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Analysis of Networks and Network Equilibrium Equation by Applying Graph theory

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Abstract
Here we describe a general technique detecting structural features in large scale network data that works by dividing the nodes of network, by making use of few fundamental laws of networks and graph theory. We give the examples demonstrating how the system of equations can be solved by making use of graph theory. Also to write the tie-set matrix and obtain the network equilibrium equation in matrix forms using KVL. Calculating the loop current and then calculating branch voltage is also discussed with an example.

Keywords: Network, directed graph, trees, incidence matrix, KCL, KVL

Introduction:
An electrical network is an interconnection of electrical network elements such as resistance, capacitances, inductances, voltage and current source. The closeness of the link between network analysis and graph theory is widely recognized, but the nature of the link is seldom discussed. Graph theory, like all other branches of mathematics, consists of a set of interconnected tautologies. Each network is associated with two variables, the voltage variable and current variable. Networks are widely used in the biological, physical and social representation of topology of systems of interacting components. The network analysis is a method to analyze, control the process and work flow.

Analysis of networks
The object is to set up a system of equations from which the independent loop currents can be determined.ie. Once this set of current is known, the currents in all the branches can be calculated, for this we have to assign arbitrarily, to each of the branches of positive direction for current flow. The direction assigned to each branch of cotree determines a positive sense of current circulation for the corresponding loop. We number the loops from 1 to l and branches from 1 to b and define matrix B of order $l*b$ having elements b_{hk} as
b_{hk} = +1, if the positive sense of current flows in branch k coincides with that of loop h.
= -1, if the positive sense of current flows in branch k coincides with that of loop h.
=0, if branch k is not associated with loop h.
The matrix B is known as the tie-set matrix. Row h of B contains non-zero entries only in places whose column numbers corresponds to branches forming part of loop h. Row h will tell us exactly which branches form loop h.

Kirchhoff's Voltage Law (KVL): Kirchhoff's voltage law states that if V_k is the potential drop in the K^{th} branch, then $\sum V_k = 0$.

For the loop h is

$$\sum_{k=1}^{b} b_{hk}\, V_k = 0, \quad (h = 1,2,3.....l).$$

Set of l equations can be written in the matrix form

$B\left[V_b\right]=0$-------------- (1)

Where $V_b=(v_1,v_2,.......v_b)^T$ and B is the fundamental circuit matrix.

$v_k=Z_k i_k - v_{sk}$ Where v_{sk} is the branch input voltage source?

i_k : The current

Z_k : The impedance of k^{th} branch.

$$\begin{bmatrix} v_1 \\ v_2 \\ . \\ . \\ v_b \end{bmatrix} = \begin{bmatrix} z_1 & 0 & ..... & . & 0 \\ 0 & z_2 & .... & . & 0 \\ 0 & ..... & z_3 & ..... & 0 \\ ... & .... & .... & .... & ... \\ 0 & 0 & ..... & .... & z_b \end{bmatrix} . \begin{bmatrix} i_1 \\ i_2 \\ .. \\ i_b \end{bmatrix} - \begin{bmatrix} v_{s1} \\ v_{s2} \\ . \\ . \\ v_{sb} \end{bmatrix}$$

The matrix $\left[Z_b\right]$ is diagonal.

If the circuit contains inductances so that is coupling between branches this matrix is no long we diagonal.

If Z_{kk} is the self-impedance of branch k, and $Z_{kh}=Z_{hk}$ is the mutual impedance of branches h and k then,

$$v_k=\sum_{n=1}^{b} Z_{kh}\; i_k - v_{sk}\;;\;\;(k=1,2,....b)$$

$$\begin{bmatrix} v_1 \\ v_2 \\ . \\ . \\ v_b \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & .... & ... & z_{1b} \\ z_{21} & z_{22} & ... & ... & z_{2b} \\ ... & ... & ... & ... & ... \\ ... & ... & ... & ... & ... \\ z_{b1} & z_{b2} & ... & ... & z_{bb} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ .. \\ .. \\ i_b \end{bmatrix} \begin{bmatrix} v_{s1} \\ v_{s2} \\ .. \\ .. \\ v_{sb} \end{bmatrix}$$

This must be written as

$\left[V_b\right]=\left[Z_b\right]\left[I_b\right]-\left[V_s\right]$----------------- (2)

Impedance matrix $\left[Z_b\right]$ is symmetric.

From (1) we get

$\left[B\right]\left[V_b\right]=\left[0\right]=\left[B\right]\left[Z_b\right]\left[I_b\right]-\left[B\right]\left[V_s\right]$

Let the loop current vector I_L be written as

$\left[I_L\right]=\left[I_1,I_2,I_3.......I_k\right]^T$

The current flowing through a branch common to two loops is either the sum or the difference of the two loop currents (with sign convention)

$k-th$ Branch current is

$i_k=b_{1k}\;I_1+b_{2k}\;I_2+........+b_{lk}\;I_l\;\;(k=1,2,3...b)$

This set of b equations can be rewritten in matrix form as,

$\left[I_b\right]=\left[B^T\right]\left[I_L\right]$------------------------ (4)

I_L : Loop current vector

B^T : Transpose of B

The branch current vector

$\left[I_b\right]=\left[i_1,i_2,i_3,......i_b\right]^T$

Let $E_1,E_2\;,E_3.......E_l$ be the loop e.m.f.

Then

$$E_h=\sum_{k=1}^{b} b_{hk} v_{sk}\;\;;(h=1,2,3....l)$$

In the matrix form

$E=BV_s$------------------ (5)

$E^T=\left[E_1,E_2.......E_l\right]$

From (2), (4) and (5)

$\left[E\right]=\left[B\right]\left[Z_b\right]\left[B\right]^T\left[I_L\right]$-------------------- (6)

$\left[B\right]\left[Z_b\right]\left[B\right]^T$ is a square matrix of order $l\times l$. Denoting this Z,

$\left[E\right]=\left[Z\right]\left[I\right]$............................ (7)

$\left[Z\right]$ is clearly a symmetric matrix

Since $Z_{hk}=Z_{kh}$,

It is known as loop impedance matrix.

Equation (7) is Kirchhoff's Voltage Law (KVL)

Kirchhoff's Current Law: Here we discuss the KCL equation of the network having only current source as input.

KCL states that if, i_k is the current in the k^{th} branch, at given node

$\sum i_k=0$ ....

$$\sum_{k=1}^{b} ahk\,i_k=0\;\;;\;h=1,2,3.....n \text{ --------------- (2)}$$

Set of, n equations can be written in matrix form

$A_a I_b=0$----------------- (3); $I_b=\left[i_1,i_2......i_b\right]^T$

A_a Is complete incidence matrix

$A\,I_b=0$------------------------ (4)

Where A is incidence matrix of order $(n-1)\times b$.

For f cut set matrix,

$QI_b=0$−−−−−−−−−−−−−(5)

If i_{sk} is the input current source and y_k the admittance of the k^{th} branch, then

$i_k=y_k v_k - i_{sk}\;;k=1,2,.....b$−−−−−−−−−−(6)

In matrix form

$I_b=Y_b V_b - I_s$−−−−−−−−−−−(7)

The matrix Y_b is diagonal.

The circuit contains inductances so that there is coupling between branches then Y_b is no longer diagonal. is the self-admittance of branch and $y_{hk}=y_{kh}$ is the mutual admittance of branches h and k.

The column of incidence matrix will give the branch voltages in terms of node voltages.

$V_b=A^T V_n$−−−−−−(8)

Same results holds for $f-$cut set matrix also.

$V_b=Q^T V_n$−−−−−−−−−−−−−−(9)

From (7) and (8)

$$I_b = Y_b V_b - I_{sc}$$

$$I_b = Y_b A^T V_n - I_s \text{ ------ } (10)$$

From (4)

$$AI_b = 0$$

From (10)

$$A\left[Y_b A^T V_n - I_s\right] = 0$$

Or $YV = I$ ---------- (11)

The admittance matrix

$Y = AY_b A^T$ ------------ (12) is of order

Symmetric matrix and

$$I = AI_s \text{ ------------- } (13)$$

Here equation (11) is KCL

Network Equilibrium Equation

The graph of planar network is drawn by keeping all points of intersection of two or more branches i.e. nodes, and representing the network elements by lines, voltage and current sources by then internal impedances. The internal impedance of an ideal voltage source is zero and is to be replaced by short circuit, and that of an ideal current source is infinite and hence to be replaced by an open circuit.

Example.

Draw the graph for the given circuit and indicate the number of all possible trees of the network.

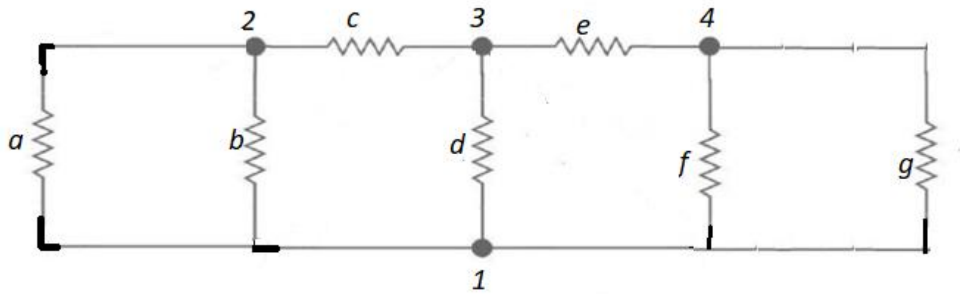


Fig. 1: represents network,

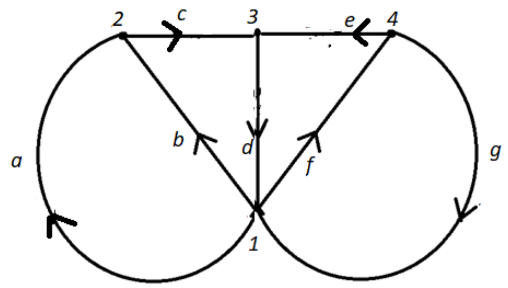
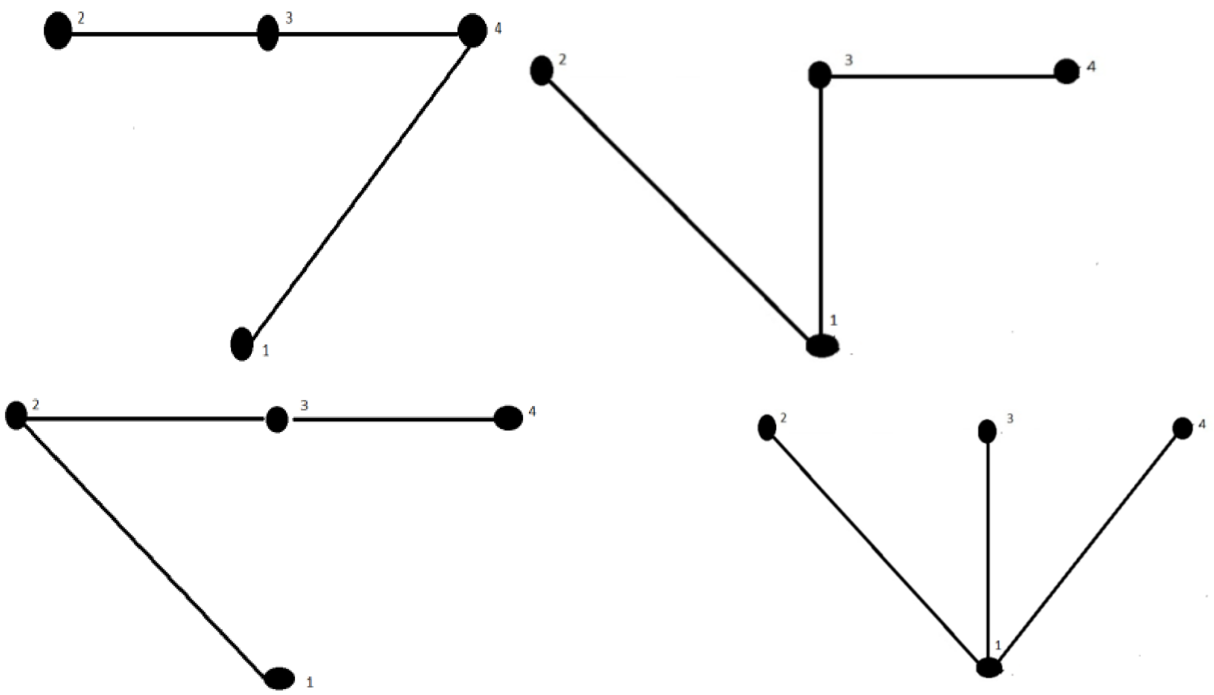
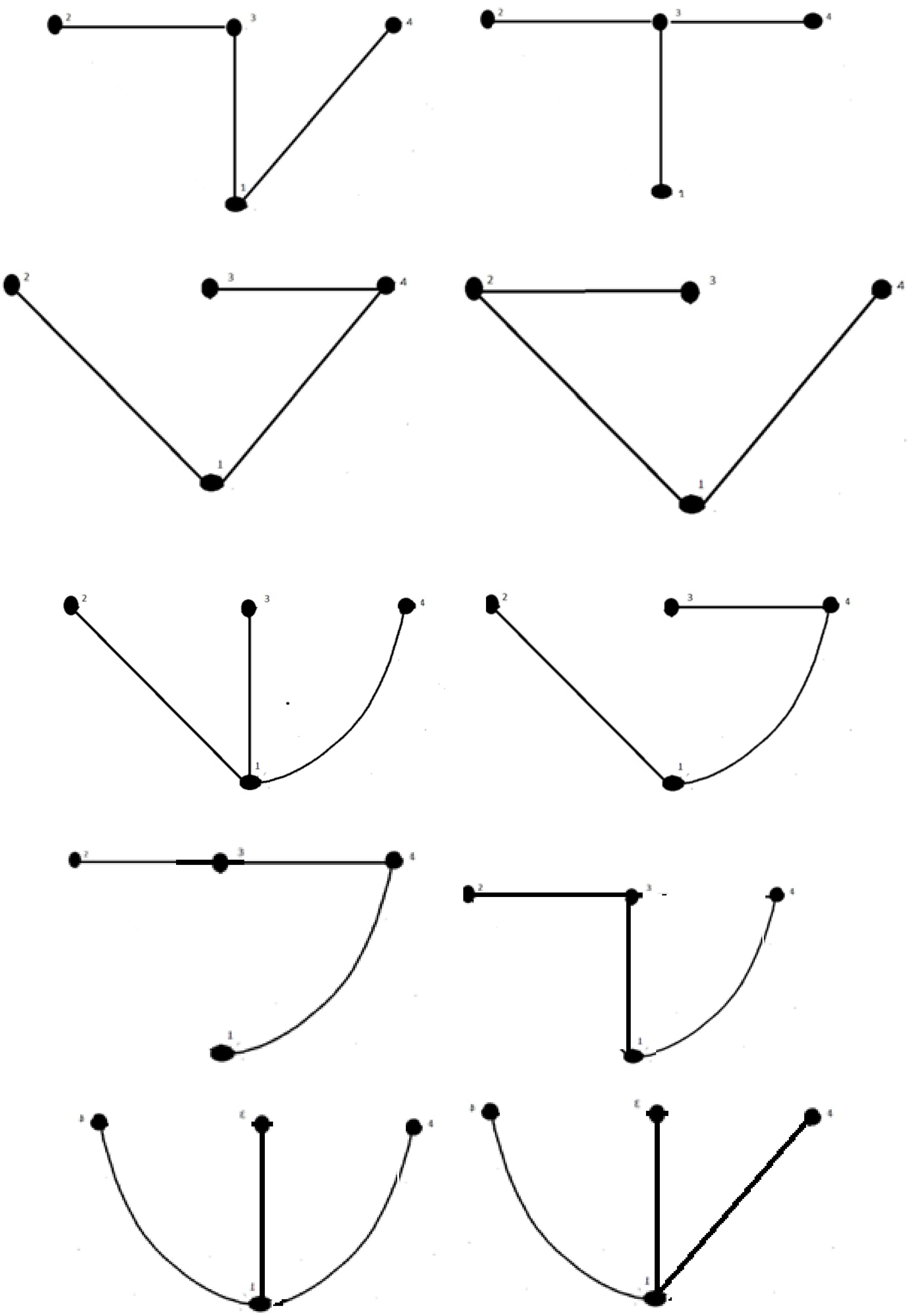


Fig. 2: Represents directed graph.





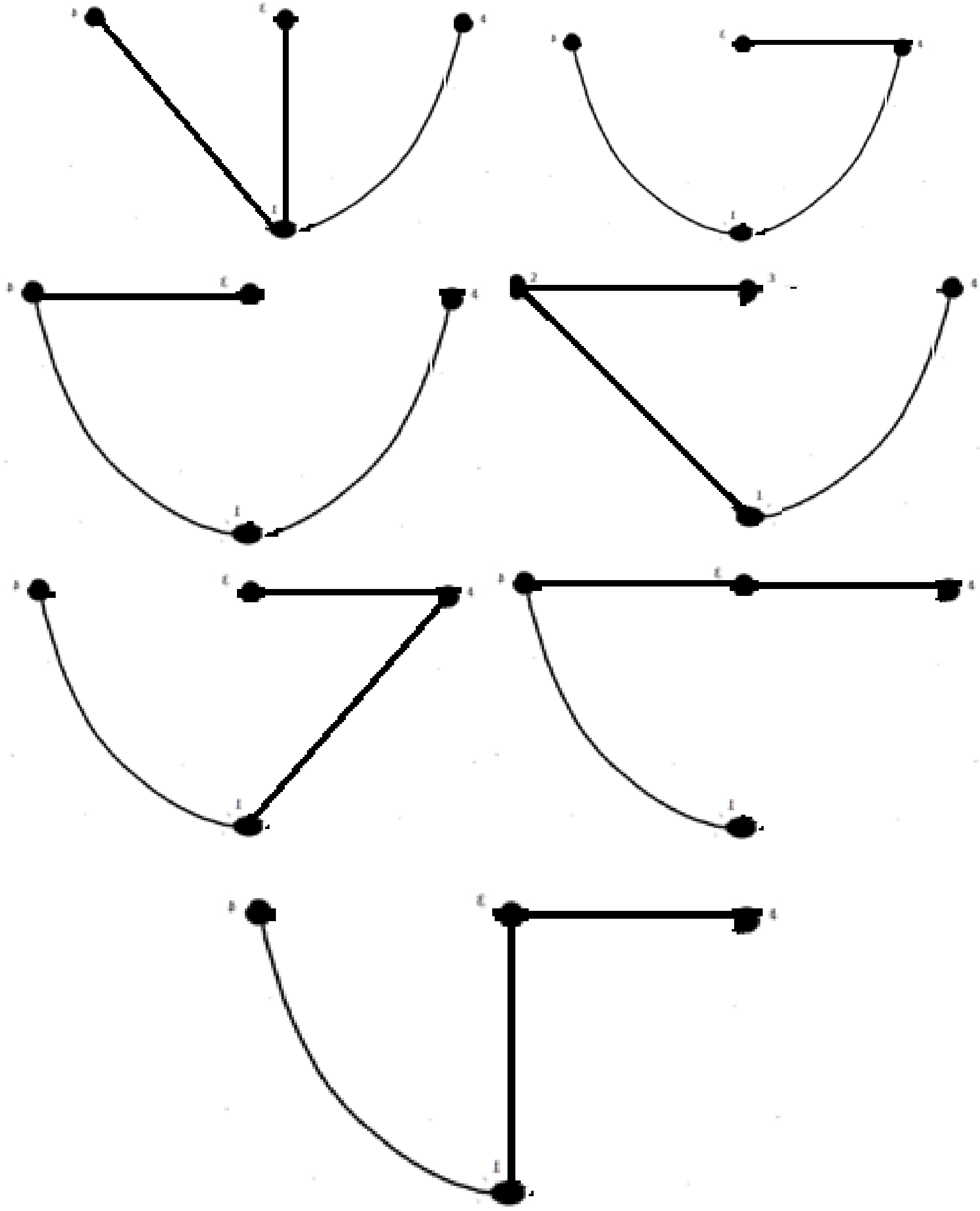


Fig. 3: Represents all possible trees.

Here

i) Current source I is replaced by open circuit.

ii) Voltage source V is replaced by short circuit.

iii) All other lines containing linear elements will be shown by lines in the graph.

iv) The graph is shown in fig 2,.It has branches $b=7$,nodes=4 therefore branches (twigs) will be $(n-1)=(4-1)=3$ and links, $(b-n+1)=(7-4+1)=4$

The complete incidence matrix is written as:

Nodes	**Branches** $\rightarrow$
	(1) (2) (3) (4) (5) (6) (7)

$$A_a=\begin{bmatrix}-1 & -1 & 0 & 1 & 0 & -1 & 1\\ 1 & 1 & -1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & -1 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & -1 & 1 & -1\end{bmatrix}$$

The reduced incidence matrix is

$$A=\begin{bmatrix}-1 & -1 & 0 & 1 & 0 & -1 & 1\\ 1 & 1 & -1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & -1 & 1 & 0 & 0\end{bmatrix}$$

$$[A]^T=\begin{bmatrix}-1 & 1 & 0\\ -1 & 1 & 0\\ 0 & -1 & 1\\ 1 & 0 & -1\\ 0 & 0 & 1\\ -1 & 0 & 0\\ 1 & 0 & 0\end{bmatrix}$$

The number of all possible trees

$$=\det\left\{[A][A]^T\right\}$$

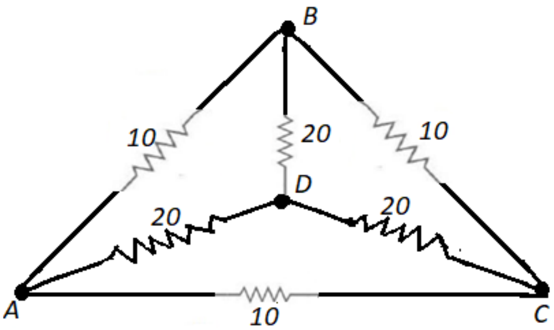
$$=\det\begin{bmatrix}5 & -2 & -1\\ -2 & 3 & -1\\ -1 & -1 & 3\end{bmatrix}=21$$

The twenty one possible trees are shown in fig 3.

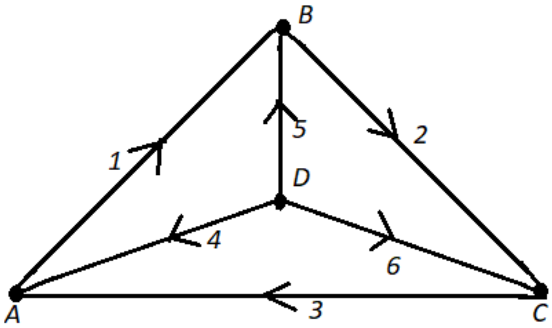
To calculate loop current and branch voltage

For the following network

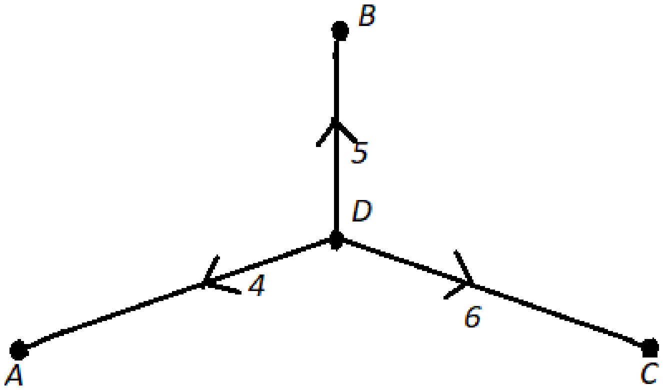
We will write the tie-set matrix and obtain the network equilibrium equation in matrix form using KVL. Then will calculate the loop current and hence branch voltage.



(a): Network



(b): Oriented graph with arbitrary direction



(c): A tree

The oriented graph and one of the possible tree is shown in fig (b) and (c), respectively.

The tree is formed with the set of twigs {4, 5, and 6} and the links are {1, 2, and 3}.

Number of branches = 6

Number of nodes = 4

Number of links = 3

The tie-set matrix can be written as:

Loop current	Branches					
	1	2	3	4	5	6
I_1	1	0	0	1	-1	0
I_2	0	1	0	0	1	-1
I_3	0	0	1	-1	0	1

Now,the equilibrium equation in matrix form,from loop current basis,using KVL,is given by

$$[B]\,[Z_b]\,[B^T]\,[I_L] = [B]\,[V_s]-[B]\,[Z_b]\,[I_s]$$

Where, $V_s =$ branch input voltage source sector

$I_s =$ branch input current source vector

Here,

$$[B]=\begin{bmatrix}1 & 0 & 0 & 1 & -1 & 0\\ 0 & 1 & 0 & 0 & 1 & -1\\ 0 & 0 & 1 & -1 & 0 & 1\end{bmatrix}$$

$$B^T = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & -1\\ -1 & 1 & 0\\ 0 & -1 & 1\end{bmatrix}$$

$$Z_b = \begin{bmatrix}10 & & & & & 0\\ & 10 & & & & \\ & & 10 & & & \\ & & & 20 & & \\ & & & & 20 & \\ 0 & & & & & 20\end{bmatrix} \quad I_L = \begin{bmatrix}I_1\\ I_2\\ I_3\end{bmatrix}$$

$$V_s = \begin{bmatrix}1\\0\\0\\0\\0\\0\end{bmatrix} \quad I_s = \begin{bmatrix}0\\0\\0\\0\\0\\0\end{bmatrix}$$

Therefore, $[B]\,[Z_b]\,[B^T]\,[I_L] = [B]\,[V_s]$

$$\begin{bmatrix}100 & -20 & -20\\ -20 & 100 & -20\\ -20 & -20 & 100\end{bmatrix}\begin{bmatrix}I_1\\ I_2\\ I_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$I_1 = \frac{\Delta_1}{\Delta}$, $I_2 = \frac{\Delta_2}{\Delta}$, $I_3 = \frac{\Delta_3}{\Delta}$ on solving gives

$$I_1 = \frac{1}{90}, I_2 = \frac{1}{360}, I_3 = \frac{1}{360}$$

The branch current is given by

$$[I_b] = [B^T][I_L]$$

The branch voltages are

$$[V_b] = [Z_b]\ [I_b]\ - [V_s]$$

$$[V_b] = [Z_b]\ [B^T][I_L] - [V_s]$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = \begin{bmatrix} 10 & & & & & 0 \\ & 10 & & & & \\ & & 10 & & & \\ & & & 20 & & \\ & & & & 20 & \\ 0 & & & & & 20 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{90} \\ \frac{1}{360} \\ \frac{1}{360} \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

On calculation it gives

$$v_1 = \frac{-8}{9}V, v_2 = \frac{1}{18}V, v_3 = \frac{1}{18}V, v_4 = \frac{1}{9}V, v_5 = \frac{1}{9}V, v_6 = \frac{-1}{9}V$$

Conclusion

In this paper we discussed analysis of networks, network equilibrium to determine the centrality of particular nodes and to detect community sub graph structure within the network. Also, to obtain network equilibrium equation and then calculate the loop current and branch voltage. This concept can be used in analysing structure of network in which the use of both graphical and statistical procedure is used.

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