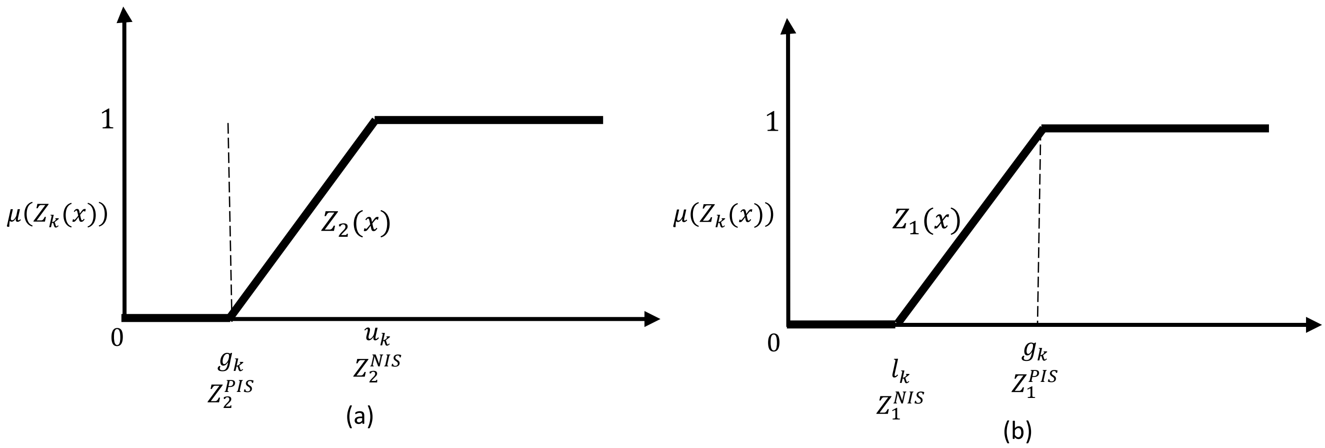


Fig 1: Linear Membership form

Hence, the associated FGP model for the multiobjective APP problem (1)-(26) is formulate as follows:

$$\text{find} \quad x$$
$$Maximize \quad \lambda$$
$$\text{to satisfy;}$$
$$\lambda \leq \mu(Z_1(x)) = \frac{u_k - Z_k(x)}{u_k - g_k}$$
$$\lambda \leq \mu(Z_2(x)) = \frac{u_k - Z_k(x)}{u_k - g_k}$$
$$\lambda \leq \mu(Z_3(x)) = \frac{Z_k(x) - l_k}{g_k - l_k}$$
$$\mu\left(Z_j(x)\right) \in [0,1], \;\; j = 1,2,3$$
$$Constraints \; (4) - (24)$$
$$x_i \geq 0, i = 1 \ldots\ldots n$$

This suggested approach states that goal weights are decided by DM, and goal aspiration levels are derived using a payout table. The positive ideal solutions (PIS) and negative ideal solutions (NIS) of the objective functions can be respectively specified as follows, (Hwang and Yoon,1981; Lai and Hwang, 1992b);

$$Z_1^{PIS} = MinZ_1; \; Z_1^{NIS} = Max\{Z_1\left(v_j^*\right)\}$$
$$Z_2^{PIS} = MinZ_2; \; Z_2^{NIS} = Max\{Z_2\left(v_j^*\right)\}$$
$$Z_3^{PIS} = MaxZ_3; \; Z_3^{NIS} = Min\{Z_3\left(v_j^*\right)\}$$

Where v_j^* is the positive ideal solution of objective function Z_k.

3.4 Model Algorithm

The following steps constitute the algorithm for building the FGP model:

Step 1: Formulate the APP problem using the FGP model.

Step 2: Solve the multi-objective APP problem as a single objective APP problem using each time only one objective. This value is the best value for this objective as other objectives are absent.

Step 3: From the results of step2 determine the corresponding values for every objective at each solution derived.

Step 4: From steps 2 and 3, for each objective function find a lower bound and an upper bound corresponding to the set of solutions for each objective Let Z_i^{PIS} and Z_i^{NIS} denote the lower bound and upper bound for the ith objective (Z_i).

Step 5: For the objective functions Specify the linear membership function of each objective function according to (26) and (27).

Step 6: Introduce the auxiliary variable λ to transform the problem into the equivalent ordinary LP pattern. The variable λ can be interpreted as representing the overall degree of DM satisfaction with the multiple fuzzy goal values.

Step 7: Find the optimal solution vector x_i, where x_i is the efficient solution of the original multiobjective APP problem with the DM's preferences.

4. Implementation

An industrial case studies

Data description

The case study of Rich Pharmaceuticals Limited (RPL) was utilized to show how useful the suggested methodology is

RPL is one of the leading producers of pharmaceuticals in Nigeria. RPL's goods are mostly sold in Southern and Middle belt of Nigeria, some parts of West and East Africa, they have recently experienced strong demand. RPL's business APP approach is to keep a stable labor force level over the planning horizon, allowing for the flexible meeting of demand through the use of inventories, overtime, and backorders. Due to the shortcomings of the graphical method, in which evaluation comparisons are only available for specific plans under specified conditions and indication for the optimal plan is ambiguous, RPL has been unable to reach the performance initially predicted.

Alternately, the DM can use a mathematical programming technique to create an aggregate production schedule for RPL factory. Based on company reports, the planning horizon spans for six months, May to October. The model includes two types of standard products. Each period, the standard payroll is ₦64. The expenses for hiring and firing employees are ₦30 and ₦40 per employee every day, respectively. Production expenses for overtime are capped at 30% of production expenses for regular hours. Additionally, it is assumed that each product has no beginning inventory and no backorders at the last period. The inventory's maximum allowed storage area is $3000m^3$. In a day, there are two working shifts. 8 hours are allotted for regular production per shift, while 3 hours allotted for overtime production. To produce these products, 10 types of raw materials are required. Repairs are done just in shift 2 (i.e., overtime). When demand for a certain period exceeds production capacity during regular hours and inventory levels are likewise insufficient to meet this demand, production is continued during overtime.

The APP decision problem for the industrial case that is discussed here focuses on the creation of a multiple fuzzy goals programming model for figuring out the best way to meet forecasted demand by modifying output rates, hiring and firing, inventory levels, overtime and backorders. The anticipated outcomes of this APP decision include minimizing total production costs and Process time and maximizing sales.

4.1 Computational Results

As already indicated, the suggested APP is coded and run using LINGO 18 solution. The payoff matrix establishes the minimum and maximum values for objectives as in Table 4. Thus, the objectives and aspirational levels have been established as $g_1 = 1368835$; $g_2 = 10266$; $g_3 = 1760481$.

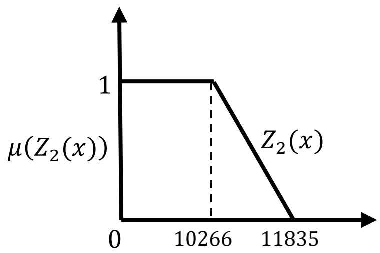
Table 4: Payoff Matrix.

		$Z_k(v_j^*)$		
Objectives	PIS			NIS
Min $Z_1(x)$	1368835	1606714	1588914	1606714
Min $Z_2(x)$	10266	10344.06	11835.14	11835.14
Max $Z_3(x)$	1760481	1650640	1638420	1638420

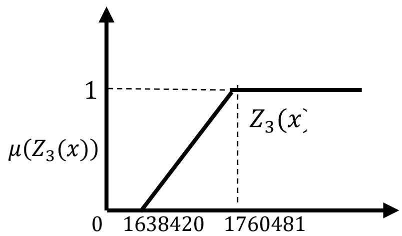
The linear membership function of each objective function is determined with its PIS and NIS as the interval of the objective values, and also to specify the equivalence of these objective values as a membership value in the interval [0, 1]. The fuzzy aspiration levels can be quantified using the linear and continuous membership function. According to Eq. 26 and 27, the relevant linear membership functions can be defined as shown below.

$$\mu(Z_1(x)) = \begin{cases} 1 & Z_1(x) \le 1368835 \\ \dfrac{1606714 - Z_1(x)}{1606714 - 1368835} & 1368835 \le Z_1(x) \le 1606714 \\ 0 & Z_1(x) \ge 1606714 \end{cases}$$

$$\mu(Z_2(x)) = \begin{cases} 1 & Z_2(x) \le 10266 \\ \dfrac{11835 - Z_2(x)}{11835 - 10266} & 10266 \le Z_2(x) \le 11835 \\ 0 & Z_2(x) \ge 11835 \end{cases}$$



$$\mu(Z_3(x)) = \begin{cases} 1 & Z_k(x) \ge 1760481 \\ \dfrac{Z_3(x) - 1638420}{1760481 - 1638420} & 1638420 \le Z_k(x) \le 1760481 \\ 0 & Z_k(x) \le 1638420 \end{cases}$$



The information in Table 5 can be used to draw the conclusion that the suggested FGP is capable of locating a high-quality compromise solution even in the face of numerous competing objective functions and constraints. As is obvious, there is a high level of satisfaction for all objective functions, and this is seen as a good Compromising solution for the problem.

Table 5: The fuzzy goal programming.

Satisfaction Level	Objective values	
μ_1 μ_2 μ_3	Z_1 Z_2 Z_3	λ
0.8809078 0.6733067 0.6733067	1397165 10778.58 1720604	0.6733067

Considering the various fuzzy goal values (Z_1, Z_2 *and* Z_3), the suggested model gives the overall levels of DM satisfaction (λ value). Each goal is fully satisfied if the answer is $\lambda = 1$. If $\lambda = 0$, none of the goals are satisfied.

If $0 < \lambda < 1$, all of the goals are satisfied at some level. For instance, the initial calculation of the overall DM satisfaction (λ) with the goal values ($Z_1 = 1397165, Z_2 = 10778.58$, and $Z_3 = 1720604$) was 0.6733067. The λ value can be adjusted to look for a set of superior compromise options if the DM did not accept the initial overall degree of this satisfaction value.

4.2 Additional Analysis

Table 6 gives the comparison and also demonstrates the interaction of trade-offs and conflicts among dependent objective functions; Minimization of cost, Maximization of Customer Satisfaction Level and Maximization of Sales Revenue have diverse meanings. For instance, the combination of the Minimization of cost and Maximization

of Customer Satisfaction in the first trade-off was $Z_1 = 1380668$ and Z_2 =10344.05 with $\lambda = 0.9502547$ giving a higher satisfaction to DM. same with second and third trade-offs. These answers show that the trade-offs and conflicts among dependent objective functions differ and interact fairly. The objective and λ values may change depending on how various arbitrary objective function combinations are combined. Since the suggested FGP model can concurrently minimize costs, maximize customer satisfaction levels, and maximize sales revenue, it satisfies the requirements of the practical application, see Table 7.

Table 6:

λ	0.9502547	0.8338611	0.6733067
Z_1	1380668	1408356	−
Z_2	10344.05	−	10778
Z_3	−	1740202	1720604

The Minimum Total cost is fuzzy due to the fuzziness of the input in the new APP. The λ values of Z, where one extreme ($= 1$) represents the most likely total cost and the other extreme ($= 0$) displays the range in which the total cost might occur, is used to represent the likelihood that the total cost of the APP will appear in the related range. The minimum total cost for the situation covered in this paper is probably 1390611; it will never be more than 1371369 or lower than 1368835. It is significant to remark that the membership function derived, as shown in Figure 1, has an approximative shape that resembles a continuous function and appears to be rather fine.

Table 7: Results of the APP.

		Product 1			
Period	**Regular time production (units)**	**Over time production (units)**	**Ending inventory (units)**	**Backorder**	**Demand**
May	234.5284	72.77206	55.30050	0	252
June	269.5261	80.85784	110.6845	0	295
July	299.4735	89.84204	0	43	430
August	230.9357	72.06427	0	26	260
September	240.2142	59.79578	174.0100	30	300
October	125.9900	0.000000	0	0	270
		Product 2			
May	183.7806	71.22794	0	0	255
June	263.8072	79.14216	48.95786	0	294
July	293.1191	87.93573	0	19.84791	430
August	279.8479	0.000	0	26	260
September	293.1191	32.88089	0	30	300
October	300.0000	0.00	0	0	270
Period	**Total hiring (persons)**	**Total Firing (persons)**	**Machine capacity**	**Warehouse space (ft^2)**	
May	0	20	90.96507	1005	
June	20	0	101.0723	1100	
July	22	0	112.3026	1800	
August	0	22	90.08033	2700	
September	0	0	90.08033	2700	
October	3	3	87.50000	3300	
Z1	1397165				
Z2	10778.58				
Z3	1720604				

The suggested solution approach allows for the simultaneous acquisition of the optimal operating plans, which are connected to the lower or upper bounds of the minimum total cost for the 11 values of λ as listed in Table 8. It is demonstrated that for any λ obtained $XH_{kt}XF_{kt} = 0$ and $B_{it}XP_{itw} = 0$ which validates the optimality of the operation plans produced by the method suggested in this study and that it is not essential to impose these two sets of constraints on the APP model. The attained results for the lower bounds of the minimum total cost show that the productivity of the production plan increases with the use of overtime production, the hiring of fewer workers, and the establishment of a lesser inventory level to satisfy the anticipated demands without exceeding the resources required for a higher level of λ. The DM should recruit more employees to increase output by utilizing regular-time production and built-up inventory, if a lower λ level is offered in order to optimize the operation plan.

The volume of output and the price per unit of a product that will maximize profits are determined by economic metrics such as the marginal cost of production and marginal revenue. The link between marginal revenue and the marginal cost of production enables rational industries to pinpoint the point at which they can generate the maximum amount of profit. Here, achieving marginal revenue equality over marginal cost is the desired outcome. This can aid the DM through Table 8 to know the extent of adjustments in the parameters of the APP. Marginal revenue increases whenever the revenue received from producing one additional unit of a good grows faster—or shrinks more slowly—than its marginal cost of production. Increasing marginal revenue is a sign that the company is producing too little relative to consumer demand, and that there are profit opportunities if production expands, as can be obtained by the difference of the Maximum Sales Revenue and Minimum Total Cost.

Table 8: The Optimal Aggregate Production Planning.

λ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
μ_1	0.9893	0.9896	0.9883	0.9847	0.9690	0.9425	0.9085	0.7740	0.8096	0.9000	1.0000
μ_2	0.9503	0.9503	0.9083	0.8587	0.8090	0.7594	0.7097	0.7000	0.8000	0.9000	1.0000
μ_3	0.0000	0.1000	0.2000	0.3000	0.4000	0.5000	0.6000	0.6196	0.4182	0.2168	0.0154
Z1	1371369	1371317	1371624	1372478	1376208	1382508	1390611	1422599	1414125	1392623	1368835
Z2	10344	10344	10410	10488	10566	10644	10721	10737	10580	10423	10266
Z3	1638420	1650626	1662832	1675038	1687244	1699450	1711657	1714043	1689462	1664881	1640300
X111	240.72	240.72	240.72	240.72	240.72	240.72	240.72	242.57	242.57	254.91	252.13
X112	267.47	267.47	267.47	267.47	267.47	267.47	267.47	269.53	269.53	283.23	280.15
X113	297.19	297.19	297.19	297.19	297.19	297.19	297.19	299.47	299.47	314.70	311.28
X114	260.01	260.01	260.01	260.01	263.33	273.32	272.52	203.18	203.18	174.90	311.28
X115	263.33	263.33	263.33	260.01	263.33	263.33	263.33	326.00	300.01	300.01	245.24
X116	100.00	100.00	121.92	125.99	125.99	125.99	125.99	0.00	0.00	12.07	100.00
X121	72.22	72.22	72.22	72.22	72.22	72.22	72.22	64.73	64.73	14.78	0.00
X122	80.24	80.24	80.24	80.24	80.24	80.24	80.24	80.86	80.86	84.97	84.04
X123	89.16	89.16	89.16	89.16	89.16	89.16	89.16	89.84	89.84	94.41	93.38
X124	0.00	0.00	0.00	0.00	0.00	4.51	30.48	99.82	99.82	94.41	3.49
X125	36.67	36.67	36.67	40.00	36.68	36.68	36.68	0.00	0.00	0.00	0.00
X126	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00	100.00	104.90	0.00
X211	239.28	239.28	239.28	239.28	239.28	239.28	239.28	183.78	183.78	225.09	227.87
X212	265.86	265.86	265.86	265.86	265.86	265.86	265.86	263.81	263.81	250.10	253.18
X213	291.06	291.06	291.06	291.06	291.06	291.06	291.06	293.12	293.12	277.89	281.32
X214	260.00	260.00	260.00	260.00	260.00	260.01	260.81	233.07	227.36	194.41	260.00
X215	270.00	270.00	270.00	273.32	270.00	270.00	270.00	241.20	224.22	242.92	288.09
X216	270.00	270.00	270.00	291.88	300.00	300.00	300.00	183.69	185.79	173.22	195.48
X221	15.73	15.73	15.73	15.73	15.73	15.73	15.73	71.23	71.23	67.53	56.29
X222	79.76	79.76	79.76	79.76	79.76	79.76	79.76	79.14	79.14	75.03	75.96
X223	87.32	87.32	87.32	87.32	87.32	87.32	87.32	87.94	87.94	83.37	84.39
X224	0.00	0.00	0.00	0.00	0.00	0.00	0.00	69.92	68.21	65.59	0.00
X225	30.00	30.00	30.00	26.69	44.54	56.00	56.00	77.69	75.79	72.88	86.43
X226	0.00	0.00	0.00	0.00	0.00	0.00	0.00	86.32	84.21	80.97	0.00
XL11	90.27	90.27	90.27	90.27	90.27	90.27	90.27	90.97	90.97	95.59	94.55
XL12	100.30	100.30	100.30	100.30	100.30	100.30	100.30	101.07	101.07	106.21	105.06
XL13	111.45	111.45	111.45	111.45	111.45	111.45	111.45	112.30	112.30	118.01	116.73
XL14	98.75	98.75	98.75	97.50	98.75	102.50	102.20	124.78	124.78	118.01	116.73
XL15	98.75	98.75	98.75	97.50	98.75	98.75	98.75	138.65	138.65	131.13	91.97
XL16	98.75	98.75	98.75	90.54	87.50	87.50	87.50	125.00	125.00	131.13	91.97
XL21	89.73	89.73	89.73	89.73	89.73	89.73	89.73	89.03	89.03	84.41	85.45
XL22	99.70	99.70	99.70	99.70	99.70	99.70	99.70	98.93	98.93	93.79	94.94
XL23	109.15	109.15	109.15	109.15	109.15	109.15	109.15	109.92	109.92	104.21	105.49
XL24	101.25	101.25	101.25	102.50	101.25	97.50	97.80	87.40	85.26	81.99	105.49
XL25	101.25	101.25	101.25	102.50	101.25	101.25	101.25	97.11	94.73	91.10	108.03
XL26	101.25	101.25	101.25	109.46	112.50	112.50	112.50	107.90	105.26	101.22	108.03
XH11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
XH12	10.03	10.03	10.03	10.03	10.03	10.03	10.03	10.11	10.11	10.62	10.51
XH13	11.14	11.14	11.14	11.14	11.14	11.14	11.14	11.23	11.23	11.80	11.67
XH14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	12.48	12.48	0.00	0.00
XH15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	13.86	13.86	13.11	0.00
XH16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
XH21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
XH22	9.97	9.97	9.97	9.97	9.97	9.97	9.97	9.89	9.89	9.38	9.49
XH23	9.45	9.45	9.45	9.45	9.45	9.45	9.45	10.99	10.99	10.42	10.55
XH24	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
XH25	0.00	0.00	0.00	0.00	0.00	3.75	3.45	9.71	9.47	9.11	2.54
XH26	0.00	0.00	0.00	6.96	11.25	11.25	11.25	10.79	10.53	10.12	0.00
XF11	9.73	9.73	9.73	9.73	9.73	9.73	9.73	9.03	9.03	4.41	5.45
XF12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
XF13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

XF14	12.70	12.70	12.70	13.94	12.70	8.95	9.25	0.00	0.00	0.00	0.00
XF15	0.00	0.00	0.00	0.00	0.00	3.75	3.45	0.00	0.00	0.00	24.76
XF16	0.00	0.00	0.00	6.96	11.25	11.25	11.25	13.65	13.65	0.00	0.00
XF21	10.27	10.27	10.27	10.27	10.27	10.27	10.27	10.97	10.97	15.59	14.55
XF22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
XF23	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
XF24	7.90	7.90	7.90	6.65	7.90	11.65	11.35	22.52	24.66	22.22	0.00
XF25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
XF26	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
XP11	60.94	60.94	60.94	60.94	60.94	60.94	60.94	55.30	55.30	17.69	0.13
XP12	113.65	113.65	113.65	113.65	113.65	113.65	113.65	110.68	110.68	90.89	69.33
XP13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
XP14	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	54.76
XP15	200.01	174.04	174.01	174.01	174.01	174.01	174.01	200.00	174.01	174.01	174.01
XP16	30.01	4.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
XP21	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	37.62	29.16
XP22	51.63	51.63	51.63	51.63	51.63	51.63	51.63	48.96	48.96	68.75	64.30
XP23	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.01	0.01	0.01	0.01
XP24	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01
XP25	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.01	15.81	74.53
XP26	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
B11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
B12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-26.01
B13	0.00	0.00	0.00	0.01	3.33	17.83	43.00	43.00	43.00	9.31	0.00
B14	0.00	25.97	26.00	26.00	26.00	26.00	26.00	26.00	26.00	26.00	26.00
B15	0.00	0.00	25.93	30.00	30.00	30.00	30.00	30.00	4.01	20.98	4.01
B16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
B21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
B22	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00
B23	0.00	0.00	0.00	0.01	0.01	0.01	0.81	43.00	35.58	0.00	0.00
B24	0.00	0.00	0.00	0.01	14.54	26.00	26.00	18.89	0.00	0.00	0.00
B25	0.00	0.00	0.01	21.88	30.00	30.00	30.00	0.01	0.01	0.01	0.01
B26	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

5. Conclusion

Fuzzy APP problem have drawn the interest of many scholars over the past few decades, inspiring the development of a variety of models. According to the cost-benefit analysis, a company should continue to increase production until marginal revenue (MR) is equal to marginal cost (MC). The FGP-APP model is transformed into a family of crisp APP models that are described by a single mathematical program. Using this method, users can determine the membership function of the fuzzy multi-objectives of the APP problem linked to the fuzzy parameters. Lingo software was used to resolve the developed FGP model for the APP problem. The result revealed RPL's optimal values for production during regular and overtime shifts, backorders, inventories, hiring and firing of staff during shifts, product prices, etc. across the planning horizon.

The DM can choose a preferred production plan with a common satisfaction level or different combinations of possibility and satisfaction levels using the suggested models and approaches, depending on the market demands and available production capacities that meets their top priorities. Also, the DM in APP can use the structure of the optimal solution shown in Table 7.

It has also become evident that a linguistic presentation is more useful and appropriate for illustrating the APP's imprecise parameters. DM may choose to ignore the loss of fuzziness from the input information when the returns are clear values, which would encourage management based on unjustifiably optimistic choices. The proposed method, on the other hand, was shown in this research to be totally capable of preserving the fuzziness that was connected to the ambiguous APP data, particularly by expressing the objective value with membership functions rather than with crispy values. As a result, when compared to prior studies, the suggested approach is able to produce more logical solutions for imprecise parameters, providing the DM with information that other approaches are unable to. For instance, If the optimal output is where the MR is equal to MC, any other cost is irrelevant. So the analysis also tells DM what not to consider when making decisions about future resource allocation. This APP strategy could not ensure an overall optimal solution, but the result is near to optimal, and it can significantly improve the likelihood that an aggregate plan can have feasible family disaggregation plans under various conditions, particularly in the presence of uncertain demands. Future studies will take into account the advancement of multiobjective FGP decision-making techniques considering loss of productivity, quality of products etc.

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