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Harmonic Oscillations of Dissipative Viscoelastic Layered Bodies on a Deformable Half-Plane

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Abstract
Vibrations of a mechanical system, consisting viscoelastic layers, lying on the half-plane and being in plane strain under the influence of vertical harmonic load is considered. Required to find the vertical and horizontal motion and tension layers and half-points. The dynamic state of each of the laminated body, and half by the equations of Lame continuum mechanics with operator coefficients. The problem is reduced to solving a system of non-homogeneous algebraic equations with complex coefficients, which is solved by Gauss. The change in amplitude displacements and stresses as a function of frequency. In studies of the dynamic stress - strain state introduced the concept of global resonance amplitude (providing the minimum value of the resonant amplitude), which changes radically to nonuniform mechanical systems.

Keywords: oscillations, dissipative heterogeneous system, half space, viscoelastic body, frequency, voltage.

Introduction

The dynamics of structures under the influence of the stress cycle of the mechanical energy is absorbed as a result of dissipation mechanism in the material. Therefore, you must take into account the effect of the viscoelastic material. Many materials, such as materials used in road construction on the basis of bitumen, cohesive soils, materials or modern himii-termoplasy have distinct viscoelastic properties. Because of the explicit actions viscoelastic materials or designs become actual problems viscoelastic layered foundation vibrations. For the first time a systematic approach to the study of an arbitrary number of layers has been proposed V.Tomsonom [1] and N.Haskellom [2]. Further development of this theory is presented in L.A.Molotkova [3], I.Dankina [4], II Safarova, V.P.Mayboroda, IE Troyanovskiy et al. [6-11]. In [14,15] proposed decision of axially symmetric problem of propagation of stress waves in a layer at the half. It is assumed viscoelastic behavior of the layer materials and half according to the concept of complex elastic moduli. The source of voltage harmonic waves is focused normal stress on the surface of the layer. Required to find the vertical $\vartheta_\kappa(x,z,t)$ and horizontal $u_\kappa(x,z,t)$ move layer pixels and half-plane (a = 1,2,3....n). Thus to the viscoelastic medium is characterized by Lamé operator modules [12]:

$$\widetilde{E}_\kappa f(t) = E_{0\kappa}\left[f(t) - \int_{-\infty}^{t} R_{E\kappa}(t-\tau) f(t) d\tau \right].$$

Here *f(t)*– arbitrary function of time; $R_{E\kappa}(t-\tau)$ - core relaxations, $E_{0\kappa}$ – Young's module of elasticity instant (k = 1...... N), if $R_{E\kappa}(t-\tau)=0$, then layered - homogeneous elastic medium is characterized by constant Lame $\lambda_\kappa, \mu_\kappa$, density ρ_κ, velocities of longitudinal and transverse waves

$$c_{p\kappa} = \left(\frac{\lambda_\kappa + 2\mu_\kappa}{\rho_\kappa}\right)^{1/2},\ c_{s\kappa} = \left(\frac{\mu_\kappa}{\rho_\kappa}\right)^{1/2} \quad [13].$$

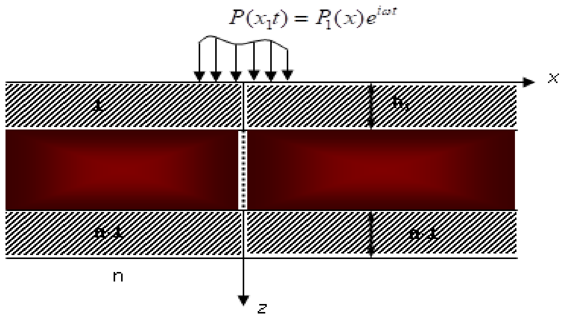

Fig.1: Design scheme

The equations of motion for the second medium can be written as

$$\rho_\kappa \frac{\partial^2 u_\kappa}{\partial t^2} = \frac{\partial \sigma_{11}^{(\kappa)}}{\partial x} + \frac{\partial \sigma_{13}^{(\kappa)}}{\partial z};$$

$$\rho_\kappa \frac{\partial^2 \vartheta_\kappa}{\partial t^2} = \frac{\partial \sigma_{33}^{(\kappa)}}{\partial z} + \frac{\partial \sigma_{13}^{(\kappa)}}{\partial x}. \qquad (\kappa = 1, 2, 3 \ldots N) \qquad (1)$$

Where ρ_κ - material density, σ_{ij}^k - components of the stress tensor

$$\sigma_{11}^{(\kappa)} = E_{1\kappa}\left(\frac{\partial u_\kappa}{\partial x} + \nu_\kappa \frac{\partial \vartheta_\kappa}{\partial z}\right);$$

$$\sigma_{22}^{(\kappa)} = E_{1\kappa}\left(\frac{\partial \vartheta_\kappa}{\partial z} + \nu_\kappa \frac{\partial u_\kappa}{\partial x}\right), \qquad (2)$$

$$\sigma_{12}^{(\kappa)} = G_\kappa\left(\frac{\partial u_\kappa}{\partial z} + \frac{\partial \vartheta_\kappa}{\partial x}\right).$$

$$E_{1\kappa} = \frac{\bar{E}_\kappa}{1-\nu_k^2}, G = \frac{\bar{E}_\kappa}{1+\nu_\kappa}.$$

Movements u_k and ϑ_k satisfy one of the following boundary conditions on the surface of the laminated body:

I. $\sigma_{33}^1 = -q_1(x,t), \sigma_{13}^1 = -q_2\ (x,t)$ at z=0;

II. $\vartheta_1 = q_1(x,t),\ \sigma_{13}^1 = -q_2\ (x,t)$ atz=0; (3,a)

III. $\vartheta_1 = q_1(x,t),\ u_1 = -q_2\ (x,t)$ at z=0.

And zero initial conditions

$$u_k = \vartheta_k = \frac{\partial u_k}{\partial t} = \frac{\partial \vartheta_k}{\partial t} = 0 \text{ at } t=0. \qquad (3,б)$$

In the case of exposure to harmonic loads on the surface of the laminated body and the boundaries between layers of viscoelastic run hard contact conditions, then the boundary conditions take the form:

$$\sigma_{33}^1(x,0,t) = -p(x)e^{-i\omega t},$$

$$\sigma_{13}^1(x,0,t) = 0,$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (4)$$

$$\sigma_{33}^k(x,h_k,t) = \sigma_{33}^{k+1}(x,h_k,t),$$

$$\sigma_{13}^k(x,h_k,t) = \sigma_{13}^{k+1}(x,h_k,t),$$

$$u_k(x,h_k,t) = u_{k+1}(x,h_k,t),$$

$$\vartheta_k(x,h_k,t) = \vartheta_{k+1}(x,h_k,t),\ \kappa = 1.2\ldots\ldots n.$$

Determining displacement and stress amplitude versus frequency for different parameters planar layered bodies when exposed harmonic load is the main goal of.

Methods of solution

The solution of this equation (1) in the form:

$$u_\kappa = U_\kappa(z)\,\mathrm{e}^{\mathrm{i}(\gamma x - \omega t)};$$

$$\vartheta_\kappa = V_\kappa(z)\,\mathrm{e}^{\mathrm{i}(\gamma x - \omega t)}, \qquad (6),$$

ω – angular frequency, which is a complex value with spectral problems and the actual value for the forced oscillations; $U_\kappa(z)$ and $V_\kappa(z)$ - displacement amplitude; λ=2π/γ - wave length, γ- wave number. Substituting (6) and (1) with (2), we obtain the following ordinary differential equation:

$$L_k \frac{dV_k}{dz} - L_{2k} U_k - G_k \frac{d^2 U_k}{dz^2} = 0;$$

$$L_k \frac{dU_k}{dz} - L_{3k} V_k - L_{4k} \frac{d^2 V_k}{dz^2} = 0. \qquad (7)$$

Where

$$L_{1k} = \left(\frac{\bar{E}_k \nu_k}{1-\nu^2{}_k} + G_k\right) i\gamma;\ L_{2k} = \rho_k \omega^2 - \frac{\bar{E}_k}{1-\nu_k^2}\gamma^2;$$

$$L_{3k} = \rho_k \omega^2 - G_k \gamma^2;\quad L_{4k} = \frac{\bar{E}_k}{1-\nu_k^2}.$$

After the introduction of the auxiliary function $\Phi_\kappa(z)$ relationships

$$U_\kappa = L_{1\kappa} \frac{d}{dz}\Phi_\kappa;\ ; V_\kappa = \left[L_{2\kappa} + G_\kappa \frac{d^2}{dz^2}\right]\Phi_\kappa, \qquad (8)$$

we obtain from (8) differential equation of the fourth order

$$\frac{d^4 \Phi_\kappa}{dz^4} + L_{5\kappa} \frac{d^2 \Phi_\kappa}{dz^2} + L_{6\kappa}\Phi_\kappa = 0, \qquad (9)$$

where $L_{5\kappa} = -2\gamma^2 + \frac{(3-\nu_\kappa)\omega^2}{2c_{s\kappa}^2 \Gamma_\kappa}$;

$$L_{6\kappa} = \gamma^2 - \frac{(3-\nu_\kappa)\omega^2\gamma^2}{2c_{s\kappa}^2 \Gamma_\kappa} + \frac{\omega^4}{c_{p\kappa}^2 c_{s\kappa}^2 (\Gamma_\kappa)^2};$$

$$\Gamma_\kappa = 1 - \Gamma_\kappa^C(\omega) - i\Gamma_\kappa^S(\omega);$$

$c_{p\kappa}^2 = (\lambda_\kappa + 2\mu_\kappa)/\rho_\kappa;\ c_{s\kappa}^2 = \mu_\kappa/\rho_\kappa -$ the speed of propagation of longitudinal and transverse waves. The solution of equations (9) is expressed by the exponential function

$$\Phi_\kappa(y) = A_\kappa e^{+\alpha_\kappa y} + B_\kappa e^{-\alpha_\kappa y} + C_\kappa e^{S_\kappa y} + D_\kappa e^{-S_\kappa y} \quad (10)$$

Where $q_\kappa^2 = \gamma^2\left(1 - \frac{c^2}{\bar{c}_{p\kappa}^2}\right);\quad S_\kappa^2 = \gamma^2\left(1 - \frac{c^2}{\bar{c}_{S\kappa}^2}\right),$

$$\bar{c}_{s\kappa}^2 = c_{s\kappa}^2 \Gamma_\kappa, \bar{c}_{p\kappa}^2 = c_{p\kappa}^2 \Gamma_\kappa,$$

A_k, B_k, C_k, D_k - complex arbitrary constants, which are determined from the boundary conditions (4). The arbitrary constants determined from the system of algebraic equations with complex coefficients

$$\left[C(\omega, c_{sk}, c_{pk}, \Gamma_\kappa, h_k)\right]\{q\} = \{p\}, \qquad (11)$$

where $[C]$- square matrix (2k x 2k), which elements consist of exponential functions with complex parameters, $\{q\}$- contains unknown, i.e., A_k, B_k, C_k, D_k; $\{p\}$- it consists of the amplitude of external loads. We define the dependence of the resonance amplitudes $A_{\eta mj}$ (η -voltage components (σ_{le} , l,e=1,2,3), and moving u_l, m- number coordinate, j-number of the resonance frequency) of the parameters of the mechanical system. Depending on the construction of this algorithm includes the construction of the amplitude-frequency characteristics for each component of the displacement and stress amplitude and finding maxima $A_{\eta mj}$ on each of them. To find the highs and lows Muller method is applicable to the resonance curve [11]. Amplitude - frequency response - a curve described by the equation $A_{\eta mj} = \left| A_{\sigma mj} \right|$. Her minima and maxima satisfy the equation

$$\frac{\partial A_{\eta mj}(\omega)}{\partial \omega} = 0. \qquad (12)$$

It is necessary to find the roots of the equation (12). For this purpose it is necessary to calculate the the left side of (12) at specified values $\omega = \omega_1, \omega_2, \ldots\ldots \omega_n$. Choosing $\Delta\omega = 10^{-4} - 10^{-6}$, by the formula derivatives

$$\frac{\partial A_{\eta mj}(\omega)}{\partial \omega} = \frac{A_{\eta mj}(\omega_j + \Delta\omega) - A_{\eta mj}(\omega_j - \Delta\omega)}{2\Delta\omega} = 0.$$

The system of algebraic equations (11) is solved by Gauss with the release of the main element. Using the solutions of (6), (8) and (10) we find the expressions for the displacement-th layer

$$u_\kappa = i\gamma[A_\kappa q_\kappa \exp(q_\kappa z) - B_\kappa q_\kappa \exp(-q_\kappa z) + C_\kappa S_\kappa \exp(S_\kappa z) - D_\kappa S_\kappa \exp(-S_\kappa z)]e^{i(\gamma x - \omega t)},$$
$$\vartheta_\kappa = [-A_\kappa q_\kappa \exp(q_\kappa z) - B_\kappa q_\kappa \exp(-q_\kappa z) - C_\kappa \gamma^2 \exp(S_\kappa z) - D_\kappa \gamma^2 \exp(-S_\kappa z)]e^{i(\gamma x - \omega t)}. \qquad (13)$$

Movements (13) for the half-satisfy exponential decay coordinate z (z $\to$ ∞). As an example, consider the fluctuations layer lying on a half harmonic of the vertical load. Then, the main determinant of the (11) (6x6) $[C]$- It takes the following form

$$[C] = \begin{vmatrix} (1+\bar{S}_1^2)\, e^{-\bar{\xi}q_1} & (1+\bar{S}_1^2)\, e^{\bar{\xi}q_1} & -2e^{\bar{\xi}q_1} & 2e^{\bar{\xi}q_1} & 0 & 0 \\ -2\bar{q}_1^{\,\xi\dot{q}_{11}} & 2\bar{q}_1^{\,\xi\dot{q}_{11}} & \left(\bar{s}_1 + \frac{1}{\bar{s}_1}\right)e^{-\bar{\xi}q_1} & \left(\bar{s}_1 + \frac{1}{\bar{s}_1}\right)e^{\bar{\xi}q_1} & 0 & 0 \\ (1+S_1^2)\, e^{\bar{\xi}q_1} & (1+\bar{S}_1^2)\, e^{-\bar{\xi}q_1} & -2e^{\xi\bar{S}_1} & 2e^{-\bar{\xi}q_1} & (1+s^2)\gamma_1 & -2/\gamma_1 \\ -2\bar{q}_1^{\,\xi\dot{q}_{11}} & 2\bar{q}_1^{\,\xi\dot{q}_{11}} & \left(\bar{s}_1 + \frac{1}{\bar{s}_1}\right)e^{\bar{\xi}q_1} & \left(\bar{s}_1 + \frac{1}{\bar{s}_1}\right)e^{-\bar{\xi}q_1} & \frac{2\bar{q}_1}{\gamma_1}{}_1 & \left(\bar{s} + \frac{1}{\bar{s}}\right)/\gamma \\ e^{\xi\bar{q}_1} & e^{\xi\bar{q}_1} & e^{\xi\bar{q}_1} & e^{\xi\bar{q}_1} & -1 & -1 \\ \bar{q}e^{\xi\bar{q}_1} & \bar{q}_{1\,1}e^{-\xi\bar{q}_1} & -\frac{1}{\bar{s}}e^{\xi\,\bar{s}} & -\frac{1}{\bar{s}}e^{\xi\,\bar{s}} & -\bar{q} & \frac{1}{s} \end{vmatrix},$$

Where ζ – dimensionless wave number, ζ = γ h, γ_1 $\bar{\mu}_{1/}$ / $\bar{\mu}$ Consider two versions of the mechanical system. For the numerical solution of the problem the following options were used:

$$c_{p1} = 5400 m/c, c_{p1} = 3195 m/c, c_{s1} = 2300 m/c,\ c_{s2} = 1311 m/c,$$
$$\nu_1 = 0.30,\ \nu_1 = 0.35,\ \rho = 0.283 МПа\, c^2 м^{-4}, \rho = 0.126 МПа\, c^2 м^{-4}$$

In the calculations acceptance dimensionless parameters:

$$c_p^1 = \frac{3195}{5400} = 0{,}5917, c_s^1 = \frac{2300}{5400} = 0{,}4259, c_s^2 = \frac{1311}{5440} = 0{,}2428, c_{ss}^2 = \frac{1311}{3195} = 0{,}4103, \rho_0 = \frac{0{,}126}{0{,}283} = 0{,}4452.$$

In the first case considered homogeneous dissipative mechanical system. The calculation results of the first embodiment shown in Figure 2, a, b, c, d. The dimensionless amplitude of the resonance

$\left|A_{\sigma 33jk}\right| = \left|\dfrac{A_{\sigma 33jk}}{\sigma_p}\right|$ ($\left|A_{\sigma 33jk}\right|$ – voltage amplitude, $\left|\sigma_p\right|$ - the amplitude of the voltage load current) () on the parameter h/λ It changed monotonically (Figure 2a). The corresponding amplitude characteristics -chastity voltages for the three sections (Figure 2a) are shown in Figure 2, b, c, d. This amplitude -chastity characteristics are fully consistent with the results of [7]. In the second embodiment, the resilient layer: $R_1 = 0$, other parameters coincide with those adopted above. The calculation results are presented in Figure 3, a, b, c, d, according to which at maximum approximation of the natural frequencies of the corresponding amplitudes $\left|A_{\sigma 3311}\right|$ and $\left|A_{\sigma 3312}\right|$ become equal [11]. Addiction $\left|A_{\sigma 33jk}\right|$ a parameter h/λ It turned out to be non-monotonic for inhomogeneous dissipative mechanical systems. The damping properties of the system as a whole are determined by the maximum amplitude of the resonance with forced vibrations (let's call it global resonance amplitude). The intensity of the dissipative processes in the system, the higher the lower the global resonance amplitude:

$$\delta_{\sigma\kappa} = \max_{\omega}(\left|A_{\sigma jk}\right|),\ \kappa=1,2,\ldots\ldots N. \qquad (14)$$

In dissipative homogeneous system performs the role of global amplitude for all values of the first resonant amplitude. In dissipative inhomogeneous system as a global amplitude acts depending on the value of the parameter as the first and second resonant amplitude. "Turn the Tables" occurs when the characteristic value of the parameter at which the real part of the natural frequencies of the closest.

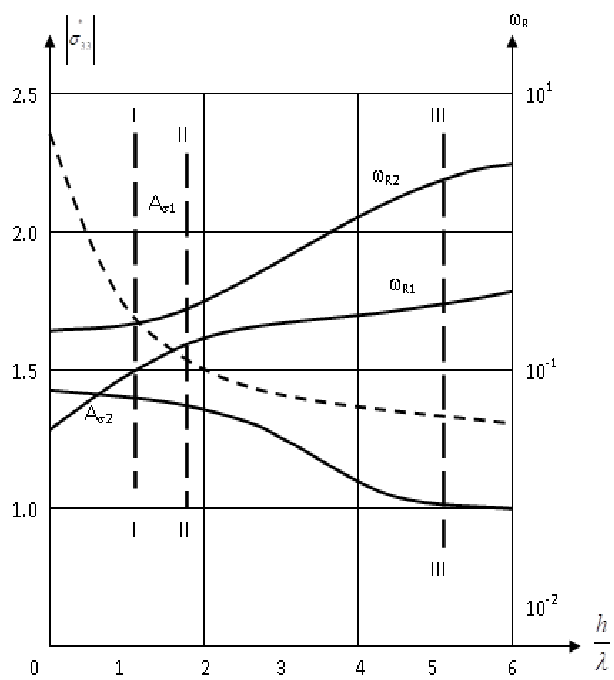

Fig.2a: The dependence of the amplitude of the resonance h/λ (dissipative homogeneous system).

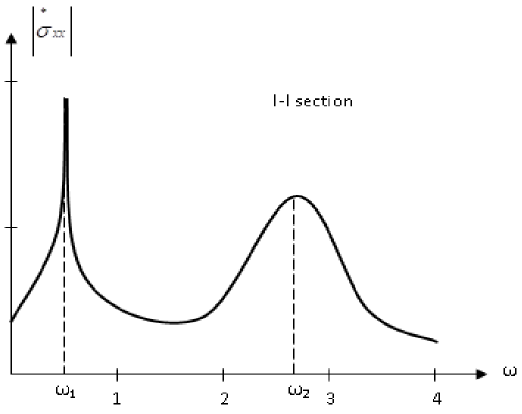

Fig.2.b: Amplitude - frequency characteristics (I-I section) (dissipative homogeneous system).

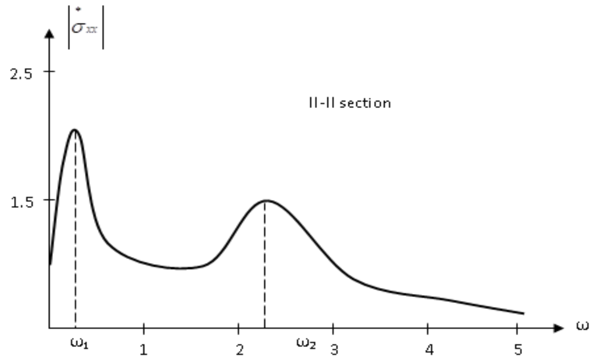

Fig.2c: Amplitude - frequency characteristics (II-II section) (dissipative homogeneous system).

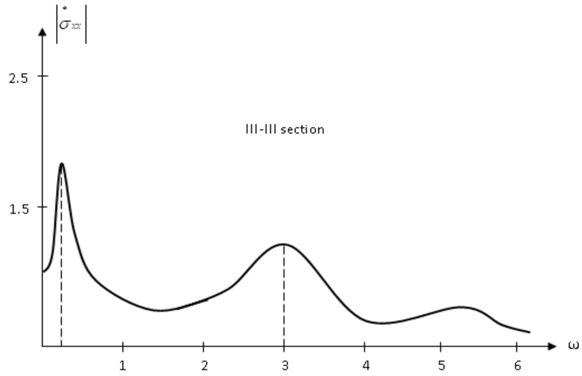

Fig.2g: Amplitude - frequency characteristics (III-III section) (Dissipative homogeneous)

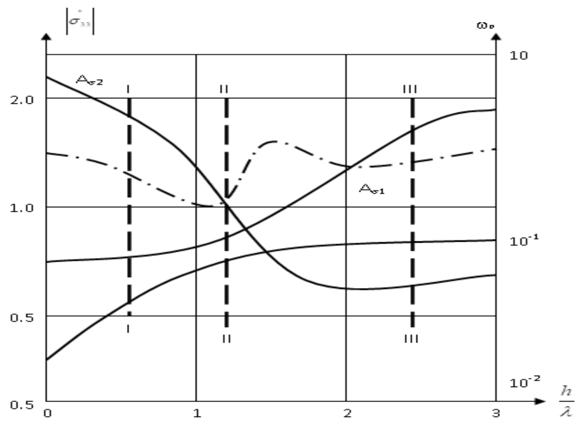

Fig. 3a: The dependence of the amplitude of the resonance h/λ (dissipative heterogeneous mechanical system).

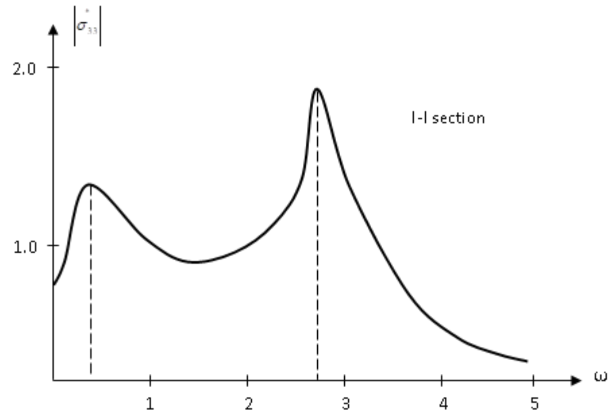

Fig 3b: Amplitude - frequency characteristics (I-I cross section) (Dissipative heterogeneous mechanical system)

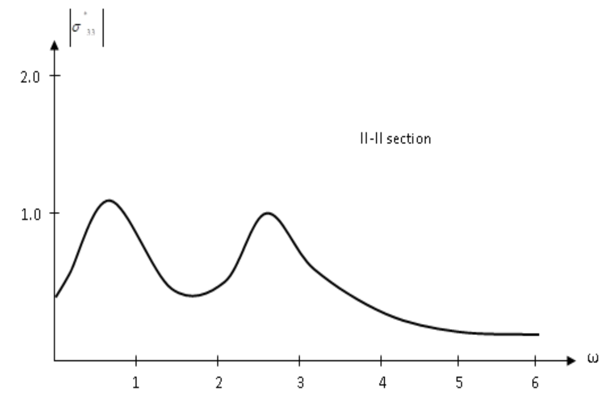

Fig 3c: Amplitude - frequency characteristics (II-II cross section) (Dissipative heterogeneous mechanical system)

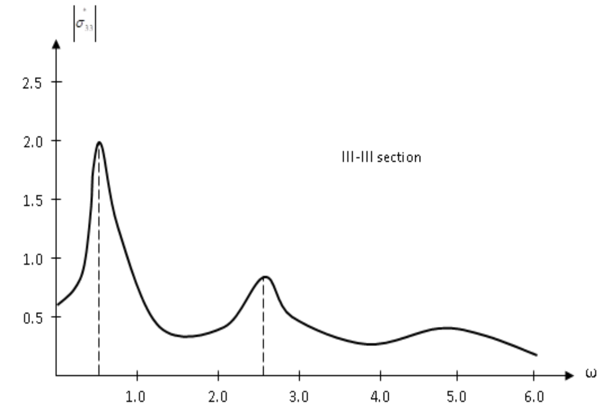

Fig.3g: Amplitude - frequency characteristics (III-III section) (Dissipative heterogeneous mechanical system)

At this value of the global resonant amplitude is minimal and therefore dissipative processes in the system occur most intensively, and global damping coefficient has a pronounced maximum (or minimum) [7,8,9,11]. From the simulation results can be seen that the minimum value of the amplitude of the voltage is achieved in the second sections for inhomogeneous dissipative mechanical systems (Figure 3, c).

Conclusions

The damping properties of the mechanical system with forced oscillations is determined by the maximum resonant amplitude (global resonance amplitude). The intensity of the dissipative processes in the system, the higher the global lower resonant amplitude (Figure 3, c). In a homogeneous dissipative mechanical system, the role of the global amplitude values for all parameters performs first resonance amplitude, and dissipative inhomogeneous -to depending on the size of (the global amplitude) setting both the first and second resonant amplitude. "Change roles" occurs when the characteristic value of the parameter at which the real part of the natural frequencies of the closest. At this value of the global resonant amplitude is minimal and therefore dissipative processes in the system occur most intensively, and global damping coefficient has a pronounced maximum [7,8,9,11].

References

1. Thomson W. T. Transmission of elastic waves through a stratified solid material//J. Appl. Phys. – 1950. - 21,№1.-P. 89-93.
2. Haskell N. A. The dispersion of surface waves in multilayered media //Bull. Seism. Soc. Amer. – 1953. - 43, №1. –P. 17-34.
3. L.A. Molotkov/ on the propagation of elastic waves in media containing thin plane-parallel layers // Problems. dyn. the theory of seismic. waves.- 1961.-№5.- pp 240-280.
4. Dunkin I. W. Computation of modal solutions in layered elastic media at high frequencies // Bull. Seism. Soc. Amer. – 1965. - 55, №2. –P. 335-358.
5. Kayumov S., Safarov I.I. Propagation and diffraction of waves in a dissipative - inhomogeneous cylindrical deformable mechanical systems. Tashkent: 2002, 214p
6. Safarov I.I. Wave propagation in the fiber lying in the visco-elastic half-space, Abstracts of Conf. "The experience of the application of composite materials in agricultural engineering", CHERNYAGO, 1985, p.91-92.
7. Safarov I.I., Mayboroda V.P., Troyanovskiy I.E., Vozagashvali M.G. Waves in the deformable layer halfspace. - Calculations of strength, vyp.25, M.: 1984. p.213-220.
8. Safarov I.I., Teshaev M.Kh., Boltaev Z.I., Axmedov M.Sh.Coommon natural in dissipative inhomogl eovs plane Bodies/Discove 2016,52,(251) 2108-2126.
9. Safarov I.I., Teshaev M.Kh., Axmedov M.Sh. Vibrations dissipative plate mechanical systems with point//The International Ouarterlu journal/Science **&** Technology, 2016, 2(8).P.437-450.
10. Safarov I.I, Akhmedov M. Sh.,Qilichov O.Dynamics of underground hiheline from the flowing fluid.. Lambert Academic Publishing (Germany). 2016. 345p.
11. Safarov I.I., Akhmedov M.Sh., Boltaev Z.I. Proper waves in layered media. Lambert Academic Publishing (Germany). 2016 192p.
12. Koltunov M.A. Creep and relaksatsiya.- M.: Higher School, 1976.
13. W. Nowacki elasticity theory. - M.: Mir, 1975, 872 p.
14. Martinchek G. Dynamic viscoelasticity in the technical application // Success Mechanics, 1983. Volume 6, no. 3/4, 1-28p.
15. Martinchek G. Determination of dynamic viscoelastic characteristics of the method of mechanical impedance // Mechanics of Composite Materials. Riga. 1080. №4.- 708-712.