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Fuzzy Decision Making for Selection of Feasible Customer Support System Based on Interval Type-2 Trapezoidal Fuzzy Numbers

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Abstract
This research mainly focuses on the combination of techniques for order performance by similarity to ideal solution (TOPSIS) and Grey relational analysis (GRA). The term linguistic is defined for alternatives and weights. Linguistic data is modeled in the structure of interval type-2 trapezoidal fuzzy numbers that have the tendency to attain intra-personal and interpersonal uncertainties associated with the linguistic term. The proposed strategy has the capability to solve decision-making issues and maybe resolved by using strategies that include ordinary fuzzy sets. In the last, an example has been illustrative to check the effectiveness and feasibility of the suggested method.

Keywords: Multi criteria decision making, interval type-2 trapezoidal fuzzy number, TOPSIS method, grey relational analysis.

1. Introduction

Multi-criteria decision making *(MCDM)* [1, 2] has been one of the fastest developing areas in the past decades, depending on the alteration in the business zone. *MCDM* [3] is the type of problem where decision-makers choose the best alternative under the set of attributes by comparing the overall performance of the given alternatives. For solving complex decision-making problems, *MCDM* is used to solve the issues related to different fields such as in the fields of engineering, economics, and social and biological sciences. The situation arises where we have to select one object from a lot of choices. Neither analytical nor any statistical method is helpful in these situations as each individual is free to choose anything.

Fuzzy set theory proposed by Zadeh [5] is considered as an evolutionary development in dealing with problems of organized complexity as pointed by Weaver [4]. The purpose of *MCDM* is to choose the appropriate candidate through alternatives in accordance to the data regarding attribute values and weights, that is offered via decision-makers [5, 6], but the data available in the linguistic terms form about value of attributes are uncertain or fuzzy. To resolve this uncertain condition, most of the approaches have been created to cope with problems of *MCDM* over "Type 1 fuzzy set *(T1FS)*" [5]. Interval type 2 fuzzy sets (*IT2FS*) is implemented regardless of the *T1FS* for covering the uncertainty or providing the flexibility [5, 7, 8]. Grey relational analysis *(GRA)* [9] technique was developed initially by Deng and has been commonly used for solving uncertainty issues and make all decisions in multi-criteria conditions under discrete facts and incomplete information. The key benefit of the *GRA* method is that the results are based on original facts and the calculations are simpler as it is one of the finest techniques to make decisions in business surroundings.

Contemporary decision-making involves many complexities and high levels of uncertainties whereas a practical decision problem involves uncertainties with respect to elements of the basic decision-making model. So, it is more convenient to use natural language is suitable more to be utilized for assessing the alternatives than numbers. There lies the certain correspondence amount between *GRA* operation and input and TOPSIS multi-criterion evaluation [10].

Gargovhu and Atanassov in [11] highly extended this "Fuzzy sets" *FSs* idea by elaborating the "Interval fuzzy sets" *IFS* concept and "Interval valued intuitionistic fuzzy sets" *IVIFS*. Initially, Zadeh in [5, 27] introduced the "Type 2 fuzzy sets" *T2FS* which is the *T1FS* extension. Membership degree in *T1FS* is crisp value in interval [0,1] [12] while the membership degree in *T2FS* is *T1FS* in [0,1]. Hence, *T2FS* offers more freedom degree to deal with uncertainty and vagueness of practical and real-world issues as contrast to the *T1FS*. Wanga and Dinga [13] elaborated the *IVTIFS*. Kumar and Joshi [14] described the Intuitionistic fuzzy parameterized fuzzy soft set theory concept. Sajjad et al [15] modified the TOPSIS method (with unknown weights environment) by employing the maximum deviation approach. *IT2FS* have more tendency than another fuzzy set dealing with uncertain and imperfect data in real world application [12, 16]. *IT2FS* is implemented in most of the practical fields like ranking with trapezoidal *IT2FS* and decision making.

In [17], the author deals with large intuitionists type 1 trapezoidal fuzzy number having no complete data regarding attribute weights or used "Grey relational projection method" *(GRPM)* to rank the alternatives. Mendel in [18, 19] employed linguistic weighted averaging for hierarchical group decision making. Lee and Chen in [7, 20] discussed *IT2FS* ranking term and fuzzy *MCDM* based on arithmetic operation. Wang et al in [6, 21] discussed the decision environment where attribute values are modelled in the form of "Interval type 2 trepezoidal fuzzy numbers" *IT2TFNs* and there is partial information on attribute weights.

The main focus of this paper is to combine the application of *GRA* for group decisions with TOPSIS. In order to do this, the paper is organized as follows, preliminaries are defined in Section 1, the *MCDM* method based on the *IT2FN*, and some important terms are defined in Section 2, the TOPSIS method for *MCDM* problems and the combination of *GRA* is discussed in section 3. In Section 4, an example is proposed to evaluate the effectiveness and feasibility of the above-mentioned method. Finally, in Section 5, the conclusion is presented.

2. Preliminary

Definition 2.1 [5, 22] A T1FS G in the universe of discourse X characterized by its membership function $\mu_G(x)$ and is represented as follows:

$$G = \{(x, \mu_G(x)) | x \in X, \mu_G(x) \in [0,1]\} \quad (1)$$

Definition 2.2 [23, 27] A T2FS F in the universe of discourse U represented by a type-2 membership function μ_F shown as follows:

$$F = \{((x, u), \mu_F(x, u)) | x \in U, u \in j_x \subseteq [0,1], \mu_F(x, u) \in [0,1]\} \quad (2)$$

where j_x denotes an interval in [0,1]. The T2FS F also represented as follows;

$$F = \int_{x \in U} \int_{u \in j_x} \frac{\mu_F(x,u)}{(x,u)} \quad (3)$$

Definition 2.3 [23] Let F be a T2FS in the universe of discourse X denoted by the type-2 membership function μ_F. If all the secondary grades $\mu_{\bar{F}}(x, u)$ of F are equal to 1, then F is called an interval type-2 fuzzy set. It symbolically can be shown as follows:

$$\bar{F} = \int_{x \in U} \int_{u \in j_x} \frac{1}{(x,u)} \quad (4)$$

Definition 2.4 [28, 29] Assume that $\tilde{F}^u$ and $\tilde{F}^l$ are both generalised trapezoidal fuzzy numbers, the relative heights of $\tilde{F}^u$ and $\tilde{F}^l$ are $h(\tilde{F})^u$ and $h(\tilde{F})^l$, and $h(\tilde{F})^u, h(\tilde{F})^l \in [0,1]$, an IT2TrFN Z is represented as below:

$$\tilde{F} = [\tilde{F}^l, \tilde{F}^u] = [(a_1^{\bar{l}}, a_2^{\bar{l}}, a_3^{\bar{l}}, a_4^{\bar{l}}; h(\tilde{F})^l), (a_1^{\bar{u}}, a_2^{\bar{u}}, a_3^{\bar{u}}, a_4^{\bar{u}}; h(\tilde{F})^u)] \quad (5)$$

where $0 \le a_1^{\bar{l}} \le a_2^{\bar{l}} \le a_3^{\bar{l}} \le a_4^{\bar{l}} \le 1$, $0 \le a_1^{\bar{u}} \le a_2^{\bar{u}} \le a_3^{\bar{u}} \le a_4^{\bar{u}} \le 1$, $a_1^{\bar{u}} \le a_1^{\bar{l}}$, $a_4^{\bar{l}} \le a_4^{\bar{u}}$, $0 \le h(\tilde{F})^l \le h(\tilde{F})^u \le 1$, and "lower membership function (LMF)" $\tilde{F}^l$ and the "upper membership function (UMF)" $\tilde{F}^u$ of F are represented as:

$$\tilde{F}^l(x) = \begin{cases} \frac{(x - a_1^{\bar{l}})h(\tilde{F})^l}{a_2^{\bar{l}} - a_1^{\bar{l}}}, & a_1^{\bar{l}} \le x \le a_2^{\bar{l}} \\ h(\tilde{F})^l, & a_2^{\bar{l}} \le x \le a_3^{\bar{l}} \\ \frac{(a_4^{\bar{l}} - x)h(\tilde{F})^l}{a_4^{\bar{l}} - a_3^{\bar{l}}}, & a_3^{\bar{l}} \le x \le a_4^{\bar{l}} \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

$$\tilde{F}^u(x) = \begin{cases} \frac{(x - a_1^{\bar{u}})h(\tilde{F})^u}{a_2^{\bar{u}} - a_1^{\bar{u}}}, & a_1^{\bar{u}} \le x \le a_2^{\bar{u}} \\ h(\tilde{F})^u, & a_2^{\bar{u}} \le x \le a_3^{\bar{u}} \\ \frac{(a_4^{\bar{u}} - x)h(\tilde{F})^u}{a_4^{\bar{u}} - a_3^{\bar{u}}}, & a_3^{\bar{u}} \le x \le a_4^{\bar{u}} \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

Definition 2.5 let F and G be two "Interval type 2 trepezoidal fuzzy numbers" (IT2TFNss).

$$\tilde{F} = (\tilde{F}^u, \tilde{F}^l) = (a_1^u, a_2^u, a_3^u, a_4^u: h(\tilde{F})^u; a_1^l, a_2^l, a_3^l, a_4^l: h(\tilde{F})^l) \quad (8)$$

and

$$\widetilde{G} = (\widetilde{G}^u, \widetilde{G}^l) = (b_1^u, b_2^u, b_3^u, b_4^u : h(\widetilde{G})^u; b_1^l, b_2^l, b_3^l, b_4^l : h(\widetilde{G})^l) \quad (9)$$

Then, hamming distance between $\tilde{F}$ and $\widetilde{G}$ is defined as

$$d(\tilde{F}, \widetilde{G}) = \sqrt[2]{\sum_p^4 (a_p^u - b_p^u)^2 + \sum_p^4 (a_p^u - b_p^u)^2 + (h(\tilde{F})^u - h(\widetilde{G})^u)^2 + (h(\tilde{F})^l - h(\widetilde{G})^l)^2} \quad (10)$$

3. *MCDM* Method Based on The *IT2FN*

Some important terms which are extensively used in the proposed method are provided as under:

Alternatives: Alternatives are the options which are to be examined for the selection of the best criteria.

Attributes or Criteria: Criteria is a set of characteristics on the basis of which attributes are selected. This criterion will base on the following conditions:

- Completeness: It means to make sure the inclusion of all important criteria.
- Redundancy: It means to remove at a very early stage the criteria that have been proved to be unimportant and or duplicates.
- Operationally: It requires each alternative to be operational against each criterion.

Weights: This decides the relative importance of the set criteria, and there is a group of decision makers and experts that accordingly grants rating points and scales to its attributes in term of *IT2TFNs*.

Decision makers: Those experts who are responsible for the task of weighting each attribute are called the decision makers.

Decision matrix: Decision matrix is a table that we use to make an objective decision about making selection from a range of options.

For making process uniform or eliminate distinct physical factors on decision solution, this decision making is standardized the first decision matrix. Here, R is a standardized decision matrix.

$$R = [r_{pq}]_{m\times n} = \tilde{F} = (a_{pq}^u, a_{pq}^u, a_{pq}^u, a_{pq}^u : h(\tilde{F})^u; a_{pq}^l, a_{pq}^l, a_{pq}^l, a_{pq}^l : h(\tilde{F})^l) \quad (11)$$

where p represents row and q represents column.

We can evaluate into 2 types of criteria [25] such as cost type, and benefit type criteria for

Cost type criteria:

$$r_{pq}^{v'} = \frac{\max_q(a_{pq}^4) - a_{pq}^{v'}}{\max_q(a_{pq}^4) - \min_q(a_{pq}^4)}, v' = 1,2,3,4 \quad (12)$$

Benefit type criteria:

$$r_{pq}^{v'} = \frac{a_{pq}^{v'} - \max_q(a_{pq}^4)}{\max_q(a_{pq}^4) - \min_q(a_{pq}^4)}, v' = 1,2,3,4, \quad (13)$$

4. TOPSIS Method for *MCDM* Problems

TOPSIS Method In this method two artificial alternatives are hypothesized; one is ideal alternatives and other is negative alternatives.

Positive ideal alternatives: The one which have best level for all attribute considered.

Negative ideal alternatives: The one which have worst attribute value. TOPSIS selects the alternatives that is closest to PIS and farthest to NIS alternatives. The algorithm of TOPSIS [26] is define as following steps:

Step 1: First define linguistic variable for importance weight on each criterion and linguistic variable for raiting at define scale.

Step 2: *MCDM* problems which can express in the form of matrix the linguistic variable which can be described in the form of *IT2TFNs* by decision maker.

Step 3: Standardize the decision matrix R into two type criteria, cost type and benefit type criteria.

For Cost type criteria is defined as:

$$r_{pq}^{v'} = \frac{\max_q(a_{pq}^4) - a_{pq}^{v'}}{\max_q(a_{pq}^4) - \min_q(a_{pq}^4)}, v' = 1,2,3,4 \quad (14)$$

For Benefit type criteria is defined as:

$$r_{pq}^{v'} = \frac{a_{pq}^{v'} - \max_q(a_{pq}^4)}{\max_q(a_{pq}^4) - \min_q(a_{pq}^4)}, v' = 1,2,3,4, \quad (15)$$

Step 4: Calculated weighted normalized fuzzy decision matrix.[26]

$$Y = [x_{pq}]_{m\times n} = [r_{pq\times w_q}]_{m\times n}, p = 1,2,3,..,m \text{and} q = 1,2,3..,n \quad (16)$$

Step 5: Define "Fuzzy positive ideal solution (FPIS)" and "Fuzzy negative ideal solution (FNIS)" from weighted normalized fuzzy decision matrix.

$$A^+ = [x_1^+, x_2^+, x_3^+, ..., x_n^+] \text{and} A^- = [x_1^-, x_2^-, x_3^-, ..., x_n^-] \quad (17)$$

Step 6: Calculate distance of each alternative from FPIS and FNIS.

$$d_p^+ = \sum_{q=1}^n \| y_p - y_{pq} \|, p = 1,2,3,..,m \text{and} q = 1,2,3..,n \quad (18)$$

and

$$d_p^- = \sum_{q=1}^n \| y_p - y_{pq} \|, p = 1,2,3,..,m \text{and} q = 1,2,3..,n \quad (19)$$

5. Combination of grey relational analysis (*GRA*) and TOPSIS for multi-criteria group decision making with IT2F information

The proposed algorithm of the modified method involving the combination of the two approaches of GRA and TOPSIS is given below:

Step 1: First define linguistic variable for importance weight on each criterion and linguistic variable for raiting.

Step 2: *MCDM* problems which can express in the form of matrix the linguistic variable which can be described in the form of *IT2TFNs* by decision maker.

$$R = [r_{pq}]_{m\times n} = \tilde{F} = (a^u_{pq}, a^u_{pq}, a^u_{pq}, a^u_{pq}\colon \mathrm{h}(\tilde{F})^u; a^l_{pq}, a^l_{pq}, a^l_{pq}, a^l_{pq}\colon \mathrm{h}(\tilde{F})^l) \tag{20}$$

Step 3: Standardize the decision matrix based on cost type and benefit type criteria [25]

For cost type criteria:

$$r^{v'}_{pq} = \frac{\max_q(a^4_{pq}) - a^{v'}_{pq}}{\max_q(a^4_{pq}) - \min_q(a^4_{pq})}, v' = 1,2,3,4 \tag{21}$$

For Benefit type criteria:

$$r^{v'}_{pq} = \frac{a^{v'}_{pq} - \max_q(a^4_{pq})}{\max_q(a^4_{pq}) - \min_q(a^4_{pq})}, v' = 1,2,3,4, \tag{22}$$

Step 4: Calculated weighted normalized fuzzy decision matrix. [26]

$$Y = [x_{pq}]_{m\times n} = [r_{pq\times w_q}]_{m\times n}, p = 1,2,3,..,m \text{ and } q = 1,2,3..,n \tag{23}$$

Step 5: Define "Fuzzy positive ideal solution" (FPIS) and "Fuzzy positive ideal solution" (FNIS) from weighted normalized fuzzy decision matrix. From normalized matrix FPIS of *IT2TFNs* is defined as

$$r^+_q = \left([r^1_q+, r^2_q+, r^3_q+, r^4_q+]; h(A_p)\right)$$
$$= \left([\underbrace{\max}_{p}(r^1_{pq}), \underbrace{\max}_{p}(r^2_{pq}), \underbrace{\max}_{p}(r^3_{pq}), \underbrace{\max}_{p}(r^4_{pq})]; \underbrace{\max}_{p} h(A_p) \right) \tag{24}$$

and FNIS of *IT2TFNs* is defined as

$$r^-_q = \left([r^1_q-, r^2_q-, r^3_q-, r^4_q-]; h(A_p)\right)$$
$$= \left([\underbrace{\min}_{p}(r^1_{pq}), \underbrace{\min}_{p}(r^2_{pq}), \underbrace{\min}_{p}(r^3_{pq}), \underbrace{\min}_{p}(r^4_{pq})]; \underbrace{\max}_{p} h(A_p) \right) \tag{25}$$

Step 6: Calculate distance of each alternative from FPIS and FNIS. Here,

$$d^+(\tilde{F}, \tilde{G}) = \left(\sum_p^4 \left(a^u_p - b^u_p\right)^2 + \sum_p^4 \left(a^l_p - b^l_p\right)^2 + \left(\mathrm{h}(\tilde{F})^u - \mathrm{h}(\tilde{G})^u\right)^2 + \left(\mathrm{h}(\tilde{F})^l - \mathrm{h}(\tilde{G})^l\right)^2\right)^{\frac{1}{2}} \tag{26}$$

$$T^+ = \underbrace{min}_{p}\underbrace{min}_{q} d^+_{pq} \text{and} S^+ = \underbrace{max}_{p}\underbrace{max}_{q} d^+_{pq} \tag{27}$$

$\lambda \in [0,1]$ represents resolution coefficient. Generally, its value is 0.5 if the value $\lambda > 0.5$ the decision maker is optimistic attitude towards problem. But if $\lambda < 0.5$ the decision maker has pessimistic attitude towards problem. And if the $\lambda = 0.5$ decision maker has moderate attitude towards problem. Therefore, we use $\lambda = 0.5$ to deal with the situation weather it is neither optimistic nor pessimistic. Similarly, the "Grey relation calculation (GRC)" of each alternative from FNIS is given as:

$$\zeta^+_{pq} = \frac{T^+ + \lambda S^+}{d^+_{pq} + \lambda S^+} \tag{28}$$

Similarly, the GRC of each alternative from FNIS is given as:

$$d^-(\tilde{F}, \tilde{G}) = \left(\sum_p^4 \left(a^u_p - b^u_p\right)^2 + \sum_p^4 \left(a^l_p - b^l_p\right)^2 + \left(\mathrm{h}(\tilde{F})^u - \mathrm{h}(\tilde{G})^u\right)^2 + \left(\mathrm{h}(\tilde{F})^l - \mathrm{h}(\tilde{G})^l\right)^2\right)^{\frac{1}{2}} \tag{29}$$

where

$$T^- = \underbrace{min}_{p}\underbrace{min}_{q} d^-_{pq} \text{and} S^- = \underbrace{max}_{p}\underbrace{max}_{q} d^-_{pq} \tag{30}$$

thus,

$$\zeta^-_{pq} = \frac{T^- + \lambda S^-}{d^-_{pq} + \lambda S^-} \tag{31}$$

Step 7: Calculate relative closeness as to rank each alternative.

$$cc_p = \frac{\zeta^+_{pq}}{\zeta^+_{pq} + \zeta^-_{pq}} \quad (32$$

6. Example

An online customer support system was established by five companies like ufone, Warid, jazz, telenor, and zong. It represented the set of 5 alternatives denoted as $(AL_1, AL_2, AL_3, AL_4, AL_5)$. Each company can be evaluated under certain criteria, these companies provided different services like m_1: Internet data, m_2: online banking m_3: call services m_4: SMS services which represented the set of criteria denoted by (m_1, m_2, m_3, m_4). these companies extract information from websites and present to the customer, now we have to see that which company provides comfort in all aspects mention above:

Step 1: First define linguistic variable for importance weight on each criterion and linguistic variable for rating. So, define linguistic terms in form of *IT2TFNs* in Table 1.

Table 1: Linguistic terms of *IT2TFNs*

Linguistic term	***IT2FN***
Absolutely Low (*AL*)	(0, 0, 0, 2, 1; 0, 0, 0, 1, 0.1)
Low (*L*)	(0, 1, 1, 3, 1; 0, 1, 1, 2, 0.2)
Slightly Low (*SL*)	(1, 3, 3, 5, 1; 2, 3, 3, 4, 0.3)
Equally(*E*)	(2, 3, 4, 5, 1; 2, 3, 5, 5, 0.5)
Slightly High (*SH*)	(5, 7, 7, 9, 1; 6, 7, 7, 8, 0.9)
High (*H*)	(7, 9, 9, 10, 1; 8, 9, 9, 10, 0.8)
Absolutely High (*AH*)	(9, 10, 10, 10, 1; 8, 9, 10, 10, 0.9)

Then, define weights linguistic terms in term of *IT2TFNs* in Table 2.

Table 2: Weight linguistic terms of *IT2TFNs*

Linguistic term	***IT2FN***
Absolutely Low (*AL*)	(0, 0, 0, 0.2, 1; 0, 0, 0, 0.1, 0.1)
Low (*L*)	(0, 0.1, 0.1, 0.3, 1; 0, 0.1, 0.1, 0.2, 0.2)
Slightly Low (*SL*)	(0.1, 0.3, 0.3, 0.5, 1; 0.2, 0.3, 0.3, 0.4, 0.3)
Equally(*E*)	(0.2, 0.3, 0.4, 0.5, 1; 0.2, 0.3, 0.5, 0.5, 0.5)
Slightly High (*SH*)	(0.5, 0.7, 0.7, 0.9, 1; 0.6, 0.7, 0.7, 0.8, 0.9)
High (*H*)	(0, 0.1, 0.1, 0.3, 1; 0, 0.1, 0.1, 0.2, 0.2)
Absolutely High (*AH*)	(0.9, 0.10, 0.10, 0.10, 1; 0.8, 0.9, 0.10, 0.10, 0.9)

Step 2: *MCDM* problems can express in the form of matrix and the linguistic variable can be described in the form of *IT2TFNs* by decision maker. Decision matrix is given in Table 3 by decision maker in term of linguistic terms.

Table 3: Decision matrix

U	m_1	m_2	m_3	m_4
AL_1	*SH*	*SL*	*AH*	*L*
AL_2	*E*	*AH*	*SH*	*L*
AL_3	*SH*	*L*	*H*	*E*
AL_4	*SL*	*AH*	*SH*	*H*
AL_5	*SH*	*SL*	*E*	*AL*

The weights of decision matrix in the form of linguistic terms are shown in Table 4.

Table 4: Weights decision matrix in the form of linguistic term

U	m_1	m_2	m_3	m_4
w	*E*	*L*	*H*	*AL*

Decision information is given in the form of *IT2TFNs* in Table 5.

Table 5: Attribute value of alternatives

U	m_1	m_2	m_3	m_4
AL_1	(5,7,7,9: 1; 6,7,7,8: 0.9)	(1 3 3 5: 1; 2 3 3 4: 0.3)	(9,10,10,10: 1; 8,9,10,10: 0.9)	(0,1,1,3: 1; 0,1,1,2: 0.2)
AL_2	(2,3,4,5: 1; 2,3,5,5: 0.5)	(9,10,10,10: 1; 8,9,10,10: 0.9)	(5,7,7,9: 1; 6,7,7,8: 0.7)	(0,1,1,3: 1; 0,1,1,2: 0.2)
AL_3	(5,7,7,9: 1; 6,7,7,8: 0.9)	(0,1,1,3: 1; 0,1,1,2: 0.5)	(7,9,9,10: 1; 8,9,9,10: 0.8)	(2,3,4,5: 1; 2,3,5,5: 0.5)
AL_4	(1,3,3,5: 1; 2,3 3 4: 0.3)	(9,10,10,10: 1; 8,9,10,10: 0.9)	(5,7,7,9: 1; 6,7,7,8: 0.7)	(7,9,9,10: 1; 8,9,9,10: 0.8)
AL_5	(5,7,7,9: 1; 6,7,7,8: 0.9)	(1 3 3 5: 1; 2 3 3 4: 0.3)	(2,3,4,5: 1; 2,3,5,5: 0.5)	(0,0,0,1: 1; 0,0,0,1: 0.1)

Step 3: Standardize the decision matrix R into two major criteria types, benefit and cost type criteria. Here, q indicates alternative and p indicates attribute. Standardize decision matrix R created by using formula (22) benefit type criteria in Table 6.

Table 6: Standardize decision matrix

U	m_1	m_2	m_3	m_4
AL_1	(0.4444, 0.6667,0.6667, 0.8889: 1; 0.5,0.6250, 0.6250,0.75: 0.7)	(0.1, 0.3, 0.3, 0.5: 1; 0.2, 0.3, 0.3, 0.4 : 0.3)	(0.8750, 1, 1, 1: 1; 0.75, 0.8750, 1,1: 0.9)	(0, 0.1,0.1,0.3: 1; 0, 0, 0 ,0.1: 0.1)
AL_2	(0.1, 1 ,1, 1: 0.2222; 0.1250,0.3750,0.3750:0.5)	(0.9,1,1,1:1; 0.8, 0.9, 1,1: 0.9)	(0.3750,0.6250,0.6250,0.8750:1; 0.5 0.6250 0.6250 0.75: 0.7)	(0,0.1,0.1, 0.3: 1; 0, 0,0, 0.1: 0.1)
AL_3	(0.8889 ,1 ,1 ,1: 1; 0.75 0.8750 1 1: 0.9)	(0 ,0.1, 0.1,0.3: 1; 0.1, 0.1,0.2: 0.2)	(0.6250,0.8750, 0.8750,1: 1; 0.75,0.8750,0.8750,1:0.8)	(0.2,0.3,0.4,0.5: 1; 0.2,0.3,0.5,0.5: 0.5)
AL_4	(0,0.2222,0.2222,0.4444: 1;	(0.9,1,1,1: 1;	(0.3750,0.6250,0.6250,0.8750:1;	(0.7,0.9,0.9, 1: 1;

	0,0.1250,0.1250,0.250:0.3)	0.8,0.9,1,1: 0.9)	0.5,0.6250,0.6250,0.75: 0.7)	0.8,0.9,1,1: 0.8)
AL_5	(0.4444,0.6667,0.6667,0.8889:1 0.5,0.6250,0.6250,0.75:0.9)	(0.1,0.3,0.,0.5:1; 0.2,0.3,0.3,0.4:0.3)	(0,0.1250,0.25,0.3750:1; 0,0.1250,0.3750,0.3750:0.5)	(0,0,0,0.2:1; 0,0,0,0.1:0.1)

Decision maker gives weights to each attribute in term of normalized *IT2TFNs*, it appear as Table 7.

Table 7: Weights decision matrix in term of linguistic values

U	m_1	m_2	m_3	m_4
w	(0.2,0.3,0.4,0.5: 1; 0.2,0.3,0.5,0.5: 0.5)	(0,0.1,0.1,0.3: 1; 0,0.1,0.1,0.2: 0.2)	(0,0.1,0.1,0.3: 1; 0,0.1,0.1,0.2: 0.2)	(0,0,0,0.2: 1; 0,0,0,0.1: 0.1)

Step 4: Weighted normalized fuzzy decision matrix by using Equation 23 is calculated in Table 8.

Table 8: Weighted standardize normal decision matrix

U	m_1	m_2	m_3	m_4
AL_1	(0.0889,0.2,0.2,0.3556:1; 0.1,0.1875,0.25,0.3750: 0.21)	(0,0.03,0.03,0.15:1; 0,0.03,0.03,0.08:0.06)	(0.6125,0.9,0.9,0.1:1; 0.6,0.7875,0.9,0.1:0.72)	(0,0,0,0.06: 1; 0,0,0,0.01: 0)
AL_2	(0.0222,0.0667,0.1,0.1778: 1; 0,0.0375,0.15,0.1875:0.15)	(0,0.1,0.1,0.3: 1; 0,0.09,0.1,0.2:0.18)	(0.2625,0.5625,0.5625,0.087:1; 0.4,0.5625,0.5625,0.0750:0.56)	(0,0,0,0.06:1; 0,0,0,0:0)
AL_3	(0.1778,0.3,0.3,0.4: 1; 0.15,0.2625,0.4,0.5: 0.27)	(0,0.01,0.01,0.09:1; 0,0.01,0.01,0.04:0.04)	(0.4375,0.7875,0.7875,0.1:1; 0.6,0.7875,0.7875,0.1:0.64)	(0,0,0,0.1: 1; 0,0,0,0: 0)
AL_4	(0,0.0667,0.0667,0.1778:1; 0,0.0375,0.05,0.1250:0.09)	(0,0.1,0.1,0.3:1; 0,0.09,0.1,0.2:0.18)	(0.2625,0.5625,0.5625,0.0875:1; 0.4,0.5625,0.5625,0.0750:0.56)	(0,0,0,0.2:1; 0,0,0,0: 0)
AL_5	(0.0889,0.2000,0.2000,0.355:1; 0.1,0.1875,0.2500,0.3750:0.27)	(0,0.03,0.03,0.15: 1); 0,0.03,0.03,0.08:0.06)	(0 0.1125,0.2250,0.0375: 1; 0,0.1125,0.3375,0.0375:0.40)	(0,0,0,0.04: 1; 0,0 ,0,0:0)

Step 5: Define FPIS and FNIS from weighted normalized fuzzy decision matrix. Calculate ($PITS$) and ($NITS$) of the *IT2TFNs* by Equations 24 and 25. Calculated results are shown in Table 9.

Table 9: Positive ideal target solution and negative ideal target solution

U	m_1	m_2	m_3	m_4
r^+	(0.178,0.3,0.3,0.4:1; 0.15,0.263,0.4,0.5:0.27)	(0,0.1,0.1,0.3: 1; 0,0.9,0.1,0.2:0.18)	(0.613,0.9,0.9,0.1:1; 0.6,0.788,0.9,0.1:0.72)	(0,0,0,0.2:1; 0,0,0,0.1:0.08)
r^-	(0,0.067,0.067,0.178:1; 0,0.038,0.05,0.125:0.27)	(0,0.01,0.1,0.09:1; 0,0.01,0.01,0.04:0.18)	(0,0.113,0.225,0.038:1 0,0.113,0.338,0.038:0.72)	(0,0,0,0.04:1; 0,0,0,0.01:0.08)

Now, we find the positive hamming distance d^+ by using Equation 26 and 27.

$$[d^+]_{4\times 5} = \begin{pmatrix} 0.9307 & 0.9976 & 0.5292 & 1.0088 \\ 1.1186 & 0.9055 & 0.9978 & 1.0088 \\ 0.8544 & 1.0298 & 0.6547 & 0.9811 \\ 1.1984 & 0.9055 & 0.9978 & 0.9592 \\ 0.8978 & 0.9976 & 1.7863 & 1.0118 \end{pmatrix}$$

calculate negative hamming distance d^- by using Equation 29 and 30

$$[d^-]_{4\times 5} = \begin{pmatrix} 1 & 0.9730 & 1.6943 & 0.9952 \\ 0.9303 & 0.9594 & 1.1146 & 0.9952 \\ 1.1208 & 0.9798 & 1.5328 & 0.9773 \\ 0.9539 & 0.9594 & 1.1146 & 0.9766 \\ 0.9696 & 0.9730 & 0.7746 & 0.9950 \end{pmatrix}$$

Step 6: To find the GRC with the help of $PITS$ and $NITS$ of each alternative by using the following equation respectively. It is shown as follows by Equation 28 and 31, respectively.

$$[\zeta_e^+]_{4\times 5} = \begin{pmatrix} 0.7798 & 0.7522 & 1 & 0.7478 \\ 0.7070 & 0.7907 & 0.7522 & 0.7478 \\ 0.8139 & 0.7397 & 0.9189 & 0.7589 \\ 0.68 & 0.7907 & 0.7522 & 0.7678 \\ 0.7941 & 0.7522 & 0.5308 & 0.7466 \end{pmatrix}$$

$$[\zeta_e^-]_{4\times 5} = \begin{pmatrix} 0.8780 & 0.8910 & 0.6381 & 0.8803 \\ 0.9124 & 0.8977 & 0.8267 & 0.8803 \\ 0.8241 & 0.8877 & 0.6814 & 0.8889 \\ 0.9004 & 0.8977 & 0.8267 & 0.8893 \\ 0.8927 & 0.8910 & 1 & 0.8804 \end{pmatrix}$$

Step 7: Calculate relative closeness as to rank each alternative by Equation 32. The value of relative closeness is

$$cc_i = (0.4994 \;\; 0.4601 \;\; 0.4961 \;\; 0.4598 \;\; 0.4352)$$

Ranking according to relative closeness is

$$cc_1 > cc_3 > cc_2 > cc_4 > cc_5$$

7. Conclusion

The combinations of *GRA* with TOPSIS have been discussed in this paper to solve the *MCDM* problem. The representational power of this method is greater than previous methods because we have used type-2 trapezoidal fuzzy numbers whereas previously simple fuzzy sets were employed for combining the two approaches. The proposed method can be used if the attribute terms are in the form of *IT2TFNs*s in order to handle with *MCDM* problems. *IT2TFNs*s has the tendency to capture intra-personal as well as interpersonal uncertainties, associated with the linguistic terms. The suggested technique has the tendency to solve decision making issues that may be solved by the employing strategies including ordinary fuzzy sets. Moreover, the approach can further be extended for the Fuzzy Pythagorean trapezoidal numbers and q-rung ortho-pair fuzzy numbers.

Author Contributions

All the authors contributed equally to this work. They all read and approved the last version of the paper.

Conflicts of Interest

The authors declare no conflict of interest.

References

1. H. Garg, K. Kumar, Some Aggregation Operators for Linguistic Intuitionistic Fuzzy Set and Its Application to Group Decision-Making Process Using the Set Pair Analysis. Arabian Journal for Science and Engineering 43 (2018) 3213-3227.
2. H. Garg, R. Arora, Dual Hesitant Fuzzy Soft Aggregation Operators and Their Application in Decision Making, Cognitive Computation 10 (2018) 1-21.
3. I. Kaya, M. Çolak, F. Terzi, A Comprehensive Review of Fuzzy Multi Criteria Decision Making Methodologies for Energy Policy Making, Energy Strategy Reviews 24 (2019) 207-228.
4. W. Weaver, Science and Complexity, Classical Papers-Science and Complexity 6 (3) (2004) 65-74.
5. L. A. Zadeh, Fuzzy Sets, Information and Control 8 (1965) 338-353.
6. K. T. Atanassov, Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems 20 (1) (1986) 87-96.
7. S-M. Chen, L-W. Lee, Fuzzy Multiple Attributes Group Decision-Making Based on the Ranking Values and the Arithmetic Operations of Interval Type-2 Fuzzy Sets, Expert Systems with Applications 37 (1) (2010) 824-833.
8. S-M. Chen, M-W. Yang, L-W. Lee, S-W. Yang, Fuzzy Multiple Attributes Group Decision-Making Based on Ranking Interval Type-2 Fuzzy Sets. Expert Systems with Applications 39 (5) (2012) 5295-5308.
9. G. Fang, Y. Guo, X. Huang, M. Rutten, Y. Yuan, Combining Grey Relational Analysis and a Bayesian Model Averaging Method to Derive Monthly Optimal Operating Rules for a Hydropower Reservoir Water 10 (8) (2018) 1099.
10. N. B. Kore, K. Ravi, S. B. Patil, A Simplified Description of Fuzzy TOPSIS Method for Multi Criteria Decision Making, International Research Journal of Engineering and Technology 4 (5) (2017) 1-4.
11. K. Atanassov, G. Gargov, Interval-Valued Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems 31 (3) (1989) 343-349.
12. J. M. Mendel, Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions, Prentice Hall PTR (2001).
13. [Q. Ding, Y-M. Wang, An Improved Aggregation Operatorsbased Method for Multiple Attribute Group Decision Making Using Interval-valued Trapezoidal Intuitionistic Fuzzy Sets, Journal of Intelligent and Fuzzy Systems 37 (1) (2019) 965-980.
14. B. P. Joshi, A. Kumar, A. Singh, P. K. Bhatt, B. K. Bharti, Intuitionistic Fuzzy Parameterized Fuzzy Soft Set Theory and Its Application, Journal of Intelligent and Fuzzy Systems 35 (5) (2018) 5217-5223.
15. M. Sajjad Ali Khan, A. Ali, S. Abdullah, F. Amin, F. Hussain, New Extension of TOPSIS Method Based on Pythagorean Hesitant Fuzzy Sets with Incomplete Weight Information, Journal of Intelligent and Fuzzy Systems 35 (5) (2018) 5435-5448.
16. J. R. Castro, O. Castillo, P. Melin, A Hybrid Learning Algorithm for a Class of Interval Type-2 Fuzzy Neural Networks, Information Sciences 179 (2009) 2175-2193.
17. X. Zhang, F. Jin, P. Liu, A Grey Relational Projection Method for Multi-Attribute Decision Making Based on Intuitionistic Trapezoidal Fuzzy Number, Applied Mathematical Modelling 37 (5) (2013) 3467-3477.
18. D. Wu, J. M. Mendel, Aggregation Using the Linguistic Weighted Average and Interval Type-2 Fuzzy Sets, IEEE Transactions on Fuzzy Systems 15 (6) (2007) 1145-1161.
19. D. Wu, J. M. Mendel, Corrections to Aggregation Using the Linguistic Weighted Average and Interval Type-2 Fuzzy Sets, IEEE Transactions on Fuzzy Systems 16 (6) (2008) 1664-1666.
20. L. W. Lee, S. M. Chen, Fuzzy Multiple Attributes Group Decision-Making Based on the Extension of TOPSIS Method and Interval Type-2 Fuzzy Sets, International Conference on Machine Learning and Cybernetics, IEEE Xplore 1 (7) (2008) 3260-3265.
21. W. Wang, X. Liu, Multi-Attribute Decision Making Models under Interval Type-2 Fuzzy Environment, 2011 IEEE International Conference on Fuzzy Systems, IEEE Xplore (2011) 1179-1184.
22. [S. Singh and H. Garg, Symmetric Triangular Interval Type-2 Intuitionistic Fuzzy Sets with Their Applications in Multi Criteria Decision Making, Symmetry 10 (2018) 401.
23. P. Kundu, S. Majumder, S. Kar, M. Maiti, A Method to Solve Linear Programming Problem with Interval Type-2 Fuzzy Parameters, Fuzzy Optimization and Decision Making 18 (2019) 103-130.
24. J. Wang, Q-H. Chen, H-Y. Zhang, X-H. Chen, J-Q. Wang, Multi-Criteria Decision-Making Method Based on Type-2 Fuzzy Sets, Filomat 31 (2) (2017) 431-450.
25. W. Yang, Y. Wu, A Novel TOPSIS Method Based on Improved Grey Relational Analysis for Multi Attribute Decision-Making Problem, Mathematical Problems in

Engineering, (2019) Article ID 8761681, 10 pages.

26. [R. Ran, B-J. Wang, Combining Grey Relational Analysis and TOPSIS Concepts for Evaluating the Technical Innovation Capability of High Technology Enterprises with Fuzzy Information, Journal of Intelligent and Fuzzy Systems 29 (4) (2015) 1301-1309.
27. L. A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning, Information Sciences 8 (3) (1975) 199-249.
28. S.-M. Chen, J.-H. Chen, Fuzzy risk analysis based on similarity measures between interval-valued fuzzy numbers and interval-valued fuzzy number arithmetic operators, Expert Systems with Applications 36 (2–3) (2009) 6309–6317.
29. T.-Y. Chen, C.-H. Chang, and J.-F. R. Lu, the extended QUALIFLEX method for multiple criteria decision analysis based on interval type-2 fuzzy sets and applications to medical decision making, European Journal of Operational Research 226 (3) (2013) 615–625.