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Mathematical Study of Diabetes Using the Homotopy Perturbation Method

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Abstract
Diabetes is one of the most widespread metabolic disorders. The aim of mathematical modeling is to help a patient and his doctor in management of diabetes.We studied a ordinary differential mathematical model of diabetes using the Homotopy. This method is very effective and convenient for solving nonlinear differential systems.equations. Perturbation method and the result obtained shows that the parameters involved played a crucial role in the size of population of diabetes at time t and the number of diabetics with complication. The proposed method does not require small parameters in the equations, so the limitations of the traditional perturbation methods can be eliminated. The approximations obtained by this method are valid not only for small parameters, but also for very large parameters

Keywords: Glucose;Insulin;Differential Systems;Diabets,Homotopy method

Introduction

Statistics related to diabetes are plain shocking. Approximately 17% of Romania's population suffers from this condition and, according to the WHO, every year 2 million people die because of diabetes or diabetes related complications. At the same time, there are only few clinics where diabetics can receive qualified help (IFD report**:** 2017).

There may be a small statistical error, but roughly 2 million people die every day in the world. In Romania, according to certain approximative calculations, between 125,000 and 230,000 people die because of diabetes. In the absence of a qualified support for the body, diabetes leads to all kinds of complications. They may include certain problems with various organs or some other kinds of related complications.

In Romania, there is no specialized program to fight diabetes and often there is no endocrinologist or qualified diabetologist in our clinics to provide specialized care. In most cases, doctors choose the easy way out and fail to carry out detailed tests, simply prescribing insulin. Therefore, people are on their own in the fight with this condition (Mayo :2004)

Diabetes is one of the most common non transmissible diseases and the most frequent endocrine disease, characterized by disorders at the level of the entire metabolism (especially carbohydrates) and by complications that affect the eyes, kidneys, nerves and blood vessels. Diabetes is of two types: type 1 and type 2 diabetes.

In type 1 or insulin-dependent diabetes, the pancreas no longer excretes insulin or excretes a very small quantity, insufficient to maintain glycemia within normal limits, requiring insulin injection. This type of diabetes usually makes its debut in young people.

Type 1 diabetes is found in 5 to 10% of the diagnosed diabetes cases (Boutayeb: 2004).. Autoimmune, genetic and environment factors play a determining role in the development of this type of diabetes. Type 2 diabetes is not insulin-dependent, most patients suffering from this affection being able to carry out normally their daily activities when they manage to keep under control the level of glycemia through physical exercise, proper diet and hypoglycemic medication.

Type 2 diabetes is found in 90 to 95% of the diagnosed diabetes cases. Risk factors for type 2 diabetes include: old age, obesity, previous diabetes cases in the family, diabetes during pregnancy, glucose intolerance, physical inactivity, breed (Afro-Americans Hispanic/Latin-American, Amerindian people and certain Americans of Asian origin have a predisposition for type 2 diabetes). In this study we extended the work of (Boutayeb :2004) by carrying out a mathematical study of diabetes and its complication using the Homotopy perturbation method.

Materials and methods

Fundamentals of Homotopy Perturbation Method (HPM), first proposed by (He:1999 and He: 2000 and He: 2003) has successfully been applied to solve many types of linear and nonlinear functional equations. This method, which is a combination of homotopy in topology and classic perturbation techniques, provides a convenient way to obtain analytic or approximate solutions for a wide variety of problems arising in different fields. This method is a powerful mathematical tool to solve any system of ordinary differential equations linear and nonlinear, it is also a promising method to solver other linear and nonlinear ordinary differential equations (Liao: 1995).

We propose the following general model for the interaction of glucose and insulin (Nayfeh :1981 and Wang: 1986) :

$$\begin{cases} \dfrac{dx}{dt} = -\alpha_1 x - \alpha_2 xy + \alpha_3 \\ \dfrac{dy}{dt} = \beta_1 x - \beta_2 y \end{cases}, x \geq 0, y \geq 0, \begin{cases} x(0) = x_0 \\ y(0) = y_0 \end{cases}, \quad (1)$$

Where:

- x represents glucose concentration
- y represents insulin concentration
- a_1 is the rate constant which represents insulin-independent glucose disappearance
- a_2 is the rate constant which represents insulin-dependent glucose disappearance
- a_3 is the glucose infusion rate
- b_1 is the rate constant which represents insulin production due to glucose stimulation
- b_2 is the rate constant which represents insulin degradation (Boutayeb,2004).

To illustrate the basic ideas of the method, He considered the following
nonlinear differential equation (He:2006) :

$$A(u) - f(r) = 0, r \in \Omega, \quad (2)$$

with boundary conditions: $B(u, \frac{dn}{du}) = 0, r \in \Gamma$ (3)

The homotopy perturbation structure is shown as follows:

$$H(p,v) = (1-p)(L(v) - L(u_0)) + p(A(u) - f(r)) = 0, \quad (4)$$

where:

- $v(r,p): \Omega \to R, p \in [0,1]$. In equation (4), p is an embedding parameter
- $v(r,0) = 0, v(r,1) = 0, r \in \Gamma$
- u_0 is the first approximation that satisfies the boundary condition, A is a general differential operator, $f(r)$ is a known analytical function and Γ is the boundary of the domain Ω. The operator A can be divided into two parts L and N, where L the linear part is, and N is the nonlinear component.

It can be assumed that the solution of equation (.4) can be written as power series as follows:

$$v = v_0 + pv_1 + p^2 v_2 + ... + p^n v_n \quad (5)$$

The best approximation for the solution is:

$$u = \lim_{p \to 1} v = v_0 + v_1 + ...v_n \quad (6)$$

The series (5) is convergent for most cases; however, the convergence rate
depends on the nonlinear operator $A(v)$.

Applying homotopy perturbation to equation differential system (1) we obtain:

$$\begin{cases} (1-p)\dfrac{dx}{dt} + p(\dfrac{dx}{dt} + \alpha_1 x + \alpha_2 xy - \alpha_3) = 0 \\ (1-p)\dfrac{dy}{dt} + p(\dfrac{dy}{dt} - \beta_1 x + \beta_2 y) = 0 \end{cases} \Leftrightarrow \begin{cases} \dfrac{dx}{dt} + p(\alpha_1 x + \alpha_2 xy - \alpha_3) = 0 \\ \dfrac{dy}{dt} + p(-\beta_1 x + \beta_2 y) = 0 \end{cases} \quad (7)$$

Let : $$\begin{cases} x = x_0 + px_1 + p^2 x_2 + ... + p^n x_n \\ y = y_0 + py_1 + p^2 y_2 + ... + p^n y_n \end{cases}, \begin{cases} \dfrac{dx}{dt} = \dfrac{dx_0}{dt} + p\dfrac{dx_1}{dt} + ... + p^n \dfrac{dx_n}{dt} \\ \dfrac{dy}{dt} = \dfrac{dy_0}{dt} + p\dfrac{dy_1}{dt} + ... + p^n \dfrac{dy_n}{dt} \end{cases} \quad (8)$$

Substituting (8) into equation system (7) we have: (9)

$$\begin{cases} \dfrac{dx_0}{dt} + p\dfrac{dx_1}{dt} + p^2 \dfrac{dx_2}{dt} + p[\alpha_1 (x_0 + px_1 + p^2 x_2)\alpha_2 (x_0 + px_1 + p^2 x_2)(y_0 + py_1 + p^2 y_2) - \alpha_3] = 0 \\ \dfrac{dy_0}{dt} + p\dfrac{dy_1}{dt} + p^2 \dfrac{dy_2}{dt} + p[-\beta_1 (x_0 + px_1 + p^2 x_2) + \beta_2 (y_0 + py_1 + p^2 y_2)] = 0 \end{cases}$$

$$p^0:\begin{cases}\dfrac{dx_0}{dt}=0\to x_0=C\to C=x_0,\\ \dfrac{dy_0}{dt}=0\to y_0=D\to D=y_0,\end{cases}\text{,when : },\begin{cases}x(0)=x_0\\ y(0)=y_0\end{cases}\tag{10}$$

$$p^1:\begin{cases}\dfrac{dx_1}{dt}+\alpha_1x_0+\alpha_2x_0y_0-\alpha_3=0\\ \dfrac{dy_1}{dt}-\beta_1x_0+\beta_2y_0=0\end{cases}\tag{11}$$

$$p^2:\begin{cases}\dfrac{dx_2}{dt}+\alpha_1x_1+\alpha_2x_1y_0+\alpha_2y_1=0\\ \dfrac{dy_2}{dt}-\beta_1x_1+\beta_2y_1=0\end{cases}\tag{12}$$

From equations (10) we get: $\begin{cases}x_0(t)=x_0\\ y_0(t)=y_0\end{cases}$ (13)

From equation (11),we obtain:

$$\begin{cases}\dfrac{dx_1}{dt}=-\alpha_1x_0-\alpha_2x_0y_0+\alpha_3\\ x_1(0)=0\\ \dfrac{dy_1}{dt}=\beta_1x_0-\beta_2y_0\\ y_1(0)=0\end{cases}\to\begin{cases}x_1(t)=(-\alpha_1x_0-\alpha_2x_0y_0+\alpha_3)t+E\\ x_1(0)=0\to E=0\\ y_1(t)=(\beta_1x_0-\beta_2y_0)t+F\\ y_1(0)=0\to F=0\end{cases}\tag{14}$$

From equations (14) we get: $\begin{cases}x_1(t)=(-\alpha_1x_0-\alpha_2x_0y_0+\alpha_3)t\\ y_1(t)=(\beta_1x_0-\beta_2y_0)t\end{cases}$ (15)

From equations (12) and (15), we obtain equations:

$$\begin{cases}\dfrac{dx_2}{dt}=-\alpha_1(-\alpha_1x_0-\alpha_2x_0y_0+\alpha_3)t-\alpha_2y_0(-\alpha_1x_0-\alpha_2x_0y_0+\alpha_3)t-\alpha_2(\beta_1x_0-\beta_2y_0)t\\ \dfrac{dy_2}{dt}=\beta_1(-\alpha_1x_0-\alpha_2x_0y_0+\alpha_3)t-\beta_2(\beta_1x_0-\beta_2y_0)t\end{cases}\tag{16}$$

Integrating with respect to t, relations (16), we have:

$$\begin{cases}x_2(t)=\dfrac{1}{2}[-\alpha_1(-\alpha_1x_0-\alpha_2x_0y_0+\alpha_3)-\alpha_2y_0(-\alpha_1x_0-\alpha_2x_0y_0+\alpha_3)-\alpha_2(\beta_1x_0-\beta_2y_0)]t^2+M\\ x_2(0)=0\to M=0\\ y_2(t)=\dfrac{1}{2}[\beta_1(-\alpha_1x_0-\alpha_2x_0y_0+\alpha_3)-\beta_2(\beta_1x_0-\beta_2y_0)]t^2+N\\ y_2(0)=0\to N=0\end{cases}\tag{17}$$

From equations (17) we get:

$$\begin{cases}x_2(t)=\dfrac{1}{2}[-\alpha_1(-\alpha_1x_0-\alpha_2x_0y_0+\alpha_3)-\alpha_2y_0(-\alpha_1x_0-\alpha_2x_0y_0+\alpha_3)-\alpha_2(\beta_1x_0-\beta_2y_0)]t^2\\ y_2(t)=\dfrac{1}{2}[\beta_1(-\alpha_1x_0-\alpha_2x_0y_0+\alpha_3)-\beta_2(\beta_1x_0-\beta_2y_0)]t^2\end{cases}\tag{18}$$

According to Homotopy Perturbation:

$$\begin{cases} x(t) = \lim_{p\to 1} x_0(t) + p x_1(t) + p^2 x_2(t) \\ y(t) = \lim_{p\to 1} y_0(t) + p y_1(t) + p^2 y_2(t) \end{cases} \leftrightarrow \begin{cases} x(t) = x_0(t) + x_1(t) + x_2(t) \\ y(t) = y_0(t) + y_1(t) + y_2(t) \end{cases} \quad (19)$$

From equations (17) we obtain the solution of differential system (1):

$$\begin{cases} x(t) = x_0 + x_1(t) + x_2(t) \\ x_1(t) = (-\alpha_1 x_0 - \alpha_2 x_0 y_0 + \alpha_3)t \\ x_2(t) = \dfrac{t^2}{2}[(-\alpha_1 x_0 - \alpha_2 x_0 y_0 + \alpha_3)(-\alpha_1 - \alpha_2 y_0) - \alpha_2(\beta_1 x_0 - \beta_2 y_0)] \\ y(t) = y_0 + y_1(t) + y_2(t) \\ y_1(t) = (\beta_1 x_0 - \beta_2 y_0)t \\ y_2(t) = \dfrac{t^2}{2}[\beta_1(-\alpha_1 x_0 - \alpha_2 x_0 y_0 + \alpha_3) - \beta_2(\beta_1 x_0 - \beta_2 y_0)] \end{cases} \quad (20)$$

Conclusions

In recent years, the application of the homotopy perturbation method in non-linear problems has been developed by scientists and engineers, because this method continuously deforms the difficult problem under study into a simple problem which is easier to solve.In this paper, the homotopy perturbation method has been successfully used to study nonlinear differential system with applications in diabetes diseases. He's homotopy perturbation method which is proved to be a powerful mathematical tool for the study of nonlinear oscillators, can be easily extended to any nonlinear oscillator problem. The solutions obtained are in good agreement with exact values.

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