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Modelling of Herschel-Bulkley fluid flow across an Aneurysm Arteries

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Abstract
In this paper, the velocity, viscosity and the flow rate of blood have been used to model the pressure drop across an aneurysm artery. The artery under consideration for the development of model is cylindrical with an axially symmetric aneurysm and the blood flow through in is couple of two layers. The results show that the size of aneurysm has an inversely proportion relation with the pressure drop. It is also observed that this model can be used to estimate pressure drop and viscosities of couple fluid flow of two layers, but in case of single flow could not.

Keywords: Herschel-Bulkley, pressure drop, Aneurysm and viscosity.

Introduction

Aneurysm is an enlarged the size of arteries caused by a weakening of the arteries wall. This weakening can lead to rupture of blood artery at aneurysm location, which can cause internal bleeding and stooped the blood supply. For instance, rupture an aneurysm in the artery which is supplying the blood to brain, can bring strokes, and likewise rupture in abdominal artery leads to death from internal bleeding (Musad 2016, 2021)

Many mathematical models are used to study hemodynamic parameters, specially, shear stress, pressure and viscosity. Herschel-Bulkley model is a generalized model of non-Newtonian fluid and considered to understand the blood flow characteristics in small arteries (Sanker and Usik 2008).

The viscosity of blood is depending on velocity, temperature and the size of blood vessel. In small blood vessels and at high velocities, blood viscosity apparently decreases with decreasing vessel size and it begins to play a role in vessels smaller than 1 mm in diameter (Fahraeus -Lindqvist effect). In principle, the pressure drops over a blood vessel and the flow through it, together with vessel size, can be used to derive viscosity on the basis of Poiseuille's law (Marc 2008)

There are two physical attribute of blood vessels one of them have a resistance to flow and so they need a pressure drop along the length of the vessel to drive the blood flow. The total volume of blood is about 4.5–5.0 L, so all the blood is pumped throughout the body every minute or about 80 cm^3/s = 4.8 L/min. The flow rate in the arteries, arterioles, capillaries, venues, and veins are all the same. (Irving 2007) have modeled, in normal case, the arteries as n parallels vessels and the flow rate in each vessel is Q_t/n, and reported that the arterioles, capillaries and venules act like resistance vessels.

Develop the Mathematical Model

We have consided the steady flow of Herschel-Bulkiey fluid and the blood flow is couple with two layers of two difference viscosities. While the artery under consideration for the development of model is rigid and cylindrical with an axially symmetric aneurysm of radii $R_1(z)$ and $R_2(z)$. The geometric of the assuming artery and its coordinates system is given in figure (1), equation (1) and equation (2).

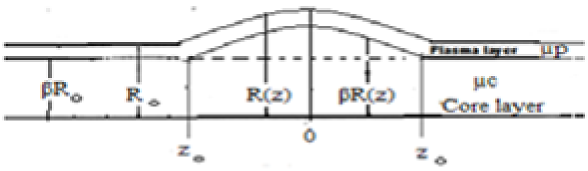

Fig.1: The boundary equation of aneurysm artery with two layers

$$R_1(z)=\beta R(z)=\begin{cases}\beta r+\dfrac{\beta h}{2}+\dfrac{\beta h}{2}COS(\pi\dfrac{z}{z_0})), & -z_0<z<z_0,\\ \beta r \quad z_0\le z\le -z_0\end{cases} \qquad (1)$$

$$R_2(z)=\begin{cases}r+\dfrac{h}{2}+\dfrac{h}{2}COS(\pi\dfrac{z}{z_0})), & -z_0<z<z_0,\\ r \quad z_0\le z\le -z_0\end{cases} \qquad (2)$$

Based on the conditions mentioned above for H-B fluid the motion can be expressed as

$$\frac{1}{r}\frac{\partial}{\partial r}(r(\mu(\frac{\partial v}{\partial r})^n+\tau_0))=p(z) \qquad (3)$$

Where p(z) is the pressure gradient, T_0 is the yield stress, μ is the viscosity of blood, h is the height of aneurysm n is related to the behavior of fluid flow and ß is the ratio of the viscosities of two layers

Integrating equation (3) we get

$$\mu(\frac{\partial v}{\partial r})^n+\tau_0=\frac{p(z)r}{2}$$

$$\mu(\frac{\partial v}{\partial r})^n=\frac{p(z)r}{2}-\tau_0 \qquad (4)$$

Where T_0 can be expressed as

$$\tau_0=\frac{p(z)R_1(z)}{2}$$

Then equation (4) can be written as

$$\mu(\frac{\partial v}{\partial r})^n=\frac{1}{2}p(z)r-\frac{1}{2}p(z)R_1(z) \qquad (5)$$

Equation (5) is the modify equation of H-B fluid flow through aneurysm artery of two layers.

The principle, in case of arterials disease such aneurysm, the viscosity, velocity and the flow through it, together with arterial size, can be used to derive the pressure drop across an aneurysm arterial, then based on equation (5)

$$(\frac{\partial v}{\partial r})=(\frac{P(z)}{2\mu})^{1/n}(r-R_1(z))^{1/n} \qquad (6)$$

Integrate equation (6) with respect to r

$$v=\frac{n}{n+1}(\frac{P(z)}{2\mu})^{1/n}(r-R_1(z))^{(1/n)+1} \qquad (7)$$

Equation (7) is the equation of velocity where

$$v=\frac{n}{n+1}(\frac{P(z)}{2\mu})^{1/n}(R_2(z)-R_1(z))^{(1/n)+1}$$ at $r=R_2(z)$

$v=0$ at $r=R_1(z)$

The flow Q can be determine by the equation

$$Q=\frac{n}{n+1}\pi(R_2)^2(z)(\frac{P(z)}{2\mu})^{1/n}(R_2(z)-R_1(z))^{(1/n)+1}$$

We know that $R_1(z)=ßR_2(z)$ where $0.75<ß<1$

$$Q=\pi\frac{n}{n+1}(\frac{P(z)}{2\mu})^{1/n}[R_2(z)(1-\beta))]^{(1/n)+3} \qquad (8)$$

Solve equation (8) for p(z)

$$P(z)=\frac{\partial p}{\partial z}=\frac{2(n+1)\mu Q^{n}}{4\pi n(1-\beta)R^{4}(1+\frac{R}{2}(1+\cos\frac{\pi z}{z_{0}})^{(1+3n)}} \quad (9)$$

Put n=1 and integrate equation (9) to modeled the pressure drop across an aneurysm arterial

$$\Delta P=\frac{z\mu\,(4+9\frac{h}{R}+3\frac{h}{R}^{2}+\frac{5}{3}\frac{h}{R}^{3})}{\pi R^{4}(1-\beta)\,(1+\frac{h}{R})^{7/2})}*\frac{Q}{m}, \quad (10)$$

Equation (10) is the equation of pressure drop Δp, of flow Q, across an arterial aneurysm of length $2z_0$, where R is the original radius ß is the ratio of viscosities, h is the height of aneurysm section and μ, is the viscosity of blood

The Results and Discussion

Equation 10 is the equation of pressure drop across an aneurysm artery and it's of two parts the resistance and the rate of flow. This equation has been solved by using mathematical software for the radius, length and size of difference arteries. The results listed in table 1 show that the resistance of flow rate in the arteriole is higher than the resistance in capillaries. While the results in table 2 is lower as compeered to that of normal per each one, this mean increase the aneurysm leads to decrease the pressure drop. Also from graph 2 it's observed that the viscosity of blood increase when the value of ß increase.

Table 1: Data of pressure drop in normal arteries based on boundary values.

Vessel	Number m	Radius cm	Length cm	Pressure Drop mmgh
Arteriole	$5*10^5$	0.0003	0.6	90
Capillaries	10^{10}	0.0000035	0.2	8

Table 2: Data of pressure drop in aneurysm arteries.

h	Capillaries	Arteriole
0	8.263224	90.56412
0.111111	8.263224	78.9073
0.222222	8.263224	69.16796
0.333333	8.263224	61.17196
0.444444	8.263224	54.62615
0.555556	8.263224	49.24874
0.666667	8.263224	44.80289
0.777778	8.263224	41.09905
0.888889	8.263224	37.98836
1	8.263224	35.35459

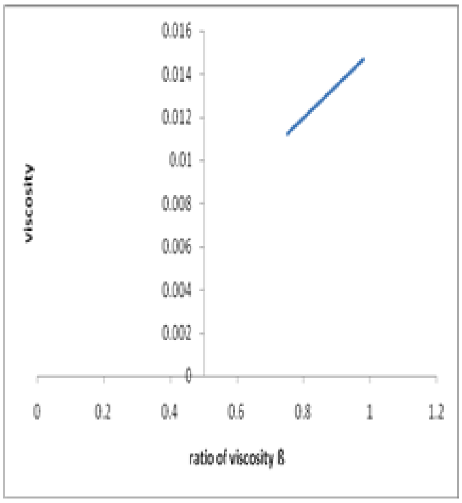

Fig. 2: the relation between ß and viscosity of noncore layer μ_p.

Conclusion

In this study the Herschel-Bulkiey model have been developed under the conditions mentioned in section 2. The results show that the data of pressure drop listed in table 1 is having a very good agreement with the results find by (Irving 2007). Also it's observed that the pressure drop along an aneurysm artery increases with increase both the size of aneurysm and the viscosity of plasma layer. The figure 2 show that increases the size of artery lead to increase the viscosity this agrees with the description mentioned by (Marc 2008).

Finally it's clear that this model con be used to estimate pressure drop and viscosity in case of blood is couple with two layers of two difference viscosities, but could not use in case of single flow.

References

1. Mohammed M. Saleh and Ghamdan M. Abdullah. Modeling of blood flow in an aorta. JMESSP Journal, vol. 7 issue 10, Oct. (2021).
2. Mohammed Musad (2016), Mathematical Modelling of Blood Flow in Aneurysm of Large Artery, journal of numerical and applied mathematics. Vol. 1,(2) 30-34
3. D.s.Sanker and Usik Lee. Two fluid Herschel-Bulkley model for blood flow in catheterized arteries.journal of mechanical science and technology 22(2008)1008-1018)
4. Marc Thiriet, Biology and Mechanics of Blood Flows, Part 1: Biology, Speringer New York (2008)
5. Irving. P. Herman, Physics of the Human Body, Springer Berlin Heidelberg (2007)