WW JMRD 2023; 9(04): 58-69
www.wwjmrd.com
International Journal
Peer Reviewed Journal
Refereed Journal
Indexed Journal
Impact Factor SJIF 2017:
5.182 2018: 5.51, (ISI) 2020-

2021: 1.361
E-ISSN: 2454-6615

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# Modelling the Compressive Strength and Water Absorption of Cement Blended with Periwinkle Shell Ash for Rigid Pavement 

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#### Abstract

Over the years, periwinkle shell ash (PSA) has proven to be a reliable option in partial replacement of cement for construction purposes. In this study, mathematical models for predicting the compressive strength and water absorption of calcined PSA cement concrete are formulated for the purpose of Rigid pavement design. The oxide composition of calcined PSA (at $800^{\circ} \mathrm{C}$ ) was determined. The percentage of PSA by weight of cement varied between $5 \%$ and $25 \%$ in experimental proceedings. The Scheffe's $(5,2)$ simplex lattice theory was adopted in formulation of trial and control mixes and also in the development of second-degree polynomial models. All cement concrete specimens produced were subjected to water absorption and 28 days compressive strength test according to standard experimental procedures. The mathematical models developed for both responses using results from trial mixes were subjected to validation test using results from the control mixes. From the F - statistics and $\mathrm{R}^{2}$ statistics conducted, the derived models for the water absorption and compressive strength proved adequate and can be relied upon in the prediction of water absorption and compressive strength of PSA cement concrete for rigid pavement.


Keywords: Calcined PSA; Rigid Pavement; Scheffe's simplex lattice; water absorption; compressive strength.

## Introduction

The importance of road networks in a locality cannot be over emphasized as it serves to bring vast development through easy accessibility and employment generation for the populace of the area [1]. Rigid pavement is a road pavement type also known as concrete pavement whose strength and durability is heavily reliant on those of the concrete slab. Cement concrete is a material basically composed of cement, aggregates and water. The cement serves to provide the binding property of the material, the fine aggregates reduce shrinkage and cracking, fills voids present in coarse aggregate, the coarse aggregates increase the crushing strength and makes the material a solid hard mass and the water helps to make hydration possible. Admixtures are sometimes added to the concrete mix to improve specific performance of the fresh, plastic or hardened concrete material or to reduce the cost of production. The performance of a concrete material is demonstrated by its inherent properties of which compressive strength and water absorption are very vital [2].
The ability of a concrete material to resist compressive forces, is one basic parameter used in its classification. This property is called compressive strength of concrete. The water absorption capacity of a concrete material is a very important property used to describe the change in mass of concrete due to water percolation. It gives an insight into how durable and permeable a concrete material can be. One of the most important properties of a good quality concrete is low permeability, especially one resistant to freezing and thawing.
Previous works on concrete showed that some of its physical and mechanical properties such as compressive strength and water absorption can be improved by partially replacing cement with pozzolanic materials ([3]; [4]). These materials are called pozzolanic materials because they contain siliceous and aluminous oxides in satisfactory percentages ([5]; [6] ).
The use of periwinkle shell ash (PSA) as a pozzolan has yielded positive results from the
outcome of earlier researches ([3]; [4]). Despite the positive potentials of using PSA in the partial replacement of cement, there are no established mathematical models for predicting PSA cement concrete properties such as compressive strength and water absorption, hence, this study.
The objective of this work is to develop mathematical models for the prediction of compressive strength and water absorption of PSA cement concrete using the Scheffe's second-degree polynomial. The average effect of using calcined PSA (at $800^{\circ} \mathrm{C}$ ) for partial replacement of cement on both responses was also studied. According to [7], the optimum calcination temperature of PSA is $800^{\circ} \mathrm{C}$. This objective was achieved using the Scheffe [8] simplex lattice theory in the development of mix design for experimental purposes.

## Materials and Methods

## Materials

- Periwinkle shell ash (PSA): The periwinkle shell ash was obtained from waste periwinkle shells after calcination. These periwinkle shells were sourced from a waste periwinkle shell assemblage in Aluu community, Rivers State. The periwinkle shells were subjected to a calcination temperature of $800^{\circ} \mathrm{C}$ in a furnace. These calcined periwinkle shells were then left to cool for about 24hours before they were granulated to fine powder producing the PSA. The powdered substance was then subjected to sieve analysis where the portion passing through sieve number 200 was used for the purpose of experiments and duly classified.
- Binder: Dangote 3X (R. 425, CB 4227) Portland cement brand which met the requirements of BS 12 [9] was used as binder. The cement was sourced from a cement distributor shop in Choba, Port Harcourt.
- Fine Aggregate: Fine River sand sourced from a construction site in the University of Port Harcourt community was used as fine aggregate for experimental purposes. The sand was sundried for 48 hours to remove every trace of moisture and was subjected to sieve analysis using an aperture size of $4.75 \mu \mathrm{~m}$ to remove unwanted materials.
- Coarse Aggregate: Granite of maximum size 20 mm was used as coarse aggregate. This material was sourced from a construction site within the University of Port Harcourt, Rivers State. The granite was subjected to cleaning and drying processes, removing all unwanted particles that may hinder granite performance.
- Water: Clean portable water of pH 7 was used for all mixing purposes. This water was obtained from a laboratory in the University of Port Harcourt.


## Methods <br> Design of the Study

This research study is engineered towards the development of mathematical models for the prediction of compressive strength and water absorption of PSA cement concrete for rigid pavement using results from standard experimental procedures. The average effect of using PSA in partially replacing cement for concrete production on the compressive strength and water absorption were also
studied from experimental investigations. The design matrix for experiment was prepared with the aid of the simplex lattice theory propounded by Scheffe (1958). Concrete moulds of size 150 mm X 150 mm X 150 mm were used in preparation of concrete specimens and compressive strength determined after 28 days of curing by complete immersion in water. The Scheffe's second-degree polynomial was used in the development of mathematical models using data from the design trial mixes. The data from design control mixes were used in models' validation.

## Materials' Classification

- Periwinkle shell ash: The oxide composition of PSA was determined in a chemical laboratory in Port Harcourt and was classified in accordance to ASTM C618 [10].
- Aggregates: Gradation, fineness modulus and USCS method of classification were used to classify the sand and granite used. The gradation and fineness modulus were determined according to IS - 383 [11] and IS 2386 [12] respectively. The USCS method uses the coefficients of uniformity and curvature obtained from the particle size distribution curve in the classification of aggregates. The coefficients were obtained using Equations (1) and (2) respectively.

Coefficient of Uniformity, $\mathrm{C}_{\mathrm{U}}=\frac{D_{60}}{D_{10}}$
Coefficient of Curvature, $\mathrm{C}_{\mathrm{C}}=\frac{D_{30}^{2}}{D_{60} X D_{10}}$
Where:
$\mathrm{D}_{60}=$ particle size corresponding to $60 \%$ finer particles
$\mathrm{D}_{30}=$ particle size corresponding to $30 \%$ finer particles
$\mathrm{D}_{10}=$ particle size corresponding to $10 \%$ finer particles
An aggregate is considered well graded when these conditions are met: $4<\mathrm{C}_{\mathrm{U}}<6$ and $1<\mathrm{C}_{\mathrm{C}}<3$, otherwise, it is considered a uniformly graded soil.

## Experimental Mix Design Development

The simplex lattice theory according to [8] was used in the development of the experimental mix design (for both trial and control mixes) used in this study. A simplex is a structural representation (shapes) of lines or planes joining assumed points of constituent materials of a mixture and which such points are equidistant from each other [13]. Figure 1 gives the simplex lattice for a $(5,2)$ mixture used in this study. The 5 represents the number of component materials while the 2 represents the maximum number of material interaction. For a (q, m) component mixture, the number of points is given by $C_{m}\left(q^{+} m^{-1}\right)$ ([8]) which produced 15 design points for five components, 2 maximum interactions mixture. The simplex lattice method of mixtures also assumed that pure substances exist at certain points which happens to be at the vertices of the simplex lattice.


Fig. 1: $(5,2)$ simplex lattice structure.

The materials used in this study are: PSA, water, cement, sand and granite. Mix proportions are being represented in theoretical (pseudo) forms with the compliance to the following basic principles;

- $X \neq$ negative;
- a pseudo mix ratio cannot be negative
- $0 \leq \mathrm{X}_{\mathrm{i}} \leq 1$;
- the pseudo mix ratio at position i must be between 0 and 1
- $\quad \Sigma \mathrm{X}_{\mathrm{i}}=1$;
- summation of all pseudo mix ratios must be equal to 1

The Scheffe method of mixtures requires all components proportions to be in pseudo forms of which all additions must sum to 1 . This cannot be achieved in reality as mixture proportions are always presented in real formats. Hence, Scheffe proposed a relationship between real and pseudo constituents represented by Equation (3).
$\mathrm{S}=[\mathrm{A}] \mathrm{X}$

Where:
$S$ = column matrix of real component ratio.
$\mathrm{X}=$ column matrix of pseudo component ratio.
$[\mathrm{A}]=$ coefficient matrix which is the transpose of the transformation matrix
The transformation matrix was obtained from smart and conservative guesses of cement concrete mix ratios. The water - binder ratio was limited to the range of $0.4-0.6$, the PSA was limited to the range of $5 \%-25 \%$ consequently limiting the cement proportion to the range of $75 \%$ to $95 \%$. The fine aggregate was limited to the range of $1-2.5$ while the coarse aggregate was limited to the range of $2-5$. At point of occurrence of pure substances (vertices), the mix ratios were chosen as; $(0.45,0.95,0.05$, $2,4),(0.5,0.90,0.10,1,2),(0.55,0.85,0.15,1.75,3.5)$, $(0.40,0.80,0.20,1.25,2.50)$ and $(0.60,0.75,0.25,2.5$, 5.00) which in matrix form becomes the transformation matrix given by Equation (4).
$[\mathrm{T}]=\left\{\begin{array}{lllll}0.45 & 0.95 & 0.05 & 2 & 4 \\ 0.50 & 0.90 & 0.10 & 1 & 2 \\ 0.55 & 0.85 & 0.15 & 1.75 & 3.5 \\ 0.40 & 0.80 & 0.20 & 1.25 & 2.50 \\ 0.60 & 0.75 & 0.25 & 2.50 & 5.00\end{array}\right]$
With a corresponding pseudo mix matrix;

$$
[\mathrm{X}]=\left\{\begin{array}{lllll}
1 & 0 & 0 & 0 & 0  \tag{5}\\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{array}\right]
$$



The transpose of [T] results in [A] as;

$$
[\mathrm{A}]=\left\{\begin{array}{lllll}
0.45 & 0.50 & 0.55 & 0.40 & 0.60  \tag{6}\\
0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\
0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\
2 & 1 & 1.75 & 1.25 & 2.50 \\
4 & 2 & 3.50 & 2.50 & 5.00
\end{array}\right]
$$

The pseudo mixes proportions of the interaction points in Figure 1 being; $\mathrm{X}_{12}\left[\begin{array}{llll}0.5 & 0.5 & 0 & 0\end{array} 0\right]$, $\mathrm{X}_{13}\left[\begin{array}{lllll}0.5 & 0 & 0.5 & 0 & 0\end{array}\right], \mathrm{X}_{14}$ [0.5 00000.50 ], $\mathrm{X}_{15}\left[\begin{array}{lllll}0.5 & 0 & 0 & 0 & 0.5\end{array}\right], \mathrm{X}_{23}\left[\begin{array}{llllll}0 & 0.5 & 0.5 & 0 & 0\end{array}\right], \mathrm{X}_{24}$ [0 0.500 .500 .5 ], $\mathrm{X}_{25}\left[\begin{array}{lllll}0 & 0.5 & 0 & 0 & 0.5\end{array}\right], \mathrm{X}_{34}\left[\begin{array}{lllllll}0 & 0 & 0.5 & 0.5 & 0\end{array}\right], \mathrm{X}_{35}$ $\left[\begin{array}{lllll}0 & 0 & 0.5 & 0 & 0.5\end{array}\right]$ and $\mathrm{X}_{45}\left[\begin{array}{lllll}0 & 0 & 0 & 0.5 & 0.5\end{array}\right]$

Tables 1 and 2 represent the experimental mix designs for the trial and control mixes respectively after proper utilization of Equation (3). As an illustration, considering point $\mathrm{X}_{12}$ on Figure 1 with pseudo proportions of [0.5 0.50 00 ], the actual mix proportions are obtained as;

$$
\mathrm{S}=\left\{\begin{array}{lllll}
0.45 & 0.50 & 0.55 & 0.40 & 0.60 \\
0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\
0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\
2 & 1 & 1.75 & 1.25 & 2.50 \\
4 & 2 & 3.50 & 2.50 & 5.00
\end{array}\right] X
$$

This results to a real matrix ratio of; $\mathrm{S}=\left[\begin{array}{ll}0.475 & 0.9250 .075\end{array}\right.$
on Tables 1 and 2 can be obtained.

Table 1: Design Matrix for Trial Mixes.

| $\mathbf{N}$ | Pseudo Component |  |  |  |  | Real component |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{X}_{1}$ | $\mathrm{X}_{2}$ | $\mathrm{X}_{3}$ | $\mathrm{X}_{4}$ | $\mathrm{X}_{5}$ | $\mathrm{~S}_{1}$ | $\mathrm{~S}_{2}$ | $\mathrm{~S}_{3}$ | $\mathrm{~S}_{4}$ | $\mathrm{~S}_{5}$ |
| $\mathbf{1}$ | 1 | 0 | 0 | 0 | 0 | 0.45 | 0.95 | 0.05 | 2 | 4 |
| $\mathbf{2}$ | 0 | 1 | 0 | 0 | 0 | 0.50 | 0.90 | 0.10 | 1 | 2 |
| $\mathbf{3}$ | 0 | 0 | 1 | 0 | 0 | 0.55 | 0.85 | 0.15 | 1.75 | 3.50 |
| $\mathbf{4}$ | 0 | 0 | 0 | 1 | 0 | 0.40 | 0.80 | 0.20 | 1.25 | 2.50 |
| $\mathbf{5}$ | 0 | 0 | 0 | 0 | 1 | 0.60 | 0.75 | 0.25 | 2.5 | 5.00 |
| $\mathbf{6}$ | $1 / 2$ | $1 / 2$ | 0 | 0 | 0 | 0.475 | 0.925 | 0.075 | 1.50 | 3.00 |
| $\mathbf{7}$ | $1 / 2$ | 0 | $1 / 2$ | 0 | 0 | 0.50 | 0.90 | 0.10 | 1.875 | 3.75 |
| $\mathbf{8}$ | $1 / 2$ | 0 | 0 | $1 / 2$ | 0 | 0.425 | 0.875 | 0.125 | 1.625 | 3.25 |
| $\mathbf{9}$ | $1 / 2$ | 0 | 0 | 0 | $1 / 2$ | 0.525 | 0.85 | 0.15 | 2.25 | 4.50 |
| $\mathbf{1 0}$ | 0 | $1 / 2$ | $1 / 2$ | 0 | 0 | 0.525 | 0.875 | 0.125 | 1.375 | 2.75 |
| $\mathbf{1 1}$ | 0 | $1 / 2$ | 0 | $1 / 2$ | 0 | 0.45 | 0.85 | 0.15 | 1.125 | 2.25 |
| $\mathbf{1 2}$ | 0 | $1 / 2$ | 0 | 0 | $1 / 2$ | 0.55 | 0.825 | 0.175 | 1.75 | 3.50 |
| $\mathbf{1 3}$ | 0 | 0 | $1 / 2$ | $1 / 2$ | 0 | 0.475 | 0.825 | 0.175 | 1.50 | 3.00 |
| $\mathbf{1 4}$ | 0 | 0 | $1 / 2$ | 0 | $1 / 2$ | 0.575 | 0.80 | 0.20 | 2.125 | 4.25 |
| $\mathbf{1 5}$ | 0 | 0 | 0 | $1 / 2$ | $1 / 2$ | 0.50 | 0.775 | 0.225 | 1.875 | 3.75 |

Table 2: Design Matrix for Control Mixes.

| $\mathbf{N}$ | Pseudo Component |  |  |  |  | Real component |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{X}_{1}$ | $\mathrm{X}_{2}$ | $\mathrm{X}_{3}$ | $\mathrm{X}_{4}$ | $\mathrm{X}_{5}$ | $\mathrm{~S}_{1}$ | $\mathrm{~S}_{2}$ | $\mathrm{~S}_{3}$ | $\mathrm{~S}_{4}$ | $\mathrm{~S}_{5}$ |
| $\mathbf{1}$ | $1 / 3$ | $1 / 3$ | $1 / 3$ | 0 | 0 | 0.50 | 0.90 | 0.10 | 1.5833 | 3.1667 |
| $\mathbf{2}$ | $1 / 3$ | $1 / 3$ | 0 | $1 / 3$ | 0 | 0.45 | 0.8833 | 0.1167 | 1.4167 | 2.8333 |
| $\mathbf{3}$ | $1 / 3$ | 0 | $1 / 3$ | $1 / 3$ | 0 | 0.4667 | 0.8667 | 0.1333 | 1.6667 | 3.3333 |
| $\mathbf{4}$ | $1 / 3$ | $1 / 3$ | 0 | 0 | $1 / 3$ | 0.5167 | 0.8667 | 0.1333 | 1.8333 | 3.6667 |
| $\mathbf{5}$ | $1 / 4$ | $1 / 4$ | $1 / 4$ | $1 / 4$ | 0 | 0.475 | 0.875 | 0.125 | 1.50 | 3.000 |
| $\mathbf{6}$ | $1 / 4$ | $1 / 4$ | $1 / 4$ | 0 | $1 / 4$ | 0.525 | 0.8625 | 0.1375 | 1.8125 | 3.625 |
| $\mathbf{7}$ | $1 / 4$ | $1 / 4$ | 0 | $1 / 4$ | $1 / 4$ | 0.4875 | 0.8500 | 0.1500 | 1.6875 | 3.375 |
| $\mathbf{8}$ | 0 | $1 / 4$ | $1 / 4$ | $1 / 4$ | $1 / 4$ | 0.5125 | 0.825 | 0.175 | 1.625 | 3.25 |
| $\mathbf{9}$ | $3 / 10$ | $1 / 10$ | $1 / 5$ | $1 / 5$ | $1 / 5$ | 0.495 | 0.855 | 0.145 | 1.8000 | 3.60 |
| $\mathbf{1 0}$ | $1 / 5$ | $1 / 5$ | $1 / 10$ | $3 / 10$ | $1 / 5$ | 0.485 | 0.845 | 0.155 | 1.6500 | 3.30 |
| $\mathbf{1 1}$ | $1 / 5$ | $1 / 5$ | $1 / 5$ | $3 / 10$ | $1 / 10$ | 0.480 | 0.855 | 0.145 | 1.5750 | 3.150 |
| $\mathbf{1 2}$ | $1 / 5$ | $1 / 5$ | $1 / 5$ | $1 / 5$ | $1 / 5$ | 0.500 | 0.850 | 0.150 | 1.700 | 3.40 |
| $\mathbf{1 3}$ | $3 / 20$ | $1 / 4$ | $1 / 5$ | $1 / 5$ | $1 / 5$ | 0.5025 | 0.8475 | 0.1525 | 1.6500 | 3.30 |
| $\mathbf{1 4}$ | $1 / 5$ | $1 / 5$ | $3 / 20$ | $1 / 4$ | $1 / 5$ | 0.4925 | 0.8475 | 0.1525 | 1.6750 | 3.350 |
| $\mathbf{1 5}$ | $1 / 4$ | $1 / 5$ | $1 / 5$ | $1 / 5$ | $3 / 20$ | 0.4925 | 0.8600 | 0.1400 | 1.675 | 3.35 |

## Experimental Techniques

- Water Absorption Test: This test was performed in accordance to ASTM D3171 [14]. PSA cement concrete cube specimens of 150 mm X 150 mm X 150 mm were prepared and allowed to set for 5 days in order to have moisture free samples. The dry weights were measured and recorded. These dried samples were later immersed in water for 24hours and their wet
weights also measured and recorded. According to [14], the water absorption of PSA cement concrete samples were calculated using Equation (7).

$$
\begin{equation*}
W_{\mathrm{a}}=\frac{W \omega-W d}{W d} \chi 100 \tag{7}
\end{equation*}
$$

Where;
$\mathrm{W}_{\mathrm{a}}=$ water absorption; W $\boldsymbol{\omega}=$ wet weight of sample; $\mathrm{W} \boldsymbol{d}=$ dry weight of sample.

- Compressive Strength Test: The compressive strength of hardened PSA cement concrete specimens was determined in accordance to BS 1881: Part 115 [15]. Cube moulds of 150 mm X 150 mm X 150 mm were used in preparation of specimens. Other apparatus used in this experiment include; Universal Testing Machine, curing tank, trowel, shovel and compacting rod. The specimens were completely immersed in water inside a curing tank and were left to cure for 28 days after which they were subjected to compressive strength test. The failure load of PSA cement concrete specimens was recorded and the compressive strength
determined according to Equation (8) [15].

$$
\begin{equation*}
F \mathrm{C}=\frac{P(\text { failure load in } N)}{A\left(\text { cross sectional area in } \mathrm{mm}^{2}\right)} \tag{8}
\end{equation*}
$$

## Derivation of Optimization Models'

According to Scheffe [8], the condition in Equation (9) must be satisfied for a ( $\mathrm{q}, \mathrm{m}$ ) simplex structure.

$$
\sum_{i=1}^{q} x_{\mathrm{i}}=1
$$

(9)

Equation (10) gives the general polynomial format for a (q, m ) polynomial, where $q$ represents the number of variables and $m$ represents the degree of the polynomial;

$$
\begin{equation*}
\stackrel{n}{Y}{ }_{i \leq 1 \leq q}=b_{0}+\sum_{i \leq 1 \geq j \leq q} b_{i} X_{i}+\sum_{1 \leq 1 \leq j \leq q} b_{i j} X_{j} X_{i j}+\ldots \ldots \ldots+\sum b_{i j k}+\sum b_{i l i 2 \ldots \ldots \ldots .} \text { in } X_{i 1} X_{i 2} \ldots \ldots X_{i n} \tag{10}
\end{equation*}
$$

Where; $1 \leq \mathrm{i} \leq \mathrm{q}, 1 \leq \mathrm{i} \leq \mathrm{j} \leq \mathrm{q}, 1 \leq \mathrm{i} \leq \mathrm{j} \leq \mathrm{k} \leq \mathrm{q}$, and b is the constant coefficient.

X is the pseudo component for constituents $\mathrm{i}, \mathrm{j}$,
and k
For a ( $\mathrm{q}, \mathrm{m}$ ) polynomial of second-degree form with five (5) number variables, Equations (9) and (10) become;
$\mathrm{X}_{1}+\mathrm{X}_{2}+\mathrm{X}_{3}+\mathrm{X}_{4}+\mathrm{X}_{5}=1$
$\tilde{Y}=b_{0}+b_{1} X_{1}+b_{2} X_{2}+b_{3} X_{3}+b_{4} X_{4}+b_{5} X_{5}+b_{12} X_{12} X_{2}+b_{13} X_{1} X_{3}+b_{14} X_{1} X_{4}+b_{15} X_{1} X_{5}+b_{25} X_{2} X_{5}+b_{24} X_{2} X_{4}+b_{23} X_{2} X_{3}+$ $\mathrm{b}_{25} \mathrm{X}_{3} \mathrm{X}_{5}+\mathrm{b}_{34} \mathrm{X}_{3} \mathrm{X}_{4}+\mathrm{b}_{11} X_{1}^{2}+b_{22} X_{2}^{2}+b_{33} X_{3}^{2}+b_{44} X_{4}^{2}+b_{55} X_{5}^{2}$

Multiplying through Equation (11) by constant $b_{0}$, yields
Equation (13).
$\mathrm{b}_{0} \mathrm{X}_{1}+\mathrm{b}_{0} \mathrm{X}_{2}+\mathrm{b}_{0} \mathrm{X}_{3}+\mathrm{b}_{0} \mathrm{X}_{4}+\mathrm{b}_{0} \mathrm{X}_{5}=\mathrm{b}_{0} \quad 1$
By multiplying Equation (11) by $\mathrm{X}_{1}, \mathrm{X}_{2}, \mathrm{X}_{3}, \mathrm{X}_{4}$, and $\mathrm{X}_{5}$ in
produced; succession and rearranging, Equations (14) to (18) were
$X_{1}^{2}=\mathrm{X}_{1}-\mathrm{X}_{1} \mathrm{X}_{2}-\mathrm{X}_{1} \mathrm{X}_{3}-\mathrm{X}_{1} \mathrm{X}_{4}-\mathrm{X}_{1} \mathrm{X}_{5}$
$X_{2}^{2}=\mathrm{X}_{2}-\mathrm{X}_{1} \mathrm{X}_{2}-\mathrm{X}_{2} \mathrm{X}_{3}-\mathrm{X}_{2} \mathrm{X}_{4}-\mathrm{X}_{2} \mathrm{X}_{5}$
$X_{3}^{2}=X_{3}-X_{1} X_{3}-X_{2} X_{3}-X_{3} X_{4}-X_{3} X_{5}$
$X_{4}^{2}=X_{4}-X_{1} X_{4}-X_{2} X_{4}-X_{3} X_{4}-X_{4} X_{5}$
$X_{5}^{2}=\mathrm{X}_{5}-\mathrm{X}_{1} \mathrm{X}_{5}-\mathrm{X}_{2} \mathrm{X}_{5}-\mathrm{X}_{3} \mathrm{X}_{5}-\mathrm{X}_{4} \mathrm{X}_{56}$
Substituting Equations (13), (14) to (18) into Equation (11),
Equation (19) was obtained after necessary transformation.
$\tilde{Y}=\left(b_{0}+b_{1}+b_{11}\right) X_{1}+\left(b_{0}+b_{2}+b_{22}\right) X_{2}+\left(b_{0}+b_{3}+b_{33}\right) X_{3}+\left(b_{0}+b_{4}+b_{44}\right) X_{4}+\left(b_{0}+b_{5}+b_{55}\right) X_{5}+\left(b_{12}-b_{11}-b_{22}\right)$ $X_{1} X_{2}+\left(b_{13}-b_{11}-b_{33}\right) X_{1} X_{3}+\left(b_{14}-b_{11}-b_{44}\right) X_{1} X_{4}+\left(b_{15}-b_{11}-b_{55}\right) X_{1} X_{5}+\left(b_{23}-b_{22}-b_{33}\right) X_{2} X_{3}+\left(b_{24}-b_{22}-b_{44}\right) X_{2} X_{4}+$ $\left(b_{25}-b_{22}-b_{55}\right) X_{2} X_{5}+\left(b_{34}-b_{33}-b_{44}\right) X_{3} X_{4}+\left(b_{35}-b_{33}-b_{55}\right) X_{3} X_{5}+\left(b_{45}-b_{44}-b_{55}\right) X_{4} X_{5}$

Denoting;

$$
\begin{align*}
& \mathrm{B}_{\mathrm{i}}=\mathrm{b}_{0}+\mathrm{b}_{\mathrm{i}}+\mathrm{b}_{\mathrm{ii}} \text { and }  \tag{19}\\
& \mathrm{B}_{\mathrm{ij}}=\mathrm{b}_{\mathrm{ij}}-\mathrm{b}_{\mathrm{ii}}-\mathrm{b}_{\mathrm{ij}}
\end{align*}
$$

The resultant reduced (5,2) polynomial is presented by Equation (20).
$\tilde{Y}=B_{1} X_{1}+B_{2} X_{2}+B_{3} X_{3}+B_{4} X_{4}+B_{5} X_{5}+B_{12} X_{1} X_{3}+B_{13} X_{1} X_{3}+B_{14} X_{1} X_{4}+B_{15} X_{1} X_{5}+B_{23} X_{2} X_{3}+B_{24} X_{2} X_{4}+B_{25} X_{2} X_{5}+$ $\mathrm{B}_{34} \mathrm{X}_{3} \mathrm{X}_{4}+\mathrm{B}_{35} \mathrm{X}_{3} \mathrm{X}_{5}+\mathrm{B}_{45} \mathrm{X}_{4} \mathrm{X}_{5}$

As can be observed, the number of coefficients has reduced from 20 in Equation (12) to 15 in Equation (20). Thus, the reduced second-degree polynomial in q-variables in general form is;
$\tilde{\mathrm{Y}}=\sum_{1 \leq i \leq q} B_{i} X_{i}+\sum_{1 \leq i \leq q} B_{i j} X_{i} X_{j}$
Where;
Y is a dependent variable (compressive strength and water
$\mathrm{Y}_{1}=\mathrm{B}_{1}$
$Y_{2}=B_{2}$
$Y_{3}=B_{3}$
absorption of PSA cement concrete).

## Determination of Models Coefficients'

According to Scheffe (1958), pure substances occur at vertices points $\left(\mathrm{X}_{1}, \mathrm{X}_{2}, \mathrm{X}_{3}, \mathrm{X}_{4}\right.$ and $\left.\mathrm{X}_{5}\right)$ with coordinates; [1
 in that order. Substituting these coordinates into Equation (20), gives the following coefficient values;
$\mathrm{Y}_{4}=\mathrm{B}_{4}$
$\mathrm{Y}_{5}=\mathrm{B}_{5}$

At the centre points, there are interactions between two variables resulting to an interacted response. For instance,
$Y_{12}=1 / 2 X_{1}+1 / 2 X_{2}+1 / 4 X_{1} X_{2}$

$$
\begin{equation*}
=1 / 2 B_{1}+1 / 2 B_{2}+1 / 4 B_{12} \tag{23}
\end{equation*}
$$

From Equation (22); $\mathrm{Bi}=\mathrm{Y}_{\mathrm{i}}$, where $\mathrm{i}=1,2,3 \ldots . \mathrm{n}$.
Then substituting $B_{i}=Y_{i}$ into Equation (23) yielded:

$$
\begin{equation*}
Y_{12}=(1 / 2) Y_{1}+(1 / 2) Y_{2}+(1 / 4) B_{12} \tag{24}
\end{equation*}
$$

On Simplification of Equation (24), Equation (25) was produced:

$$
\begin{equation*}
\mathrm{B}_{12}=4 \mathrm{Y}_{12}-2 \mathrm{Y}_{1}-2 \mathrm{Y}_{2} \tag{25}
\end{equation*}
$$

Similarly, Equations (26) to (29) were developed. Thus:

$$
\begin{gather*}
\mathrm{B}_{13}=4 \mathrm{Y}_{13}-2 \mathrm{Y}_{1}-2 \mathrm{Y}_{3}  \tag{26}\\
\mathrm{~B}_{14}=4 \mathrm{Y}_{14}-2 \mathrm{Y}_{1}-2 \mathrm{Y}_{4}  \tag{27}\\
\mathrm{~B}_{15}=4 \mathrm{Y}_{15}-2 \mathrm{Y}_{1}-2 \mathrm{Y}_{5}  \tag{28}\\
\mathrm{~B}_{23}=4 \mathrm{Y}_{23}-2 \mathrm{Y}_{2}-2 \mathrm{Y}_{3} \tag{29}
\end{gather*}
$$

considering centre point $\mathrm{X}_{12}$;

Generalizing, Equations (22) to (29), Equation (30) was formed.

$$
\left.\begin{array}{l}
\mathrm{B}_{\mathrm{i}}=\mathrm{Y}_{\mathrm{i}} \\
\mathrm{~B}_{\mathrm{ij}}=4 \mathrm{Y}_{\mathrm{ij}}-2 \mathrm{Y}_{\mathrm{i}}-2 \mathrm{Y}_{\mathrm{j}}
\end{array}\right\}
$$

## Validation of Derived Models

## - F-Statistics

These models were validated using F- statistics at 5\% level of significance. The F-statistics is given as the ratio of variance between the predicted/model response value and that of experimental value. The following hypothesis were adopted in validation of models;
Null Hypothesis: $\quad H_{0}=$ there is no significant difference between the experimental and predicted responses.
Alternate Hypothesis: $\mathrm{H}_{1}=$ there is a significant difference between the experimental and predicted responses.
Mathematically, the F-test is represented by Equation (31).
$\mathrm{F}=\frac{s_{1}^{2}}{s_{2}^{2}}$
Where; $S_{1}^{2}=$ Larger of both variances
$S_{2}^{2}=$ Smaller of both variance
$\mathrm{S}^{2}$ is obtained from Equation (32)
$\mathrm{S}^{2}=\frac{1}{n-1}\left[\Sigma(Y-\bar{Y})^{2}\right]$
Where : $\bar{Y}=$ Average mean of response, Y
$\mathrm{Y}=$ Means of response
$\mathrm{n}=$ number of observations
From F- distribution table and considering 5\% level of significance, the F -value for 14 degree of freedom is 2.483 . If the F-value calculated in accordance to Equation (31) is
(29)
less than 2.483 , then the null hypothesis is accepted and the model is declared adequate. Otherwise, the alternate hypothesis is accepted and the model is considered inadequate.

## - $\mathbf{R}^{2}$ Statistics

These models were also subjected to $R^{2}$ analysis for further adequacy test. The $R^{2}$ values were calculated in accordance to Equation (33).

$$
\begin{align*}
\mathrm{R}^{2} & =\frac{\Sigma(y e s t-\dot{\bar{y}})^{2}}{\Sigma(y-\overline{\mathrm{y}})^{2}}  \tag{33}\\
\text { Where, yest } & =\text { model value, } \\
\mathrm{y} & =\text { experimental value } \\
\overline{\mathrm{y}} & =\text { mean experimental value. } .
\end{align*}
$$

## Results and Discussion

## Classification of Periwinkle Shell Ash (PSA)

The oxide composition of the pozzolan used in this study is shown in Table 3. According to ASTM C618 [10] with all the oxide contents exceeding the minimum requirements according to the standard with the exception of sulphur trioxide. In classification of pozzolan, the combined acidic oxides $\left(\left(\mathrm{Al}_{2} \mathrm{O}_{3}+\mathrm{SiO}_{2}+\mathrm{Fe}_{2} \mathrm{O}_{3}\right)\right.$ sums to $50.89 \%$, meeting the requirements of ASTMC618 [10] for a Class C pozzolan.

Table 3: Oxide Composition Results of PSA.

| Oxide | CaO | $\mathrm{Al}_{2} \mathrm{O}_{3}$ | $\mathrm{Fe}_{2} \mathrm{O}_{3}$ | MgO | $\mathbf{S i O}_{2}$ | $\mathrm{Na}_{2} \mathrm{O}$ | $\mathrm{K}_{2} \mathrm{O}$ | $\mathrm{SO}_{3}$ | $\mathrm{TiO}_{2}$ | LOI | $\left(\mathrm{Al}_{2} \mathrm{O}_{3}+\mathrm{SiO}_{2}+\mathrm{Fe}_{2} \mathrm{O}_{3}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Value (\%) | 38.85 | 11.04 | 5.3 | 1.13 | 34.55 | 0.11 | 0.15 | 1.22 | 0.18 | 6.89 | 50.89 |

## Classification of Aggregates

The sieve analysis test result for the sand and granite used in this study is presented in Tables 4 and 5 respectively. The fineness modulus was also calculated for both aggregates as 2.694 for sand and 4.384 for granite as shown in Tables 4 and 5 respectively.

Figures 2 and 3 presents the particle size distribution curves for the aggregates from which the coefficients of uniformity and curvature were computed as 2.965 and 0.882 and 1.821 and 1.122 for sand and granite respectively leading to the classification that both aggregates are uniformly graded materials.

Table 4: Sieve Analysis Test Result of Sand.

| Sieve size (mm) | Weight retained (g) | Cumulative Weight retained (g) | retained (\%) | Percentage Weight passing (\%) |
| :---: | :---: | :---: | :---: | :---: |
| 4.75 | - | - | - | 100 |
| 2.36 | 67 | 67 | 6.7 | 93.3 |
| 1.18 | 146 | 213 | 21.3 | 78.7 |
| 0.60 | 324 | 537 | 53.7 | 46.3 |
| 0.30 | 353 | 890 | 89.0 | 11.0 |
| 0.15 | 97 | 987 | 98.7 | 1.3 |
| 0.075 | 2 | 989 | 98.9 | 1.1 |
| Pan | 1 | 1000 | 100 | - |
| Total cumm. weigth retained (4.75mm-150 $\mu \mathrm{m}$ ) |  |  | 269.4 |  |
| Fineness modulus |  |  | 269.4/100 $=2.694$ |  |

Table 5: Sieve Analysis Test Result of Granite.

| Sieve size <br> $(\mathbf{m m})$ | Weight retained <br> $(\mathrm{g})$ | Cumulative Weight <br> retained (g) | Cumulative Percentage Weight <br> retained (\%) | Percentage Weight <br> passing (\%) |
| :---: | :---: | :---: | :---: | :---: |
| 25 | - | - | - | 100 |
| 19 | 1289 | 1289 | 51.56 | 48.44 |
| 13.2 | 882 | 2171 | 86.84 | 13.16 |
| 4.75 | 329 | 2500 | 100 | - |
| 2.36 | 0 | - | 100 | - |
| 1.18 | 0 | - | 100 | - |
| Pan | 0 | - | 100 | - |
| Total cumm. weigth retained (20mm-1.18mm) |  |  |  |  |
| Fineness modulus |  |  |  |  |



Fig. 2: Particle Size Distribution of sand.


Fig. 3: Particle Size Distribution of granite.
Table 6: $28^{\mathrm{TH}}$ Day Compressive Strength Test Result of Cement Concrete.

| S/N | 28 ${ }^{\text {th }}$ day compressive strength with PSA |  |  |  |  |  | 28 ${ }^{\text {th }}$ day compressive strength without PSA |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{Z}_{1}$ | $\mathrm{Z}_{2}$ | $\mathrm{Z}_{3}$ | $\mathrm{Z}_{4}$ | $\mathrm{Z}_{5}$ | Av. strength ( $\mathrm{N} / \mathrm{mm}^{2}$ | $\mathrm{Z}_{1}$ | $\mathrm{Z}_{2}$ | $\mathrm{Z}_{4}$ | $\mathrm{Z}_{5}$ | Av. strength ( $\mathrm{N} / \mathrm{mm}^{2}$ ) |
| TP1 | 0.45 | 0.95 | 0.05 | 2 | 4 | 17.35 | 0.45 | 1.0 | 2 | 4 | 16.48 |
| TP2 | 0.50 | 0.90 | 0.10 | 1 | 2 | 25.90 | 0.50 | 1.0 | 1 | 2 | 20.05 |
| TP3 | 0.55 | 0.85 | 0.15 | 1.75 | 3.50 | 20.65 | 0.55 | 1.0 | 1.75 | 3.50 | 20.85 |
| TP4 | 0.40 | 0.80 | 0.20 | 1.25 | 2.50 | 22.26 | 0.40 | 1.0 | 1.25 | 2.50 | 23.20 |
| TP5 | 0.60 | 0.75 | 0.25 | 2.5 | 5.00 | 16.80 | 0.60 | 1.0 | 2.5 | 5.00 | 21.90 |
| TP6 | 0.475 | 0.925 | 0.075 | 1.50 | 3.00 | 27.55 | 0.475 | 1.0 | 1.5 | 3.0 | 27.05 |
| TP7 | 0.50 | 0.90 | 0.10 | 1.875 | 3.75 | 21.24 | 0.50 | 1.0 | 1.875 | 3.75 | 18.22 |
| TP8 | 0.425 | 0.875 | 0.125 | 1.625 | 3.25 | 24.44 | 0.425 | 1.0 | 1.625 | 3.25 | 25.93 |
| TP9 | 0.525 | 0.85 | 0.15 | 2.25 | 4.5 | 15.03 | 0.525 | 1.0 | 2.25 | 4.5 | 19.85 |
| TP10 | 0.525 | 0.875 | 0.125 | 1.375 | 2.75 | 24.90 | 0.525 | 1.0 | 1.375 | 2.75 | 25.90 |
| TP11 | 0.45 | 0.85 | 0.15 | 1.125 | 2.25 | 23.58 | 0.45 | 1.0 | 1.125 | 2.25 | 29.48 |
| TP12 | 0.55 | 0.825 | 0.175 | 1.75 | 3.50 | 15.86 | 0.55 | 1.0 | 1.75 | 3.50 | 20.79 |
| TP13 | 0.475 | 0.825 | 0.175 | 1.50 | 3.00 | 16.94 | 0.475 | 1.0 | 1.50 | 3.00 | 27.18 |
| TP14 | 0.575 | 0.80 | 0.20 | 2.125 | 4.25 | 15.25 | 0.575 | 1.0 | 2.125 | 4.25 | 16.06 |
| TP15 | 0.50 | 0.775 | 0.225 | 1.875 | 3.75 | 12.94 | 0.50 | 1.0 | 1.875 | 3.75 | 18.10 |
|  |  |  |  | Average |  | 20.05 |  |  |  |  | 22.07 |

Table 7: Water Absorption Test Result of Cement Concrete.

| S/N | Water Absorption test result with PSA |  |  |  |  |  | Water Absorption test result without PSA |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{Z}_{1}$ | $\mathrm{Z}_{2}$ | $\mathrm{Z}_{3}$ | $\mathrm{Z}_{4}$ | $\mathrm{Z}_{5}$ | Av. Water Abso-rption (\%) | $\mathrm{Z}_{1}$ | $\mathrm{Z}_{2}$ | $\mathrm{Z}_{4}$ | $\mathrm{Z}_{5}$ | Av. Water Absor-ption (\%) |
| TP1 | 0.45 | 0.95 | 0.05 | 2 | 4 | 2.89 | 0.45 | 1.0 | 2 | 4 | 2.41 |
| TP2 | 0.50 | 0.90 | 0.10 | 1 | 2 | 3.86 | 0.50 | 1.0 | 1 | 2 | 2.64 |
| TP3 | 0.55 | 0.85 | 0.15 | 1.75 | 3.50 | 2.57 | 0.55 | 1.0 | 1.75 | 3.50 | 2.21 |
| TP4 | 0.40 | 0.80 | 0.20 | 1.25 | 2.50 | 2.59 | 0.40 | 1.0 | 1.25 | 2.50 | 2.13 |
| TP5 | 0.60 | 0.75 | 0.25 | 2.5 | 5.00 | 3.88 | 0.60 | 1.0 | 2.5 | 5.00 | 3.59 |
| TP6 | 0.475 | 0.925 | 0.075 | 1.5 | 3.0 | 2.19 | 0.475 | 1.0 | 1.5 | 3.0 | 1.14 |
| TP7 | 0.50 | 0.90 | 0.10 | 1.875 | 3.75 | 2.30 | 0.50 | 1.0 | 1.875 | 3.75 | 1.80 |
| TP8 | 0.425 | 0.875 | 0.125 | 1.625 | 3.25 | 2.56 | 0.425 | 1.0 | 1.625 | 3.25 | 1.95 |
| TP9 | 0.525 | 0.85 | 0.15 | 2.25 | 4.5 | 3.69 | 0.525 | 1.0 | 2.25 | 4.5 | 3.03 |
| TP10 | 0.525 | 0.875 | 0.125 | 1.375 | 2.75 | 3.37 | 0.525 | 1.0 | 1.375 | 2.75 | 2.28 |
| TP11 | 0.45 | 0.85 | 0.15 | 1.125 | 2.25 | 2.58 | 0.45 | 1.0 | 1.125 | 2.25 | 2.19 |
| TP12 | 0.55 | 0.825 | 0.175 | 1.75 | 3.50 | 2.61 | 0.55 | 1.0 | 1.75 | 3.50 | 2.21 |
| TP13 | 0.475 | 0.825 | 0.175 | 1.50 | 3.00 | 2.22 | 0.475 | 1.0 | 1.50 | 3.00 | 1.04 |
| TP14 | 0.575 | 0.80 | 0.20 | 2.125 | 4.25 | 2.47 | 0.575 | 1.0 | 2.125 | 4.25 | 2.00 |
| TP15 | 0.50 | 0.775 | 0.225 | 1.875 | 3.75 | 2.35 | 0.50 | 1.0 | 1.875 | 3.75 | 1.86 |
|  |  |  |  | Average |  | 2.81 |  |  |  |  | 2.17 |

## Average Effect of PSA usage on Responses

Tables 6 and 7 present the results revealing the effect of cement replacement with PSA on the compressive strength and water absorption respectively. On the average, the unmodified cement concrete produced a higher strength value of $22.07 \mathrm{~N} / \mathrm{mm}^{2}$ in comparison to the PSA cement concrete average strength of $20.05 \mathrm{~N} / \mathrm{mm}^{2}$ giving rise to an average strength difference of $2.02 \mathrm{~N} / \mathrm{mm}^{2}$. This shows that overall, the unmodified cement concrete performed better than the modified cement concrete. Although, the unmodified cement concrete performed better on a macroscopic scale, the PSA cement concrete is better within a certain bracket. A close observation of Table 6 revealed that the performance of cement concrete improved with introduction of PSA within the bracket of $5 \%$ to $10 \%$. The average water absorption of unmodified cement
concrete was lower than its modified counterpart (Table 7). The PSA cement concrete produced an average water absorption of $2.81 \%$ higher than the $2.17 \%$ by the unmodified cement concrete. This signifies that the unmodified cement concrete is more durable and less permeable than the modified cement concrete. Although, the PSA cement concrete might be less durable, it meets the water absorption criterion of a concrete material according to ASTM C140 [16] which specifies a maximum average water absorption percentage of $4 \%$.

## Models' Development

Table 8 presents the trial mix results of responses for the PSA cement concrete used for responses' models derivation.

Table 8: Trial Test Result for Model Development.

| S/N | Response test result with PSA |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{Z}_{1}$ | $\mathrm{Z}_{2}$ | $\mathrm{Z}_{3}$ | Z4 | Z5 | Response symbol | Response C.Strength ( $\mathrm{N} / \mathrm{mm}^{2}$ ) | Response Water Abs.(\%) |
| TP1 | 0.45 | 0.95 | 0.05 | 2 | 4 | $\mathrm{Y}_{1}$ | 17.35 | 2.89 |
| TP2 | 0.50 | 0.90 | 0.10 | 1 | 2 | $\mathrm{Y}_{2}$ | 25.90 | 3.86 |
| TP3 | 0.55 | 0.85 | 0.15 | 1.75 | 3.50 | $\mathrm{Y}_{3}$ | 20.65 | 2.57 |
| TP4 | 0. 40 | 0.80 | 0.20 | 1.25 | 2.50 | $\mathrm{Y}_{4}$ | 22.26 | 2.59 |


| TP5 | 0.60 | 0.75 | 0.25 | 2.5 | 5.00 | $Y_{5}$ | 16.80 | 3.88 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| TP6 | 0.475 | 0.925 | 0.075 | 1.50 | 3.00 | $Y_{12}$ | 27.55 | 2.19 |
| TP7 | 0.50 | 0.90 | 0.10 | 1.875 | 3.75 | $Y_{13}$ | 21.24 | 2.30 |
| TP8 | 0.425 | 0.875 | 0.125 | 1.625 | 3.25 | $Y_{14}$ | 24.44 | 2.56 |
| TP9 | 0.525 | 0.85 | 0.15 | 2.25 | 4.50 | $Y_{15}$ | 15.03 | 3.69 |
| TP10 | 0.525 | 0.875 | 0.125 | 1.375 | 2.75 | $Y_{23}$ | 24.90 | 3.37 |
| TP11 | 0.45 | 0.85 | 0.15 | 1.125 | 2.25 | $Y_{24}$ | 23.58 | 2.58 |
| TP12 | 0.55 | 0.825 | 0.175 | 1.75 | 3.50 | $Y_{25}$ | 15.86 | 2.61 |
| TP13 | 0.475 | 0.825 | 0.175 | 1.50 | 3.00 | $Y_{34}$ | 16.94 | 2.22 |
| TP14 | 0.575 | 0.80 | 0.20 | 2.125 | 4.25 | $Y_{35}$ | 15.25 | 2.47 |
| TP15 | 0.50 | 0.775 | 0.225 | 1.875 | 3.75 | $Y_{45}$ | 12.94 | 2.35 |

## Water Absorption Model

Table 8 in association with Equation (30), was used to
derive the optimization model coefficients of water absorption model of PSA cement concrete. Thus:
$\mathrm{B}_{1}=\mathrm{Y}_{1}=2.89$
$\mathrm{B}_{12}=4 \mathrm{Y}_{12}-2 \mathrm{Y}_{1}-2 \mathrm{Y}_{2}=-4.74$
$\mathrm{B}_{24}=4 \mathrm{Y}_{24}-2 \mathrm{Y}_{2}-2 \mathrm{Y}_{4}=-2.58$
$\mathrm{B}_{2}=\mathrm{Y}_{2}=3.86$
$\mathrm{B}_{13}=4 \mathrm{Y}_{13}-2 \mathrm{Y}_{1}-2 \mathrm{Y}_{3}=-1.72$
$\mathrm{B}_{25}=4 \mathrm{Y}_{25}-2 \mathrm{Y}_{2}-2 \mathrm{Y}_{5}=-5.04$
$\mathrm{B}_{3}=\mathrm{Y}_{3}=2.57$
$\mathrm{B}_{14}=4 \mathrm{Y}_{14}-2 \mathrm{Y}_{1}-2 \mathrm{Y}_{4}=-0.72$
$\mathrm{B}_{34}=4 \mathrm{Y}_{34}-2 \mathrm{Y}_{3}-2 \mathrm{Y}_{4}=-1.44$
$\mathrm{B}_{4}=\mathrm{Y}_{4}=2.59$
$\mathrm{B}_{15}=4 \mathrm{Y}_{15}-2 \mathrm{Y}_{1}-2 \mathrm{Y}_{5}=1.22$
$\mathrm{B}_{35}=4 \mathrm{Y}_{35}-2 \mathrm{Y}_{3}-2 \mathrm{Y}_{5}=-3.02$
$\mathrm{B}_{5}=\mathrm{Y}_{5}=3.88$
$\mathrm{B}_{23}=4 \mathrm{Y}_{23}-2 \mathrm{Y}_{2}-2 \mathrm{Y}_{3}=0.62$
$\mathrm{B}_{45}=4 \mathrm{Y}_{45}-2 \mathrm{Y}_{4}-2 \mathrm{Y}_{5}=-3.54$

Substituting the above values into Equation (20), the concrete becomes; optimization model for water absorption of PSA cement
$\tilde{\mathrm{Y}}_{\mathrm{wa}}=2.89 \mathrm{X}_{1}+3.86 \mathrm{X}_{2}+2.57 \mathrm{X}_{3}+2.59 \mathrm{X}_{4}+3.88 \mathrm{X}_{5}-4.74 \mathrm{X}_{1} \mathrm{X}_{2}-1.72 \mathrm{X}_{1} \mathrm{X}_{3}-0.72 \mathrm{X}_{1} \mathrm{X}_{4}+1.22 \mathrm{X}_{1} \mathrm{X}_{5}+0.62 \mathrm{X}_{2} \mathrm{X}_{3}-2.58 \mathrm{X}_{2} \mathrm{X}_{4}-5.04 \mathrm{X}_{2} \mathrm{X}_{5}-$ $1.44 \mathrm{X}_{3} \mathrm{X}_{4}-3.02 \mathrm{X}_{3} \mathrm{X}_{5}-3.54 \mathrm{X}_{4} \mathrm{X}_{5}$

Equation (34) represents the optimization model for predicting the water absorption of PSA cement concrete. This model can be used in predicting the water absorption of any arbitrarily given PSA cement constituents ratio and vice versa.

## Compressive Strength Model

Table 8 in association with Equation (30), was used to derive the optimization model coefficients of $28^{\text {th }}$ day compressive strength model of PSA cement concrete. Thus:
$\mathrm{B}_{1}=\mathrm{Y}_{1}=17.35$
$\mathrm{B}_{12}=4 \mathrm{Y}_{12}-2 \mathrm{Y}_{1}-2 \mathrm{Y}_{2}=23.70$
$\mathrm{B}_{24}=4 \mathrm{Y}_{24}-2 \mathrm{Y}_{2}-2 \mathrm{Y}_{4}=-2.00$
$\mathrm{B}_{2}=\mathrm{Y}_{2}=25.90$
$\mathrm{B}_{13}=4 \mathrm{Y}_{13}-2 \mathrm{Y}_{1}-2 \mathrm{Y}_{3}=8.96$
$\mathrm{B}_{25}=4 \mathrm{Y}_{25}-2 \mathrm{Y}_{2}-2 \mathrm{Y}_{5}=-22.68$
$\mathrm{B}_{34}=4 \mathrm{Y}_{34}-2 \mathrm{Y}_{3}-2 \mathrm{Y}_{4}=-18.06$
$\mathrm{B}_{35}=4 \mathrm{Y}_{35}-2 \mathrm{Y}_{3}-2 \mathrm{Y}_{5}=-13.90$
$\mathrm{B}_{45}=4 \mathrm{Y}_{45}-2 \mathrm{Y}_{4}-2 \mathrm{Y}_{5}=-26.36$
$\mathrm{B}_{4}=\mathrm{Y}_{4}=22.26$
$\mathrm{B}_{15}=4 \mathrm{Y}_{15}-2 \mathrm{Y}_{1}-2 \mathrm{Y}_{5}=-8.18$

Substituting the above values into Equation (20), the PSA cement concrete becomes; optimization model for the $28^{\text {th }}$ day compressive strength of
$\tilde{\mathrm{Y}}_{\mathrm{c} 28}=17.35 \mathrm{X}_{1}+25.90 \mathrm{X}_{2}+20.65 \mathrm{X}_{3}+22.26 \mathrm{X}_{4}+16.80 \mathrm{X}_{5}+23.70 \mathrm{X}_{1} \mathrm{X}_{2}+8.96 \mathrm{X}_{1} \mathrm{X}_{3}+18.54 \mathrm{X}_{1} \mathrm{X}_{4}-8.18 \mathrm{X}_{1} \mathrm{X}_{5}+6.50 \mathrm{X}_{2} \mathrm{X}_{3}-2.00 \mathrm{X}_{2} \mathrm{X}_{4}-$ $22.68 \mathrm{X}_{2} \mathrm{X}_{5}-18.06 \mathrm{X}_{3} \mathrm{X}_{4}-13.90 \mathrm{X}_{3} \mathrm{X}_{5}-26.36 \mathrm{X}_{4} \mathrm{X}_{5}$

Equation (35) represents the optimization model for predicting the $28^{\text {th }}$ day compressive strength of PSA cement concrete. This model can be used in predicting the $28^{\text {th }}$ day compressive strength of any arbitrarily given PSA cement constituents ratio and vice versa.

## Models Validation

Table 9 presents control mix results used for the validation of derived models of PSA cement concrete.

Table 9: Control Test Result for Models' Validation.

| $\mathrm{S} / \mathrm{N}$ | Responses Control Test Result |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{Z}_{1}$ | $\mathrm{Z}_{2}$ | $\mathrm{Z}_{3}$ | $\mathrm{Z}_{4}$ | $\mathrm{Z}_{5}$ | Water Abs. <br> $(\%)$ | C.Strength <br> $\left(\mathrm{N} / \mathrm{mm}^{2}\right)$ |
| CP1 | 0.50 | 0.90 | 0.10 | 1.5833 | 3.1667 | 2.53 | 24.18 |
| CP2 | 0.45 | 0.8833 | 0.1167 | 1.4167 | 2.8333 | 2.44 | 23.32 |
| CP3 | 0.4667 | 0.8667 | 0.1333 | 1.6667 | 3.3333 | 2.22 | 22.45 |
| CP4 | 0.5167 | 0.8667 | 0.1333 | 1.8333 | 3.6667 | 2.59 | 20.65 |
| CP5 | 0.475 | 0.875 | 0.125 | 1.50 | 3.000 | 2.42 | 23.15 |
| CP6 | 0.525 | 0.8625 | 0.1375 | 1.8125 | 3.625 | 2.55 | 19.59 |
| CP7 | 0.4875 | 0.8500 | 0.1500 | 1.6875 | 3.375 | 2.38 | 17.96 |
| CP8 | 0.5125 | 0.825 | 0.175 | 1.625 | 3.25 | 2.31 | 16.05 |
| CP9 | 0.495 | 0.855 | 0.145 | 1.8000 | 3.60 | 2.39 | 19.82 |
| CP10 | 0.485 | 0.845 | 0.155 | 1.6500 | 3.30 | 2.28 | 19.52 |


| CP11 | 0.480 | 0.855 | 0.145 | 1.5750 | 3.150 | 2.23 | 20.68 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CP12 | 0.500 | 0.850 | 0.150 | 1.700 | 3.40 | 2.36 | 19.55 |
| CP13 | 0.5025 | 0.8475 | 0.1525 | 1.6500 | 3.30 | 2.38 | 19.50 |
| CP14 | 0.4925 | 0.8475 | 0.1525 | 1.6750 | 3.350 | 2.3 | 19.02 |
| CP15 | 0.4925 | 0.8600 | 0.1400 | 1.675 | 3.35 | 2.32 | 20.05 |

## Water Absorption Model Validation

The F- statistics for the validation of the water absorption model is presented in Table 10. The variances were obtained with the aid of Table 10 and Equation (32) as 0.0124 and 0.0104 which after application of Equation (31) resulted to an F -value of 1.192 . Because this calculated F value of 1.192 is less than the tabulated F - value of 2.483 ,
the null hypothesis was accepted the model considered adequate.
Figure 4 presents the plot of predicted values of water absorption against the experimental values with the associated $R^{2}$ value displayed on chart. The $R^{2}$ value of 0.6828 indicates that the model will predict values significantly close to the actual or experimental values.

Table 10: F-Statistics for Water Absorption Model Validation.

| $\mathbf{S} / \mathbf{N}$ | Exp.Value $=\mathbf{Y}_{\mathbf{e}}$ | Pred. Value $=\mathbf{Y}^{\mathbf{m}}$ | $\mathbf{Y}_{\mathbf{e}} \hat{\mathbf{Y}}_{\mathbf{e}}$ | $\mathbf{Y}_{\mathbf{m}}^{\mathbf{-}} \hat{\mathbf{Y}}^{\mathbf{m}}$ | $\left(\mathbf{Y}_{\mathbf{e}} \hat{\mathbf{Y}}_{\mathbf{e}} \mathbf{e}^{\mathbf{2}}\right.$ | $\left(\mathbf{Y}^{\mathbf{m}} \hat{\mathbf{Y}}^{\mathbf{m}}\right)^{\mathbf{2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2.53 | 2.456 | 0.150 | 0.112 | 0.023 | 0.013 |
| 2 | 2.44 | 2.219 | 0.060 | -0.125 | 0.004 | 0.016 |
| 3 | 2.22 | 2.25 | -0.160 | -0.094 | 0.026 | 0.009 |
| 4 | 2.59 | 2.591 | 0.210 | 0.247 | 0.044 | 0.061 |
| 5 | 2.42 | 2.316 | 0.040 | -0.028 | 0.002 | 0.001 |
| 6 | 2.55 | 2.508 | 0.170 | 0.164 | 0.029 | 0.027 |
| 7 | 2.38 | 2.343 | 0.000 | -0.001 | 0.000 | 0.000 |
| 8 | 2.31 | 2.288 | -0.070 | -0.056 | 0.005 | 0.003 |
| 9 | 2.39 | 2.386 | 0.010 | 0.042 | 0.000 | 0.002 |
| 10 | 2.28 | 2.282 | -0.100 | -0.062 | 0.010 | 0.004 |
| 11 | 2.23 | 2.268 | -0.150 | -0.076 | 0.023 | 0.006 |
| 12 | 2.36 | 2.32 | -0.020 | -0.024 | $4 \mathrm{E}-04$ | 0.001 |
| 13 | 2.38 | 2.322 | 0.000 | -0.022 | $0 \mathrm{E}+00$ | 0.000 |
| 14 | 2.3 | 2.297 | -0.080 | -0.047 | 0.006 | 0.002 |
| 15 | 2.32 | 2.311 | -0.060 | -0.033 | 0.004 | 0.001 |
|  | $\hat{Y}_{\mathrm{e}}=2.380$ | $\hat{\mathrm{Y}}^{\mathrm{m}}=2.344$ |  |  | $\sum=0.174$ | $\sum=0.145$ |



Fig. 4: A plot of Predictive Water Absorption values against Experimental Values Compressive Strength Model Validation.

Table 11 presents the F-statistics used in the validation of the compressive strength model. Table 11 in association with Equation (32) was used in the determination of the variances as 4.636 and 7.002. The F-calculated value was then obtained with the aid of Equation (31) as 1.510 which is less than the tabulated F - value of 2.483 . The null hypothesis was thus accepted and the model declared adequate.

The plot of predicted compressive strength value against experimental values is given by Figure 5. From the plot it can be observed that the predicted values correlated well with the actual or experimental values as indicated by the $\mathrm{R}^{2}$ value of 0.8223 displayed in Figure 4.

Table 11: F-Statistics of Compressive Strength Model Validation.

| S/N | Exp.Value $=\mathrm{Y}_{\mathrm{e}}$ | Pred. Value $=\mathrm{Y}^{\mathrm{m}}$ | $\mathrm{Y}_{\mathrm{e}}-\hat{\mathrm{Y}}_{\mathrm{e}}$ | $\mathrm{Y}^{\mathrm{m}}-\hat{\mathrm{Y}}^{\mathrm{m}}$ | $\left(\mathrm{Y}_{\mathrm{e}}-\hat{\mathrm{Y}}_{\mathrm{e}}\right)^{2}$ | $\left(\mathrm{Y}^{\mathrm{m}}-\hat{\mathrm{Y}}^{\mathrm{m}}\right)^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 24.18 | 25.621 | 3.814 | 4.989 | 14.547 | 24.890 |
| 2 | 23.32 | 26.277 | 2.954 | 5.645 | 8.726 | 31.866 |
| 3 | 22.45 | 21.113 | 2.084 | 0.481 | 4.343 | 0.231 |
| 4 | 20.65 | 19.203 | 0.284 | -1.429 | 0.081 | 2.042 |
| 5 | 23.15 | 23.893 | 2.784 | 3.261 | 7.751 | 10.634 |
| 6 | 19.59 | 19.825 | -0.776 | -0.807 | 0.602 | 0.651 |
| 7 | 17.96 | 19.516 | -2.406 | -1.116 | 5.789 | 1.245 |
| 8 | 16.05 | 16.621 | -4.316 | -4.011 | 18.628 | 16.088 |
| 9 | 19.82 | 18.911 | -0.546 | -1.721 | 0.298 | 2.962 |
| 10 | 19.52 | 19.367 | -0.846 | -1.265 | 0.716 | 1.600 |
| 11 | 20.68 | 20.927 | 0.314 | 0.295 | 0.099 | 0.087 |
| 12 | 19.55 | 19.253 | -0.816 | -1.379 | $7 \mathrm{E}-01$ | 1.902 |
| 13 | 19.5 | 19.246 | -0.866 | -1.386 | $7 \mathrm{E}-01$ | 1.921 |
| 14 | 19.02 | 19.265 | -1.346 | -1.367 | 1.812 | 1.869 |
| 15 | 20.05 | 20.442 | -0.316 | -0.190 | 0.100 | 0.036 |
|  | $\hat{\mathrm{Y}}_{\mathrm{e}}=20.366$ | $\hat{\mathrm{Y}}^{\mathrm{m}}=20.632$ |  |  | $\sum=64.906$ | $\sum=98.025$ |



Fig. 5: A plot of Predictive Compressive Strength Values against Experimental Values
Conclusions.

The following relevant conclusions have being drawn based on the outcome of this study, modelling the compressive strength and water absorption of cement concrete blended with periwinkle shell ash;

- The calcined PSA is a Class ' C ' pozzolan which can be used in the partial replacement of cement.
- Although, the unmodified cement concrete performed better on a macroscopic scale, the PSA cement concrete offers a better option within the optimum replacement bracket.
- The mathematical models derived for both responses compressive strength and water absorption of PSA cement concrete can be used satisfactorily in the prediction of these responses for any mix design and vice versa as evident from the validation tests carried out.


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