World Wide Journal of Multidisciplinary Research and Development

WWJMRD 2018; 4(2): 80-85
www.wwjmrd.com
International Journal
Peer Reviewed Journal
Refereed Journal
Indexed Journal
UGC Approved Journal
Impact Factor MJIF: 4.25
e-ISSN: 2454-6615

Akhmedov M.Sh.
Department, Bukhara Technological Institute of Engineering, Bukhara, Republic of Uzbekistan.

Shodiyev Z.O.
Department, Bukhara Technological Institute of Engineering, Bukhara, Republic of Uzbekistan.

Atoyev A.B.
Department, Bukhara Technological Institute of Engineering, Bukhara, Republic of Uzbekistan.

Correspondence:
Akhmedov M.Sh.
Department, Bukhara Technological Institute of Engineering, Bukhara, Republic of Uzbekistan.

Distributed Own Waves in Dissipative Inhomogeneous Flat Bodies

Akhmedov M.Sh., Shodiyev Z.O., Atoyev A.B.

Abstract
The paper deals with the propagation of natural waves in dissipative inhomogeneous planar bodies. Wave motions are described by linear Integra-differential equations. Solving this problem, we obtain a relationship between the wave velocity and its length. The task of this kind is of great interest for geophysicists, in the field of engineering and construction.

Keywords: layer, visco elastic half-space, layer, equation, phase velocity.

Introduction

Many construction and engineering structures that operate under dynamic conditions consist of deformable bodies with different visco elastic properties [1,2,3]. In addition, wave processes in elastic bodies play an important role in connection with the tasks of signal processing, in particular in connection with the creation of mechanical resonators and filters [4,5,6]. The mechanisms by which the energy of elastic waves is converted into heat are not entirely clear. Various loss mechanisms are proposed [7 ± 11], but not one of them does not fully meet all the requirements. Probably the most important mechanisms are internal friction in the form of sliding friction (or sticking, and then slipping) and viscous losses in pore fluids; the latter mechanism is most significant in strongly permeable rocks. Other effects that are probably generally less significant are the loss of some of the heat generated in the phase of compression of wave motion by thermal conductivity, piezoelectric and thermoelectric effects and the energy going to the formation of new surfaces (which plays an important role only near the source). Therefore, the development of a unified methodology and algorithm for calculating the wave fields of dissipative inhomogeneous layered bodies is an actual problem of the mechanics of a deformable solid [12, 13].

Formulation of the problem.

Suppose that in a Cartesian (x, y, z) coordinate system, with the origin and the OZ axis, a sequence of parallel planes is given (Fig. 1)

$$z = 0,\ z = h_1,\ z = h_1 + h_2, \ldots\ldots,\ z = h_1 + h_2 + \ldots + h_n$$

The plane, $z = h_1 + h_2 + \ldots + h_n$ (for n = 2) will be called the nth horizon. Suppose that the spaces between the planes mentioned are filled with isotropic elastic media forming parallel layers. Layers $0 < z < h$, characterized by permanent $\lambda_0, \mu_0, \rho_{0,}$, will be called zero. Wednesday, however, $h_1 + h_2 + \ldots + h_n < z < h_1 + h_2 + \ldots + h_n + h_{n+1}$ filling the space between the *n-m* and *n+1–m* horizons, characterized by constant $\lambda_1, \mu_1, \rho_{1,}$, will be called the *n-m* layer. It will always be assumed that adjacent layers differ from each other in at least one of the constants λ, μ and ρ. In the theoretical study of the described processes, we shall assume that within each layer the wave propagation is described by the usual equations of the theory of elasticity. As for the conditions on the interfaces of adjacent layer, we assume that the components of the vector of elastic displacements and the stress tensor remain continuous when passing through them. This contact is called hard [15]. The dynamics of dissipative inhomogeneous two-layer flat structures is investigated in the article.

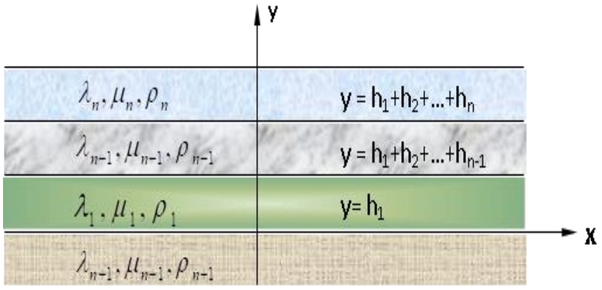


Fig. 1: The design scheme: the body on the half-space

Accounting for internal friction, caused by the dissipation of energy in the material of structures, is a more difficult task. Soft layers of multilayer structures (aggregates), as a rule, are made of materials with developed rheological properties. Therefore, the scattering of energy must first of all be taken into account for soft layers, since it mainly occurs precisely when deforming these layers. Mechanical systems, for which the visco elastic properties of their elements are identical, will be called dissipative homogeneous, systems with different rheological characteristics are dissipative heterogeneous [1,8,9]. Equations of motion of the deformed layer in the absence of mass forces have the form [1]

$$\tilde{\mu}_j \nabla^2 \vec{u} + (\tilde{\lambda}_j + \tilde{\mu}_j)\, grad\ di\vartheta\, \vec{u} = \rho_j \frac{\partial^2 \vec{u}}{\partial t^2}, \text{ (j =1, 2, 3..) (1)}$$

Here $\vec{u}(u_x, u_y, u_z)$ - vector of displacement of points of the medium; ρ_j - material density; u_i – displacement components; v_j - Poisson's ratio;

$$\tilde{\lambda}_j = \frac{v_j \tilde{E}_j}{(1+v_j)(1-2v_j)}; \quad \tilde{\mu}_j = \frac{v_j \tilde{E}_j}{2(1+v_j)}, \text{ где}$$

$\tilde{E}$ – Operational modulus of elasticity, which have the form [9,13]:

$$\tilde{E}_j \varphi(t) = E_{0j}\left[\varphi(t) - \int_0^t R_{Ej}(t-\tau)\varphi(t)d\tau\right] \qquad (2)$$

$\varphi(t)$ – arbitrary time function; $R_{Ej}(t-\tau)$ – relaxation core; E_{01} – instantaneous modulus of elasticity; We assume the integral terms in (2) to be small, then the functions $\varphi(t) = \psi(t)e^{-i\omega_R t}$, where $\psi(t)$ - slowly varying function of time, ω_R - real constant. Further, applying the freezing procedure [9], we note relations (2) with approximations of the form

$$\bar{E}_j \varphi = E_{0j}\left[1 - \Gamma_j^C(\omega_R) - i\Gamma_j^S(\omega_R)\right]\varphi,$$

Where $\Gamma_j^C(\omega_R) = \int_0^\infty R_j(\tau)\cos\omega_R\tau\, d\tau$,

$\Gamma_j^S(\omega_R) = \int_0^\infty R_j(\tau)\sin\omega_R\tau\, d\tau$, respectively, the cosine and sine Fourier images of the relaxation core of the material. As an example of a visco elastic material, we take three parametric relaxation nuclei $R_j(t) = A_j e^{-\beta_j t}/t^{1-\alpha_j}$.

On the influence function $R_j(t-\tau)$ the usual requirements of inerrability, continuity (except for $t = \tau$), signs - certainty and monotony:

$$R > 0, \qquad \frac{dR(t)}{dt} \le 0, \quad 0 < \int_0^\infty R(t)dt < 1.$$

$\vec{u}$ - Vector of displacements of the environment of the j-th layer. On the boundary of two bodies, we can specify two types of conditions:

- In the case of a rigid contact at the interface, the condition of continuity of the corresponding components of the stress tensor and displacement vector is set, i.e.

$$\sigma_{yy}^{(1)} = \sigma_{yy}^{(2)}; \quad \sigma_{xy}^{(1)} = \sigma_{xy}^{(2)}; \qquad \text{(3a)}$$
$$u_x^{(1)} = u_x^{(2)}; \quad u_y^{(1)} = u_y^{(2)}.$$

If there is no friction at the interface,

$$\sigma_{yy}^{(1)} = \sigma_{yy}^{(2)}; \quad \sigma_{xy}^{(1)} = \sigma_{xy}^{(2)} = 0; \quad u_y^{(1)} = u_y^{(2)}; \text{ (2,б)}$$

- On the free surface, the condition of freedom from stress is set, i.e.

$$\sigma_{yy}^{(1)} = 0; \quad \sigma_{yx}^{(1)} = 0, \text{ (2,c)}$$

where

$$\sigma_{xx}^{(j)} = \lambda_j\theta_j + 2\mu_j\frac{\partial u_j}{\partial x}; \quad \sigma_{xy}^{(j)} = \mu_j\left(\frac{\partial u_j}{\partial y} + \frac{\partial \vartheta_j}{\partial x}\right).$$

$$\sigma_{yy}^{(j)} = \lambda_j\theta_j + 2\mu_j\frac{\partial \vartheta_j}{\partial y} \quad \theta_j = \frac{\partial u_j}{\partial x} + \frac{\partial \vartheta_j}{\partial y}.$$

Methods of solution

Now consider the solution of the differential equation (1) - (2) for one layer. The equation of motion in displacements reduces to the following form:

$$\begin{aligned}
&\bar{\mu}_n\left(\frac{\partial^2 u_n}{\partial x^2} + \frac{\partial u^2{}_n}{\partial y^2}\right) + (\bar{\lambda}_n + \bar{\mu}_n)\frac{\partial}{\partial x}\left(\frac{\partial\ u_n}{\partial x} + \frac{\partial \vartheta_n}{\partial y}\right) - \rho_n\frac{\partial^2 u_n}{\partial t^2} = 0; \\
&\bar{\mu}_n\left(\frac{\partial^2 \vartheta_n}{\partial x^2} + \frac{\partial \vartheta^2{}_n}{\partial y^2}\right) + (\bar{\lambda}_n + \bar{\mu}_n)\frac{\partial}{\partial y}\left(\frac{\partial\ u_n}{\partial x} + \frac{\partial \vartheta_n}{\partial y}\right) - \rho_n\frac{\partial^2 \vartheta_n}{\partial t^2} = 0;
\end{aligned} \qquad (3)$$

where ρ_n - density of the material. We find the solution of the problem in the form:

$$u_n = U_n(y)e^{k(x-ct)}; \quad \vartheta_n = V_n(y)e^{k(x-ct)}; \; n = 1,2,\ldots, N \qquad (4)$$

where $U_n(y)$ and $V_n(y)$ Is the amplitude complex vector-function; k – wave number; $C = C_R + iC_I$, – complex phase velocity; a ω – Complex frequency.
To clarify their physical meaning, consider two cases:

- $k = k_R$; $C = C_R + iC_I$, then the solution (4) has the form of a sinusoid with respect to *x*, whose amplitude decays in time;
- $k = k_R + ik_I$; $C = C_R$, Then at each point x the oscillations are steady, but with respect to x they decay.

In both cases, the imaginary parts k_I or C_I, characterize the intensity of dissipative processes. Substituting (4) into (3), we obtain:

$$\bar{\mu}_n\left(U_n'' - k^2 U_n\right) + \left(\bar{\lambda}_n + \bar{\mu}_n\right) ik\left(ikU_n + V_n'\right) + \rho_n k^2 C^2 U_n = 0; \qquad (5)$$

$$\bar{\mu}_n\left(U_n'' - k^2 U_n\right) + \left(\bar{\lambda}_n + \bar{\mu}_n\right) ik\left(ikU_n + V_n'\right) + \rho_n k^2 C^2 U_n = 0.$$

Thus, we have equations (5) of the second order for two domains each. We solve the problem directly, without reducing the equation to a fourth-order equation. All the arguments are given for the layer. We find the particular solution of system (5) in the form

$$\begin{pmatrix} U_n \\ V_n \end{pmatrix} = \begin{pmatrix} A_n \\ B_n \end{pmatrix} e^{r_n y},$$

Where r_n – constant. A homogeneous algebraic system with respect to A_n and B_n has non-trivial solutions if its determinant is zero

$$\begin{vmatrix} \bar{C}_{Tn}^2(r_n^2 - k^2) - (\bar{C}_{Ln}^2 - \bar{C}_{Tn}^2)k^2 + C^2k^2 & i(\bar{C}_{Ln}^2 - \bar{C}_{Tn}^2)kr_n \\ i(\bar{C}_{Ln}^2 - \bar{C}_{TLn}^2)kr_n & \bar{C}_{TLn}^2(r_n^2 - k^2) + (\bar{C}_{Ln}^2 - \bar{C}_{Tn}^2)r_n^2 + C^2k^2 \end{vmatrix} = 0, \qquad (6)$$

$\bar{C}_{Ln}^2 = (\bar{\lambda}_n + 2\bar{\mu}_n)/\rho_n$. $\bar{C}_{Tn}^2 = \bar{\mu}_n/\rho_n$. at η = 0

величины $\bar{C}_{Ln}^2$ *and* $\bar{C}_{Tn}^2$ are respectively the velocity of the waves of compression and shear in an elastic medium [4]. Equation (6) can have four roots

$$(r_n)_{1,3} = \pm k\sqrt{1 - C^2/\bar{C}_{Ln}^2}; (r_n)_{3,4} = \pm k\sqrt{1 - C^2/\bar{C}_{Tn}^2}; n = 0,1; \quad i = 1\ldots\ldots,4.$$

As a result, we find four particular solutions of the form

$$\begin{pmatrix} U_n \\ V_n \end{pmatrix} = \sum_{i=1}^{4} C_{ni} \begin{pmatrix} A_{ni} \\ B_{ni} \end{pmatrix} e^{(r_n)_i y}, \quad n = 0,1. \qquad (7)$$

Подставив значения $(r_n)_i$ in (7) we find A_{ni}, B_{ni} at $r_n = (r_n)_i$. The expressions for the displacement are:

$$\begin{pmatrix} U_1 \\ V_1 \end{pmatrix} = C_{11}\begin{pmatrix} ik \\ -k\bar{q}_1 \end{pmatrix} e^{-k\bar{q}_1 y} + C_{12}\begin{pmatrix} ik \\ k\bar{q}_1 \end{pmatrix} e^{k\bar{q}_1 y} + C_{13}\begin{pmatrix} -k\bar{S}_1 \\ -ik \end{pmatrix} e^{-ks_{11}y} + C_{14}\begin{pmatrix} -k\bar{S}_1 \\ -ik \end{pmatrix} e^{+ks_{11}y};$$

$$\begin{pmatrix} U \\ V \end{pmatrix} = C_{22}\begin{pmatrix} ik \\ -k\bar{q}_1 \end{pmatrix} e^{k\bar{q}_1 y} + C_{24}\begin{pmatrix} k\bar{s} \\ -ik \end{pmatrix} e^{k\bar{s}y}.$$

Consequently, for both the hard and sliding contacts, we obtain a set of six boundary conditions that lead to six homogeneous equations with six unknowns $C_{11}, C_{12}, C_{13}, C_{14}, C_{22}, C_{24}$. For such a system of equations to have non-trivial solutions, the determinant of the coefficients must be zero. The last equation gives the dispersion equation for dissipative systems, where $\bar{S}_n = (1 - C^2/\bar{C}^2{}_n)^{1/2}; \bar{q}_n = (1 - C^2/\bar{C}^2{}_{Ln}), n = 0,1$

As an example, let us consider the problem of propagation of natural waves in a viscoelastic layer on a half-space.

Hard contact

The dispersion equation has the following form

$$\begin{vmatrix} (1+\bar{S}_1^2)\, e^{-\xi\bar{q}_1} & (1+\bar{S}_1^2)\, e^{\xi\bar{q}_1} & -2e^{\xi\bar{q}_1} & 2e^{\xi\bar{q}_1} & 0 & 0 \\ -2\bar{q}_1^{\xi q_{11}} & 2\bar{q}_1^{\xi q_{11}} & \left(\bar{s}_1+\frac{1}{\bar{s}_1}\right)e^{-\xi\bar{q}_1} & \left(\bar{s}_1+\frac{1}{\bar{s}_1}\right)e^{\xi\bar{q}_1} & 0 & 0 \\ (1+S_1^2)\, e^{\xi\bar{q}_1} & (1+\bar{S}_1^2)\, e^{-\xi\bar{q}_1} & -2e^{\xi\bar{S}_1} & 2e^{-\xi\bar{q}_1} & (1+s^2)\gamma_1 & -2/\gamma_1 \\ -2\bar{q}_1^{\xi q_{11}} & 2\bar{q}_1^{\xi q_{11}} & \left(\bar{s}_1+\frac{1}{\bar{s}_1}\right)e^{\xi\bar{q}_1} & \left(\bar{s}_1+\frac{1}{\bar{s}_1}\right)e^{-\xi\bar{q}_1} & \frac{2\bar{q}_1}{\gamma_{1\ 1}} & \left(\bar{s}+\frac{1}{\bar{s}}\right)/\gamma \\ e^{\xi\bar{q}_1} & e^{\xi\bar{q}_1} & e^{\xi\bar{q}_1} & e^{\xi\bar{q}_1} & -1 & -1 \\ \bar{q}e^{\xi\bar{q}_1} & \bar{q}_{1\,1}e^{-\xi\bar{q}_1} & -\frac{1}{\bar{s}}e^{\xi\bar{S}} & -\frac{1}{\bar{s}}e^{\xi\bar{S}} & -\bar{q} & \frac{1}{s} \end{vmatrix} = 0 \qquad (8)$$

Where ζ – dimensionless wave number $\zeta = kh,\ \gamma_1 = \frac{\bar{\mu}_1}{\bar{\mu}}$ or $\lambda + 2\mu = \frac{2(1-\nu)}{1-2\nu}$. As the relaxation nucleus of a viscoelastic material, we take a three-parameter core $R(t) = \frac{Ae^{-\beta t}}{t^{1-\alpha}}$ Rizhanitena-Koltunova [13], which has a weak singularity, where A, α, β - parameters materials [13]. We take the following parameters: $A = 0{,}048;\ \ \beta = 0{,}05;\ \ \alpha = 0{,}1$. Using the complex representation for the elastic modulus, described earlier. The roots of the frequency equation are solved by the Mueller method, at each iteration of the Muller method is applied by the Gauss method with the separation of the principal element. Thus, the solution of equation (8) does not require the disclosure of the determinant. As the initial approximation, we choose the phase velocities of the waves of the elastic system. For free waves with $R_j = 0$ the phase velocities and the wave number are real quantities. In the calculations, we accept the following parameter values:

$$\theta = \frac{\rho_1}{\rho_2} = 0{,}75; \quad \beta = 10^{-4}; \quad n = 1.$$

Let us consider two variants of the dissipative system. In the first variant, the dissipative system is structurally homogeneous. The wave number ξ varies from 0 to 3. The results of the calculations are shown in Fig. 2a. The dependence of the frequencies and damping on the dimensionless wave number ξ turned out to be monotonic, and the character of the dependence is the same for the frequencies and damping coefficients. In the second variant, the dissipative system is structurally inhomogeneous: the half-space under consideration, equation (8), and elastic parameters coincide with those adopted above. The results of the calculations are shown in Fig. The frequency dependence of ξ is the same as for a homogeneous system: the corresponding curves coincide with an accuracy of up to 5%. Dependence of the damping coefficients on ξ is no monotonic.

Of particular interest is the minimum value of ξ for a fixed damping coefficient:

$$\delta = \min(-\omega_{Ik}),\ \ k = 1,2,\ldots,K \qquad (9)$$

Here δ is a coefficient that determines the damping properties of the system (we call it the global damping coefficient).

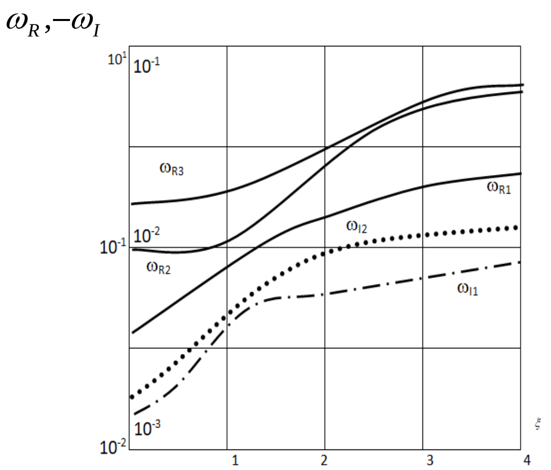


Fig. 2.a: Variation of complex Eigen frequencies from the wave number.

A) Dissipatedly homogeneous mechanical system

For a homogeneous system, the coefficient δ is entirely determined by the imaginary part of the first complex-frequency modulus. For an inhomogeneous system, the imaginary parts of both the first and second frequencies may act as the coefficient δ depending on their values. "Change of roles" occurs at a characteristic value of ξ, at this value the real parts of the first and second frequencies are the closest. The coefficient δ at the indicated characteristic value has a pronounced maximum.

Sliding contact

The dispersion equation is similar in form to equation (8). All parameter values are the same as those used above. Figures 3a and b show the dependence of the frequencies and damping coefficients on the wave number ξ, respectively, for a structurally homogeneous and inhomogeneous system. The obtained results confirm the earlier conclusions. Change the parameter, from

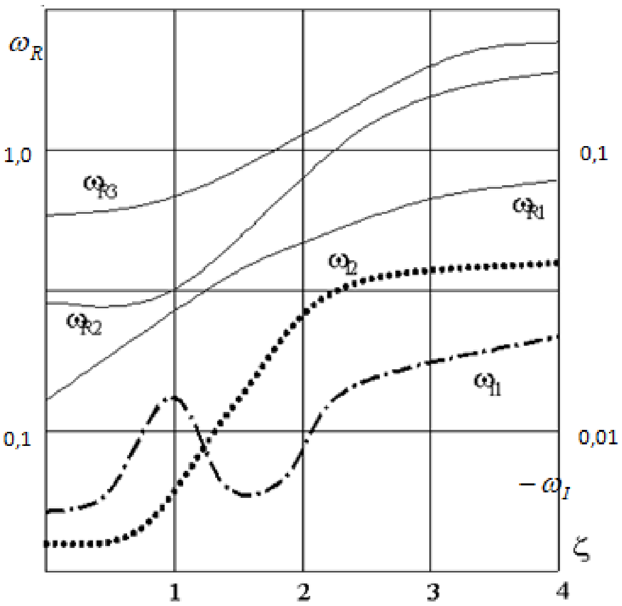
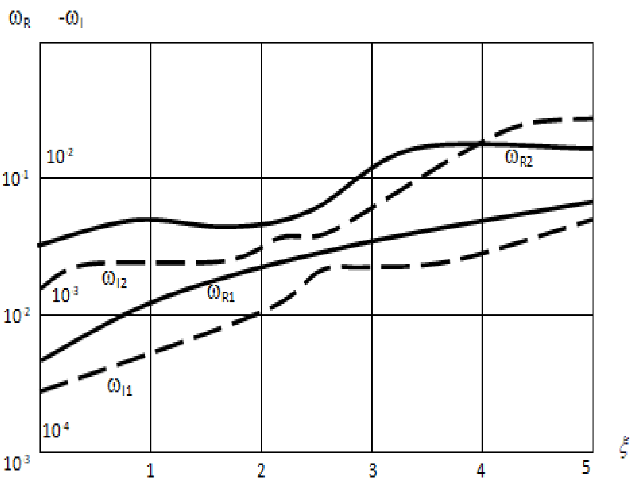


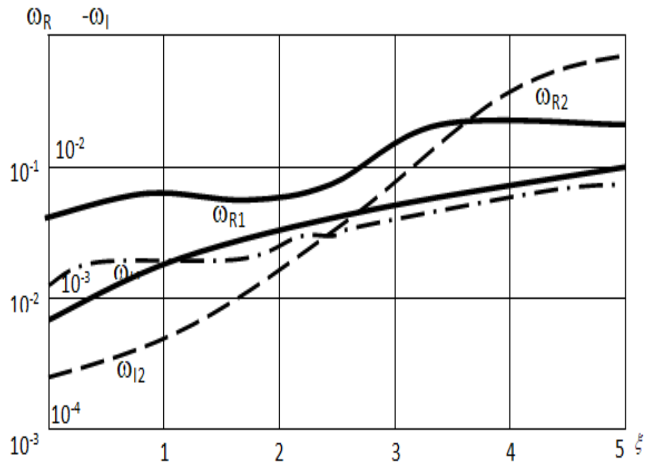
Fig.2.b: Change in complex Eigen frequencies from the wave number

b) Dissipatedly inhomogeneous mechanical system

Which depends so much on the value of the coefficient δ, can be achieved by varying the geometric dimensions of the elements without changing their mechanical properties.



a) Dissipative homogeneous mechanical system.



b) Dissipative inhomogeneous system.

Fig.3: The change in complex eigenfrequencies from the wave number

b) Dissipative inhomogeneous system Thus, a promising opportunity for effective control of the damping characteristics of structurally inhomogeneous viscoelastic systems opens up by changing their inhomogeneous systems with close frequencies. As a second example, let us consider the propagation of natural waves in a plane layer located in a deformable (viscoelastic) medium (Fig. 7). The results of the calculation are shown in Fig. 7. The frequency dependence of ξ turned out to be the same as for a dissipatedly homogeneous system: the corresponding curves coincide with an accuracy of up to 5%. As for the coefficients of damping, their behavior has changed radically: the dependence $\omega_I \sim \xi$ became no monotonic. The global damping coefficient for a given characteristic value of ξ has a pronounced maximum.

Conclusions

– In the course of solving the problem, the propagation of waves in dissipative-inhomogeneous media revealed no monotonic dependences of the damping rate on the physic mechanical and geometric parameters of the system. In dissipatedly inhomogeneous media, the dependence of the phase velocity and the damping rate on the geometric and physic-mechanical parameters of the system turned out to be no monotonic;

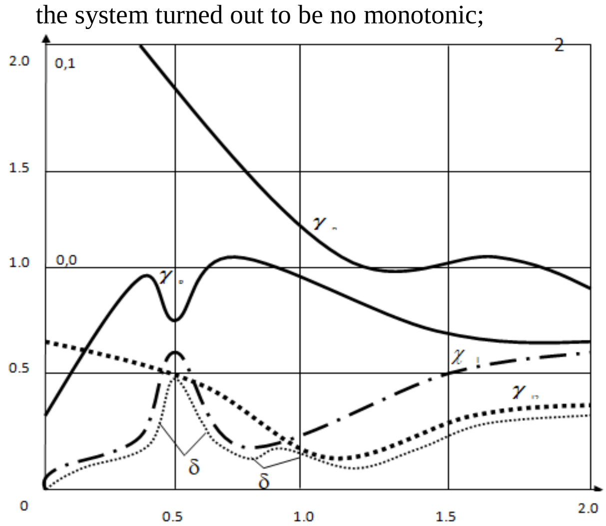


Fig. 7. Dispersion curves of phase velocities χ_{R1}, χ_{R2} and attenuation coefficient χ_{I1}, χ_{I2} for the duralumin strip in contact with the acrylic medium

C_{p1}=5400 m/s; ν_1=0.35; C_{p2}=2300 m/s; ν_1=0.35;
C_{S1}=1311m/s; ρ_1 =0.138 kg/m^3; C_{s2}=1311 m/s; ρ_2 =0.126 kg/m^3.

- Based on the obtained numerical results, it is revealed that the possibility of detachment of thin-walled structures from a soft layer and the effect of magnitude up to the resonance speed on the dimensions of the contact area. Also taking into account the viscous properties of the material, 15 - 10% increases the values of the phase velocities;
- revealed that the phase higher forms of the expansion and torsion waves exceed the highest possible speed (C) of waves in an infinite medium, the group velocity never exceeds C. Also found that the group velocity of 10-15% exceeds the non-dispersive medium, comparison by a dispersive medium. In other words, the forms of the pulses do not remain unchanged, as in homogeneous elastic bodies.

References

1. Kayumov S.S., Safarov I.I. Dissemination and diffraction of waves in dissipative-non-homogeneous cylindrical deformable mechanical systems.-Toshkent; Fan, 2004, -210p.
2. Aki K., Richard P. Quantitative seismology. - Moscow: Mir, 1983, vol 1. -519 p.
3. Brepta, R. Propagation of longitudinal waves in thin belts with rheological properties / -Krakov, Stroi i chas., XXII, 5, 1971. 433-447 p.
4. Grinchenko V.T., Maleshko V.V. Harmonic vibrations and waves of elastic bodies, - K.: Dumka Science, 1981, -283p.
5. Davies R.M. Stress waves in solids. -M.: Because of foreign language. Liter., 1961, -104p.
6. Sheriff R., Geldart L. Seismic prospecting: in 2 t.T.1. Trans. with English. -M. Peace., 1987, -448 p.
7. Novatsky V. The theory of elasticity. -M.: Mir, 1975, -872 p.
8. Safarov I.I... Propagation of waves in a layer lying in a viscoelastic half-space // - Abstracts of conferences. "Experience in the use of composite materials in agricultural engineering", Chernigov, 1985, p.91-92.
9. Safarov I.I., Boltaev Z.I., Axmedov M.Sh.Ducting in Extended Plates of Variable Thickness// Global Journal of Science Frontier Research: F Mathematics & Decision Sciences 2016. Volume 16 Issue 2 (Ver.1.0). P.33-66.
10. Safarov I.I., Teshaev M.Kh., Boltaev Z.I., Axmedov M.Sh. Spread of Natural Waves in Cylindrical Panel. Impact Factor 3.582 Case Studies Journal ISSN (2305-509X) – Volume 4, Issue 3 - March-2015. P.34-39.
11. Safarov I.I., Boltaev Z.I., Axmedov M.Sh. Impact of longitudinal and transverse waves by cylindrical layers were liquid. Journal of Multidisciplinary Engineering Science and Technology (JMEST) ISSN: 3159-0040 Vol. 1 Issue 4, November – 2014. P. 273-281.
12. Safarov I.I.,Teshaev M.Kh., Boltaev Z.I. Wave propagation in visco elastic wedge with an arbitrary angle peaks. International Journal of Research in Engineering and Science (IJRES). Volume 2 Issue 11 November. 2014. P.32-37.
13. 13 Koltunov M.A.. Creep and relaxation. - Moscow: Higher School, 1976. 277 p.