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Multi-Attributes Group Decision Making Involving Promotion Assessment Method with Partial Constraints

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Abstract
Interval type-2 fuzzy sets can articulate the indistinctness and vagueness more competently and have more powerful processing capabilities. They are characterized by the footprint of uncertainty and are incredibly helpful to portray the decision information in the course of decision-making. In this piece of writing, we study the group decision-making problems with partially known information about attribute weights presented by the decision-makers as interval type-2 fuzzy decision matrices. Initially, we employ the interval type-2 fuzzy weighted arithmetic averaging operator to aggregate all individual interval type-2 fuzzy decision matrices into the collective one and then we utilize the ranking value measure to compute the ranking value of each attribute and build the ranking value matrix of the collective interval type-2 fuzzy decision matrix. Afterward, we set up some optimization models to determine the attribute weights. Moreover, we develop a procedure to spot the best alternative. Finally, we provide an illustrative Example to ensure the feasibility of proposed method.

Keywords: Fuzzy set, interval type-2 fuzzy set, ranking values, ranking value matrix, collective normalized fuzzy decision matrix, weighted arithmetic average operator.

1. Introduction

Problems related to our daily life can be solved by using mathematical expression such as precise reasoning. Weaver [1] classified problems relevant to life into "organized simplicity" and "disorganized complexity". Organized simplicity related problems can be solved by using simple calculations and analytical approach while problems of disorganized complexity need a more precise and critical approach like statistics for dealing with physical problems which involves multiple variables and randomness at molecular level. Under some situations these problems are complementary to each other for example if one works the other fails. Most of the problems lies in the category of organized complexity as defined by Weaver. During World War II the development of computers help mankind to solve the problems related to organized complexity to a great extent but still there are some limitations that can't help to solve all of the problems related to organized complexity by either computers or human reasoning. Usually, we develop a model of artificial objects or reality aspects. Credibility, complexity and uncertainty are some factors that affect the usefulness of the model. Increasing the uncertainty helps to overcome the complexity of the model and increases its credibility. Therefore, the challenge was to develop techniques which can be used to estimate allowable uncertainty for such type of resulting models. The idea of FS by Zadeh [2] in 1965 is considered as an evolution for dealing with uncertainty as his concept of fuzzy sets are the sets which do not have price boundaries like the typical sets have.Though fuzzy set theory has served as the best tool for dealing with uncertainties but scarcity of criterion for modeling different linguistic uncertainties limits its use as is pointed out by Molodtsov [3]. To provide a rich platform for parameterizations by overcoming the deficiencies in thefuzzy set theory, The idea that the soft sets are the generalization of FS is given by Molodtsov. Fuzzy set theory in connection with soft sets have proved to be one of the most effective tools for dealing with uncertain situations some of which are discussed h

here. Maji et al. [4] propounded the perception of fuzzy soft sets and its applications in a decision-making problem.
The main prospect of decision making is selecting to the best object/alternative among the different objects/alternatives. For this purpose, decision makers analyze different techniques to evaluate the alternatives by setting up different criteria that gives the best choice. Multi-criteria decision making (MCDM) techniques are good for decision making problems now a days. In MCDM [16] we observe that mostly one object/alternative is the best choice under all the the criteria but sometime it doesn't happen. So, to sort out that kind of situation we use different techniques that assist MCDM. Mathematician proposed different techniques to overcome the increasingly complexity. Zadeh [2, 6] developed some methods to assist MCDM problems related to type-1 fuzzy sets (T1FSs). T1FSs has such type of elements whose membership degree is a crisp number (in the interval [0, 1]). After that various extensions are made to extend the criteria of T1FSs to dealt with vague/uncertain problems more precisely. These extensions include different techniques i.e. interval type-2 fuzzy sets (IT2FSs) [5, 7], Interval-valued fuzzy sets (IVFSs) [8], Intuitionistic fuzzy sets (IFSs)[9], and interval-valued intuitionistic fuzzy sets (IVIFSs) [10]. All these techniques are different from each other according to membership degree/nonmembership degree. Mendel et.al [11] gave some operations of T2FSs. To simplify the computational work Linda et al. [12] used IT2FSSs (a type T1FSSs) is used. Interval type-2 trapezoidal fuzzy number (IT2TFNs) presented by Wang et al. [13] are mostly used for MCDM.
In this work we presented MCDM for completely known information about attributes weight. Further we discussed MAGDM for completely known information using ranking vlues and attributes weight with partial known information.

2 Preliminary

Defination 2.1. [2] For a non-empty universal set X we consider $\Lambda \subseteq X$. Then, we define a fuzzy set Λ through membership function (MF) denoted by μ_Λ i.e, $\mu_\Lambda, X \rightarrow [0,1]$ where $\mu_\Lambda(x)$ denotes the degree of membership of the element x to the set Λ.
Here, we can see that Λ is fuzzy subset of X. $\mathbb{FP}(X)$ represents the set of all fuzzy subsets of a set X.

Defination 2.2. [14, 15] For a universal set X, we consider a fuzzy set Λ contained in X. If $x \in X$ and $\mu_\Lambda(x)$ is the respective MF. The highest membership degree of MF is known as height of FS. In Example 3, height of FS Λ is represented by $h(\Lambda)$ and is equal to 1.

Defination 2.3. [15, 7] Let Λ be a FS contained in X. If $x \in X$ and $\mu_\Lambda(x)$ are its MF. Then, the set Λ is known as normalized FS if $h(\Lambda) = 1$.

Defination 2.4. [7] A T1FS Λ in X is a normal T1FS iff $\exists\, x \in X$, such that $\mu_\Lambda(x) = 1$, where μ_Λ is the MF of the T1FS Λ. [15] A T1FS Λ in X is said to be a subnormal T1FS if Λ is not a normal T1FS. [15, 7] A T1FS Λ in X is known as a convex T1FS iff for all $x_1, x_2 \in X$,

$$\mu_\Lambda(\lambda x_1 + (1-\lambda)x_2) \geq min(\mu_\Lambda(x_1), \mu_\Lambda(x_2)), \lambda \in [0,1]$$

Defination 2.5. [15] A type-1 fuzzy set (T1FS) Λ in X is said to be non-convex T1FS if Λ is not convex T1FS.

Defination 2.6. [5, 16] A type-2 fuzzy set (T2FS) $\widetilde{\Lambda}$ over a X expressed by its MF $\mu_{\widetilde{\Lambda}}(x)$, presented as

$$\widetilde{\Lambda} = \{((x,\mu), \mu_{\widetilde{\Lambda}}(x,\mu)) | \forall x \in X, \forall \mu \in J_x \subseteq [0,1]\}$$

where $0 \leq \mu_{\widetilde{\Lambda}}(x,\mu) \leq 1$, in the interval [0, 1] the subinterval J_x is considered as primary membership of x, $\mu_{\widetilde{\Lambda}}(x,\mu)$ is considered as secondary membership degree.

3 Fuzzy Numbers (FNs)

Defination 3.1. [5] A fuzzy number (FN) is a particular type of FS. A FN is a FS that is normal and convex. And $\mathbb{R}$(a set of real numbers) is its universe of discourse, i.e.,
$\Lambda, \mathbb{R} \rightarrow [0,1]$
Λ is normal, i.e., $\exists x \in \mathbb{R}, \mu_\Lambda(x) = 1$.
Λ is convex, i.e., $\forall x_1, x_2 \in \mathbb{R}, \mu_\Lambda(tx_1 + (1-t)x_2) \geq \min\{\mu_\Lambda(x_1), \mu_\Lambda(x_2)\}, 0 \leq t \leq 1$. **Defination 3.2.** [5] The set of all FNs is known as fuzzy number space and denoted by $\mathbb{E}$ and defined as $\mathbb{E} = \{\Lambda | \Lambda, \mathbb{R} \rightarrow [0,1]\}$.

Defination 3.3. [10] A closed subinterval $\mu_\Lambda = [{\mu_\Lambda}^-, {\mu_\Lambda}^+]$ of F where F=[0,1] is called an interval number (IN, where $0 \leq {\mu_\Lambda}^- \leq {\mu_\Lambda}^+ \leq 1$. The set of all INs is represented by [F]. Suppose X is non-empty set. A mapping $\lambda, X \rightarrow [F]$ is known as IVFS in X. Suppose set of all $IVFSs$ in X is represented by $[F]^X$ for each $\mu_\Lambda \in [F]^X$ and $x \in X$, $\mu_\Lambda = [{\mu_\Lambda}^-, {\mu_\Lambda}^+]$ is read as the membership degree of a member x to μ_Λ, where ${\mu_\Lambda}^+, X \rightarrow F$ and ${\mu_\Lambda}^-, X \rightarrow F$ are FSs in X, which are defined as upper FS and lower FS in X, respectively.

Defination 3.4. [5, 16] A FN $\widetilde{\Lambda} = [a_1, a_2, a_3, a_4, h(\Lambda)]$ is a type-1 trapezoidal fuzzy number (T1TZFN) if its MF is defined by

$$\mu_{\widetilde{\Lambda}}(x) = \begin{cases} h(\Lambda)\frac{x-a_1}{a_2-a_1}, & a_1 \leq x < a_2 \\ h(\Lambda), & a_2 \leq x \leq a_3 \\ h(\Lambda)\frac{a_4-x}{a_4-a_3}, & a_3 < x \leq a_4 \\ 0, & otherwise \end{cases}$$

where $0 < h(\Lambda) < 1$ is the height of the T1TZFN.
$\widetilde{\Lambda}$ is said to be a normal FN if $h(\Lambda) = 1$. And $\widetilde{\Lambda}$ is said to be a non-normal if $h(\Lambda) < 1$.
$\widetilde{\Lambda}$ can be reducible to TFN by the condition $a_2 = a_3$.
$\widetilde{\Lambda}$ can be reducible to an interval number by the condition $a_1 = a_2$ and $a_3 = a_4$.
$\widetilde{\Lambda}$ can be reducible to crisp number by the condition $a_1 = a_2 = a_3 = a_4$.

Defination 3.5. [5, 16] An interval type-2 fuzzy number (IT2FN) is an (IT2FS) on $\mathbb{R}$ with MF μ is defined by $\mu(z_1) = [\underline{a_1}(z_1), \overline{a_2}(z_1)]$, where the function $a_1, a_2, \mathbb{R} \rightarrow [0,1]$ are T1FN. The function $\underline{a_1}$ is lower MF and the function $\overline{a_2}$ is upper MF.

Defination 3.6. [5] $\widetilde{\Lambda} = (\Lambda^{\mathbb{U}}; \Lambda^{\mathbb{L}}) = (a_1^{\mathbb{U}}, a_2^{\mathbb{U}}, a_3^{\mathbb{U}}, a_4^{\mathbb{U}}, h(\Lambda^{\mathbb{U}}); a_1^{\mathbb{L}}, a_2^{\mathbb{L}}, a_3^{\mathbb{L}}, a_4^{\mathbb{L}}, h(\Lambda^{\mathbb{L}}))$ is an IT2FS on $\mathbb{R}$. $\widetilde{\Lambda}$ is a IT2TFN if it can be defined by its MF $\mu_{\Lambda^{\mathbb{U}}}(x)$ and $\mu_{\Lambda^{\mathbb{L}}}(x)$. Where $\mu_{\Lambda^{\mathbb{U}}}(x)$ is a UMF and $\mu_{\Lambda^{\mathbb{L}}}(x)$ is a LMF. i.e

$$\mu_{\Lambda^{\mathbb{U}}}(x)=\begin{cases} h(\Lambda^{\mathbb{U}})\frac{x-a_1^{\mathbb{U}}}{a_2^{\mathbb{U}}-a_1^{\mathbb{U}}}, & a_1^{\mathbb{U}}\le x< a_2^{\mathbb{U}}\\ h(\Lambda^{\mathbb{U}}), & a_2^{\mathbb{U}}\le x\le a_3^{\mathbb{U}}\\ h(\Lambda^{\mathbb{U}})\frac{a_4^{\mathbb{U}}-x}{a_4^{\mathbb{U}}-a_3^{\mathbb{U}}}, & a_3^{\mathbb{U}}< x\le a_4^{\mathbb{U}}\\ 0, & otherwise\end{cases}$$

and

$$\mu_{\Lambda^{\mathbb{L}}}(x)=\begin{cases} h(\Lambda^{\mathbb{L}})\frac{x-a_1^{\mathbb{L}}}{a_2^{\mathbb{L}}-a_1^{\mathbb{L}}}, & a_1^{\mathbb{L}}\le x< a_2^{\mathbb{L}}\\ h(\Lambda^{\mathbb{L}}), & a_2^{\mathbb{L}}\le x\le a_3^{\mathbb{L}}\\ h(\Lambda^{\mathbb{L}})\frac{a_4^{\mathbb{L}}-x^{\mathbb{L}}}{a_4^{\mathbb{L}}-a_3^{\mathbb{L}}}, & a_3^{\mathbb{L}}< x\le \Lambda_4^{\mathbb{L}}\\ 0, & otherwise\end{cases}$$

3.1 Operations for IT2TFNs

Defination 3.7. Let $\widetilde{\Lambda}=(\Lambda_1^{\mathbb{U}};\Lambda_1^{\mathbb{L}})=(a_{11}^{\mathbb{U}},a_{12}^{\mathbb{U}},a_{13}^{\mathbb{U}},a_{14}^{\mathbb{U}},h(\Lambda^{\mathbb{U}});a_{11}^{\mathbb{L}},a_{12}^{\mathbb{L}},a_{13}^{\mathbb{L}},a_{14}^{\mathbb{L}},h(\Lambda^{\mathbb{L}}))$ and $\widetilde{\Lambda}_2=(\Lambda_2^{\mathbb{U}};\Lambda_1^{\mathbb{L}})=(a_{21}^{\mathbb{U}},a_{22}^{\mathbb{U}},a_{23}^{\mathbb{U}},a_{24}^{\mathbb{U}},h(\Lambda_2^{\mathbb{U}});a_{21}^{\mathbb{L}},a_{22}^{\mathbb{L}},a_{23}^{\mathbb{L}},a_{24}^{\mathbb{L}},h(\Lambda_2^{\mathbb{L}}))$ be two IT2TzFNs. Then, addition, scalar multiplication and multiplication are defined by

1. $\widetilde{\Lambda}_1+\widetilde{\Lambda}_2=(a_{11}^{\mathbb{U}}+a_{21}^{\mathbb{U}},a_{12}^{\mathbb{U}}+a_{22}^{\mathbb{U}},a_{13}^{\mathbb{U}}+a_{23}^{\mathbb{U}},a_{14}^{\mathbb{U}}+a_{24}^{\mathbb{U}},\min(h(\Lambda_1^{\mathbb{U}}),h(\Lambda_2^{\mathbb{U}}));a_{11}^{\mathbb{L}}+a_{21}^{\mathbb{L}},a_{12}^{\mathbb{L}}+a_{22}^{\mathbb{L}},a_{13}^{\mathbb{L}}+a_{23}^{\mathbb{L}},a_{14}^{\mathbb{L}}+a_{24}^{\mathbb{L}},\min(h(\Lambda_1^{L}),h(\Lambda_2^{\mathbb{L}})))$
2. $\lambda\widetilde{\Lambda}=(\lambda\Lambda^{\mathbb{U}};\lambda\Lambda^{\mathbb{L}})=(\lambda a_{11}^{\mathbb{U}},\lambda a_{12}^{\mathbb{U}},\lambda a_{13}^{\mathbb{U}},\lambda a_{14}^{\mathbb{U}},h(\Lambda^{\mathbb{U}});\lambda a_{11}^{\mathbb{L}},\lambda a_{12}^{\mathbb{L}},\lambda a_{13}^{\mathbb{L}},\lambda a_{14}^{\mathbb{L}},h(\Lambda^{\mathbb{L}}))$
3. $\widetilde{\Lambda}_1\times\widetilde{\Lambda}_2=(a_{11}^{\mathbb{U}}\cdot a_{21}^{\mathbb{U}},a_{12}^{\mathbb{U}}\cdot a_{22}^{\mathbb{U}},a_{13}^{\mathbb{U}}\cdot a_{23}^{\mathbb{U}},a_{14}^{\mathbb{U}}\cdot a_{24}^{\mathbb{U}},\min(h(\Lambda_1^{\mathbb{U}}),h(\Lambda_2^{\mathbb{U}}));a_{11}^{\mathbb{L}}\cdot a_{21}^{\mathbb{L}},a_{12}^{\mathbb{L}}\cdot a_{22}^{\mathbb{L}},a_{13}^{\mathbb{L}}\cdot a_{23}^{\mathbb{L}},a_{14}^{\mathbb{L}}\cdot a_{24}^{\mathbb{L}},\min(h(\Lambda_1^{\mathbb{L}}),h(\Lambda_2^{\mathbb{L}})))$

Defination 3.8. [5] Let $\widetilde{\Lambda}$ and $\widetilde{\Lambda}_2$ are two IT2TFNs and $\lambda\ge 0$. Then, the addition, scalar multiplication and exponentiation of IT2TzFNs presented as follows;

(1) Addition:

$\widetilde{\Lambda}_1+\widetilde{\Lambda}_2=(\Lambda_1^{\mathbb{U}}+\Lambda_2^{\mathbb{U}},\Lambda_1^{\mathbb{L}}+\Lambda_2^{\mathbb{L}})=(a_{11}^{\mathbb{U}}+a_{21}^{\mathbb{U}},a_{12}^{\mathbb{U}}+a_{22}^{\mathbb{U}},a_{13}^{\mathbb{U}}+a_{23}^{\mathbb{U}},a_{14}^{\mathbb{U}}+a_{24}^{\mathbb{U}},\frac{h(\Lambda_1^{\mathbb{U}})\cdot\|\Lambda_1^{\mathbb{U}}\|+h(\Lambda_2^{\mathbb{U}})\cdot\|\Lambda_2^{\mathbb{U}}\|}{\|\Lambda_1^{\mathbb{U}}\|+\|\Lambda_2^{\mathbb{U}}\|};a_{11}^{\mathbb{L}}+a_{21}^{\mathbb{L}},a_{12}^{\mathbb{L}}+a_{22}^{\mathbb{L}},a_{13}^{\mathbb{L}}+a_{23}^{\mathbb{L}},a_{14}^{\mathbb{L}}+a_{24}^{\mathbb{L}},\frac{h(\Lambda_1^{\mathbb{L}})\cdot\|\Lambda_1^{\mathbb{L}}\|+,h(\Lambda_2^{\mathbb{L}})\cdot\|\Lambda_2^{\mathbb{L}}\|}{\|\Lambda_1^{\mathbb{L}}\|+\|\Lambda_2^{\mathbb{L}}\|})$

where $\|\Lambda_j^{\mathbb{U}}\|=\frac{a_{j1}^{\mathbb{U}}+a_{j2}^{\mathbb{U}}+a_{j3}^{\mathbb{U}}+a_{j4}^{\mathbb{U}}}{4}$ and $\|\Lambda_j^{\mathbb{L}}\|=\frac{a_{j1}^{\mathbb{L}}+a_{j2}^{\mathbb{L}}+a_{j3}^{\mathbb{L}}+a_{j4}^{\mathbb{L}}}{4}$

(2) Scalar Multiplication:

$\lambda\widetilde{\Lambda}=(\lambda\Lambda^{\mathbb{U}};\lambda\Lambda^{\mathbb{L}})=(\lambda a_{11}^{\mathbb{U}},\lambda a_{12}^{\mathbb{U}},\lambda a_{13}^{\mathbb{U}},\lambda a_{14}^{\mathbb{U}},h(\Lambda^{\mathbb{U}});\lambda a_{11}^{\mathbb{L}},\lambda a_{12}^{\mathbb{L}},\lambda a_{13}^{\mathbb{L}},\lambda a_{14}^{\mathbb{L}},h(\Lambda^{\mathbb{L}}))$

(3) Multiplication:

$\widetilde{\Lambda}_1\times\widetilde{\Lambda}_2=(\Lambda_1^{\mathbb{U}}\cdot\Lambda_2^{\mathbb{U}};\Lambda_1^{\mathbb{L}}\cdot\Lambda_2^{\mathbb{L}})=(a_{11}^{\mathbb{U}}\cdot a_{21}^{\mathbb{U}},a_{12}^{\mathbb{U}}\cdot a_{22}^{\mathbb{U}},a_{13}^{\mathbb{U}}\cdot a_{23}^{\mathbb{U}},a_{14}^{\mathbb{U}}\cdot a_{24}^{\mathbb{U}},h(\Lambda_1^{\mathbb{U}})\cdot h(\Lambda_2^{\mathbb{U}});a_{11}^{\mathbb{L}}\cdot a_{21}^{\mathbb{L}},a_{12}^{\mathbb{L}}\cdot a_{22}^{\mathbb{L}},a_{13}^{\mathbb{L}}\cdot a_{23}^{\mathbb{L}},a_{14}^{\mathbb{L}}\cdot a_{24}^{\mathbb{L}},h(\Lambda_1^{\mathbb{L}})\cdot h(\Lambda_2^{\mathbb{L}}))$

(4) Exponentiation:

$\widetilde{\Lambda}^{\lambda}=((\Lambda^{\mathbb{U}})^{\lambda};(\Lambda^{\mathbb{L}})^{\lambda})=((a_{11}^{\mathbb{U}})^{\lambda},(a_{12}^{\mathbb{U}})^{\lambda},(a_{13}^{\mathbb{U}})^{\lambda},(a_{14}^{\mathbb{U}})^{\lambda},(h(\Lambda^{\mathbb{U}}))^{\lambda};(a_{11}^{\mathbb{L}})^{\lambda},(a_{12}^{\mathbb{L}})^{\lambda},(a_{13}^{\mathbb{L}})^{\lambda},(a_{14}^{\mathbb{L}})^{\lambda}(h(\Lambda^{\mathbb{L}}))^{\lambda})$

Defination 3.9. [5, 16] Consider three non-negative interval type-2 trapezoidal fuzzy numbers (IT2TFNS) $\widetilde{\Lambda}_i$ (i=1,2,3), then the arithmetic operations in above definition satisfy the followings,

(1) $\widetilde{\Lambda}_1+\widetilde{\Lambda}_2=\widetilde{\Lambda}_2+\widetilde{\Lambda}_1$
(2) $(\widetilde{\Lambda}_1+\widetilde{\Lambda}_2)+\widetilde{\Lambda}_3=\widetilde{\Lambda}_1+(\widetilde{\Lambda}_2+\widetilde{\Lambda}_3)$
(3) $\widetilde{\Lambda}_1\times\widetilde{\Lambda}_2=\widetilde{\Lambda}_2\times\widetilde{\Lambda}_1$
(4) $(\widetilde{\Lambda}_1\times\widetilde{\Lambda}_2)\times\widetilde{\Lambda}_3=\widetilde{\Lambda}_1\times(\widetilde{\Lambda}_2\times\widetilde{\Lambda}_3)$
(5) $\lambda_1\widetilde{\Lambda}+\lambda_2\widetilde{\Lambda}=(\lambda_1+\lambda_2)\widetilde{\Lambda},\lambda_1,\lambda_2\ge 0$
(6) $\lambda\widetilde{\Lambda}_1+\lambda\widetilde{\Lambda}_2=\lambda(\widetilde{\Lambda}_1+\widetilde{\Lambda}_2),\lambda\ge 0$
(7) $(\widetilde{\Lambda})^{\lambda_1}\times(\widetilde{\Lambda})^{\lambda_2}=(\widetilde{\Lambda})^{\lambda_1+\lambda_2},\lambda_1,\lambda_2\ge 0$
(8) $(\widetilde{\Lambda}_1)^{\lambda}\times(\widetilde{\Lambda}_2)^{\lambda}=(\widetilde{\Lambda}_1+\widetilde{\Lambda}_2)^{\lambda},\lambda\ge 0$

4 Ranking value formula for IT2TFSs

Let $\widetilde{\Lambda}_i=[(a_{i1}^u,a_{i2}^u,a_{i3}^u,a_{i4}^u,h_1(\Lambda_i^u),h_2(\widetilde{\Lambda}_i^u));(a_{i1}^l,a_{i2}^l,a_{i3}^l,a_{i4}^l,h_1(\Lambda_i^l),h_2(\Lambda_i^l))]$ be a IT2TFN. Then, the rank of $\widetilde{\Lambda}_i$ is represented by $Rank(\widetilde{\Lambda}_i)$ and defined as

$$\begin{aligned} Rank(\widetilde{\Lambda}_i)= \; & M_1(\widetilde{\Lambda}_i^u)+M_1(\widetilde{\Lambda}_i^l)+M_2(\widetilde{\Lambda}_i^u)+M_2(\widetilde{\Lambda}_i^l)+M_3(\widetilde{\Lambda}_i^u)+M\\ & -\frac{1}{4}[\mathcal{S}d_1(\widetilde{\Lambda}_i^u)+\mathcal{S}d_1(\widetilde{\Lambda}_i^l)+\mathcal{S}d_2(\widetilde{\Lambda}_i^u)+\mathcal{S}d_2(\widetilde{\Lambda}_i^l)+\mathcal{S}d\\ & +h_1(\widetilde{\Lambda}_i^u)+h_2(\widetilde{\Lambda}_i^u)+h_1(\widetilde{\Lambda}_i^l)+h_2(\widetilde{\Lambda}_i^l)\end{aligned}$$

Here, for $1\le p\le 3$, $1\le q\le 2$, and $j\in\{u,l\}$,

$M_p(\widetilde{\Lambda}_i^j)$ shows average of a_{ip}^j and $a_{i(p+1)}^j$, i.e., $M_p(\widetilde{\Lambda}_i^j)=\frac{a_{ip}^j+a_{i(p+1)}^j}{2}$,

$\mathcal{S}d_p(\widetilde{\Lambda}_i^j)$ shows the standard deviation (SD) of a_{ip}^j and $a_{i(p+1)}^j$,

$$S_p(\widetilde{\Lambda}_i^j)=\sqrt{\frac{1}{2}\sum_{k=p}^{p+1}(a_{ik}^j-M_p(\widetilde{\Lambda}_i^j))^2},$$

$\mathcal{S}d_4(\widetilde{\Lambda}_i^j)=\sqrt{\frac{1}{4}\sum_{k=1}^{4}(a_{ik}^j-\frac{1}{4}\sum_{k=1}^{4}a_{ik}^j)^2}$ shows the SD of the elements $a_{i1}^j,a_{i2}^j,a_{i3}^j$ and a_{i4}^j,

and $h(\widetilde{\Lambda}_i^j)$ represents the value of membership of the member $a_{i(q+1)}^j$ in trapezoidal MF $\widetilde{\Lambda}_i^j$.

5 Decision making with completely known information about attributes weight

Consider a set of objects/alternatives $\mathcal{O}=\{o_1,o_2,\ldots,o_l\}$ for a MAGDM problem. Also consider $D_m=\{Dm_1,Dm_2,\ldots,Dm_n\}$ and $\lambda=(\Lambda,\lambda_2,\ldots,\lambda_l)^t$ are the sets of DMs and weight vectors of those DMs respectively. And a set $U=\{u_1,u_2,\ldots,u_m\}$ shows attributes. Let us consider DMs give the complete information about attribute weights. The information set about attributes weight given by DMs presented by H. Let us consider a IT2F decision matrix $\tilde{R}^{(k)}=(\widetilde{\Lambda}_{ij}^{(k)})_{n\times m}$, where $\Lambda_{ij}^{(k)}$ is an IT2FS given by the DMs $dm_k\in D_m$ for the alternatives $o_i\in\mathcal{O}$ concerning the attribute $u_j\in U$. Generally, attributes can be divided in to benefit type and cost type.

Here, the attributes set U classified in to 2 subsets, U_1 (the subset of benefit type) and U_2 (the subset of cost type). Also $U_1\cup U_2=U$ and $U_1\cap U_2=\phi$, where ϕ is a void set. DMts $\tilde{\mathcal{R}}^{(k)}$ need to be normalized unless all the attributes/parametrs are of the same type. Here we write normalization formula to modify the DMts $\tilde{\mathcal{R}}^{(k)}$.

$$\widetilde{\Lambda}_{ij}^{(k)}=\begin{cases}\Lambda_{ij}^{(k)}, & j\in U_1\\ (\Lambda_{ij}^{(k)})^c, & j\in U_2\end{cases}$$

where $(\Lambda_{ij}^{(k)})^c$ shows complement of $\Lambda_{ij}^{(k)}$. So, we get normalized DMts $\tilde{\mathcal{R}}^{(k)}=(\widetilde{\Lambda}_{ij}^{(k)})_{n\times m}$.

First of all we will convert the decision opinion of individuals into group opinion to make a final decision. Here IT2 fuzzy (WAAO) are used to sum up all the individual normalized DMts $\tilde{\mathcal{R}}^{(k)}=(\widetilde{\Lambda}_{ij}^{(k)})_{n\times m}$, $k=1,2,\ldots,l$ into the CNFDMt $\tilde{\mathcal{R}}=(\widetilde{\Lambda}_{ij})_{n\times m}$, where

$\widetilde{\Lambda}_{ij}=\bigoplus_{k=1}^{l}\lambda_k\widetilde{\Lambda}_{ij}^{(k)}$

Here we know the complete information about attributes weight i.e the weight vector

$\mathcal{W}t = (w_1, w_1, \ldots, w_m)^t$ of the attributes u_j, $(j = 1,2,\ldots,m)$. Then, based on $\tilde{\mathcal{R}} = (\widetilde{\Lambda}_{ij})_{n\times m}$ by utilizing fuzzy ranking matrix and arithmetic operation between IT2FSs we get the value $\widetilde{\Lambda}_i$ of the alternative $o_i, (i = 1,2,\ldots,l)$;

$$\widetilde{\Lambda}_i = \oplus_{j=1}^{m} (w_j \widetilde{\Lambda}_{ij}), i = 1,2,\ldots,n$$

Higher value of $Rank(\widetilde{\Lambda}_i)$, means the best alternative is o_i.
The information about attributes weight given by DMs is normally incomplete because of increasing complexity of the socio-economic environment make it very difficult for DMs to consider all relevant conditions of the problem. To find the best alternative by utilizing IT2F decision matrix and known weights information is a very interesting issue. For this purpose,we establish an approach to determine the attributes weight.

6 Decision making through ranking value with completely Known weights

For interval type-2 fuzzy decision matrix (IT2F DMt) $\tilde{R} = (\widetilde{\Lambda}_{ij})_{n\times m}$, we consider RVMt $\mathcal{R} = (r_{ij})_{n\times m}$ of $\tilde{\mathcal{R}} = (\widetilde{\Lambda}_{ij})_{n\times m}$, where $r_{ij} = Rank(\widetilde{\Lambda}_{ij})$ is the RV of $\widetilde{\Lambda}_{ij}, 1 \le i \le n, 1 \le j \le m$. On the basis o normalized RVMt, we express the RV of each alternative as;

$r_i(w) = \sum_{j=1}^{m} w_j r_{ij},\ i = 1,2,\ldots,n.$

Certainly, the bigger value $r_i(w)$ is the best alternative o_i. When we consider only the alternative o_i, then a suitable vector of attributes weight $\mathcal{W}t = (w_1, w_2, \ldots, w_m)^t$ should be determined to maximize $r_i(w)$ using following optimization model;

Model-1

Maximize $r_i(w)$ such that $\mathcal{W} = (w_1, w_2, \ldots, w_m)^t \in H$, $w_j \ge 0, j = 1,2,\ldots,m$, and $\sum_{j=1}^{m} w_j = 1$

From model-1, we obtained the best choice $\omega^{(i)} = (w_1^{(i)}, w_2^{(i)}, \ldots, w_m^{(i)})^t$ correspond to alternative o_i, $1 \le i \le n$. Generally, the process of determining the weight vector $\mathcal{W}t = (w_1, w_2, \ldots, w_m)^t$, consider all alternatives o_i, $(i = 1,2,\ldots,n)$. So, construct a combined weight vector as follows;

$$\mathcal{W}t = \mathcal{W}_1\, w^{(1)} + \mathcal{W}_2\, w^{(2)} + \cdots \mathcal{W}_n$$

$$w^{(n)} = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \ldots & w_1^{(n)} \\ w_2^{(1)} & w_2^{(2)} & \ldots & w_2^{(n)} \\ \ldots & \ldots & \ldots & \ldots \\ w_m^{(1)} & w_m^{(2)} & \ldots & w_m^{(n)} \end{pmatrix} \begin{pmatrix} \mathcal{W}_1 \\ \mathcal{W}_2 \\ \ldots \\ \mathcal{W}_m \end{pmatrix} = \Omega W$$

where

$$\Omega = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \ldots & w_1^{(n)} \\ w_2^{(1)} & w_2^{(2)} & \ldots & w_2^{(n)} \\ \ldots & \ldots & \ldots & \ldots \\ w_m^{(1)} & w_m^{(2)} & \ldots & w_m^{(n)} \end{pmatrix}$$

and $\mathcal{W} = (\mathcal{W}_1, \mathcal{W}_2, \ldots, \mathcal{W}_n)^t$ is undetermined non-negative vector satisfies the condition; $\mathcal{W}\,\mathcal{W}^t = 1$

Let $\bar{r}_i = (r_{i1}, r_{i2}, \ldots, r_{im})$, $(i = 1,2,\ldots,n)$, then the RVMt R can be represented as $\mathcal{R} = (\bar{r}_1, \bar{r}_2, \ldots, \bar{r}_n)^t$. Since

$r_i(w) = \sum_{j=1}^{m} w_j r_{ij} = \bar{r}_i\, \omega = \bar{r}_i \Omega \mathcal{W}, i = 1,2,\ldots,n$

To find the combined weight vector $\mathcal{W}t = (w_1, w_2, \ldots, w_m)^t$, we should make overall RVs $r_i(w)$, $(i = 1,2,\ldots,n)$ greater as possible, to maximize the following vector $r(w) = (r_1(w), r_2(w), \ldots, r_n(w))$ under the condition $\mathcal{W}\,\mathcal{W}^t = 1$. To compute we construct a model;

Model-2

Max $r(w) = (r_1(w), r_2(w), \ldots, r_n(w))$

Subject to $\mathcal{W}\,\mathcal{W}^t = 1$

By equal weighted summation method, model-2 can be changed into a single objective optimization model;

Max $r(w)\, r(w)^t$

Subject to $\mathcal{W}\,\mathcal{W}^t = 1$

Let $g(w) = r(w)\, r(w)^t$, then we have

$g(w) = r(w)\, r(w)^t = \mathcal{W}^t (R\,\Omega)^t\, (R\,\Omega)\, \mathcal{W})$

Let $Sm = (\mathcal{R}\,\Omega)^t\, (\mathcal{R}\,\Omega)$, then $Sm^t = (\mathcal{R}\,\Omega)^t\, (\mathcal{R}\,\Omega) = \mathcal{S}$, i.e Sm is RSM. Moreover, $\mathcal{R} \ge 0$ and Sm is non-negative definite matrix.

Theorem-1

Consider $\mathbf{C} = (c_{ij})_{n\times n}$ be a RSM, i.e $\mathbf{C}^t = \mathbf{C}$, then $max \frac{x^t C x}{x^t x} = \lambda_{max}$ where λ_{max} is the biggest eigenvalue of $\mathbf{C}$ and x is a nonzero vector.

Theorem-2

Let $\mathbf{C} = (c_{ij})_{n\times n}$ be a real irreducible non-negative matrix, then

(1) $\mathbf{C}$ has a biggest eigenvalue λ_{max}, which is also a unique eigenvalue of C.

(2) Let $V = (v_1, v_2, \ldots, v_n)^t$ be the eigenvectors of λ_{max}, then all $v_j \ge 0$, $(j = 1,2,\ldots,n)$, i.e V is a positive eigenvector.

By Theorem 1 and 2, knowing that that g(w) has a biggest value, i.e, max$f(w)$, also the greatest eigenvalue $\bar{\lambda}_{max}$ of Sm. $Ve = (v_1, v_2, \ldots, v_n)^t$ is the eigenvector of $\bar{\lambda}_{max}$, where $\bar{\lambda}_{max}$ is unique, and all w_i, $(i = 1,2,\ldots,n)$. We can utilize $\Omega\,\mathcal{W}$ to find weight vector $\mathcal{W}t$ after normalizing Ve.

7 MAGDM approach based on IT2 fuzzy environment

Multi attribute group decision making (MAGDM) approach based on IT2 fuzzy environment for incomplete attributes weight information is follows.

Step 1, In first step we normalize IT2 DMts $\bar{\mathcal{R}}^k = (\bar{\Lambda}_{ij}^k)_{n\times m}$, where $i = 1,2,\ldots,l$ given by DMs, Dm_k, where $k = 1,2,\ldots,l$ utilizing IT2 fuzzy WAAO i.e $\widetilde{\Lambda}_{ij} = \oplus_{k=1}^{l} \lambda_k \widetilde{\Lambda}_{ij}^{(k)}$. Its particulars are to sum up all individual $\bar{\mathcal{R}}^k = (\bar{\Lambda}_{ij}^k)_{n\times m}$ $(i = 1,2,\ldots,l)$ into a CNIT2FDMt $\bar{\mathcal{R}} = (\bar{\Lambda}_{ij})_{n\times m}$.

Step 2, On the basis of defined rank of IT2FN we find the RVMt $\mathcal{R} = (r_{ij})_{n\times m}$ of $\bar{\mathcal{R}} = (\bar{\Lambda}_{ij})_{n\times m}$.

Step 3, To get best weight vectors $\omega^{(i)} = (w_1^{(i)}, w_2^{(i)}, \ldots, w_m^{(i)})^t$, where $i = 1,2,\ldots,n$ corresponding to the alternatives o_i, where $i = 1,2,\ldots,n$ by utilizing model-1 to construct the weight matrix Ω.

Step 4, Compute normalized eigenvector $\mathcal{W} = (w_1, w_2, \ldots, w_n)^t$ for the matrix $(R\,\Omega)^t\, (\mathcal{R}\,\Omega)$.

Step 5, Utilize $\Omega\,\mathcal{W}$ to drive the weight vectors $\mathcal{W}t = (w_1, w_2, \ldots, w_m)^t$.

Step 6, On the basis of $\bar{\mathcal{R}} = (\widetilde{\Lambda}_{ij})_{n\times m}$, use the WAAO i.e $\widetilde{\Lambda}_i = \oplus_{j=1}^{m} (w_j \widetilde{\Lambda}_{ij})$, where $i = 1,2,\ldots,n$ to obtain the overall value $\widetilde{\Lambda}_i$ of the alternative o_i, $i = 1,2,\ldots,n$.

Step 7, Compute the RV $Rank(\widetilde{\Lambda}_i)$ of the overall value $\widetilde{\Lambda}_i$, $1 \le i \le n$.

Step 8, Find the rank of each o_i, where $(i = 1,2,\ldots,n)$ on the basis of RV $Rank(\widetilde{\Lambda}_i)$, $1 \le i \le n$. Maximum value of $Rank(\widetilde{\Lambda}_i)$ shows best alternative o_i.

We elaborate the model with the help of an Example. In Table 1, we define the linguistic terms Very Bad (VB), Bad

(B), Unsatisfactory (US), Satisfactory (S), Good (G), Very Good (VG), Excellent (E), and their corresponding IT2FSs. Moreover, we present the complementary relations of linguistic terms (or IT2FSs) in Table 2.

Table 1: Linguistic terms and their corresponding IT2FSs

Linguistic Term	IT2FSs
Very Bad (VB)	(0.05,0.05,0.05,0.15,1; 0.05,0.05,0.05,0.1, 0.9)
Bad (B)	(0.05,0.15,0.15,0.35,1; 0.1,0.15,0.15,0.25, 0.9)
Unsatisfactory (US)	(0.15,0.35,0.35,0.55,1; 0.25,0.35,0.35,0.45, 0.9)
Satisfactory (S)	(0.35,0.55,0.55,0.75,1; 0.45,0.55,0.55,0.65, 0.9)
Good (G)	(0.55,0.65,0.65,0.85,1; 0.65,0.65,0.65,0.75, 0.9)
Very Good (VG)	(0.75,0.85,0.85,0.95,1; 0.85,0.85,0.85,0.9, 0.9)
Excellent (E)	(0.85,0.95,0.95,1,1; 0.95,0.95,0.95,0.95, 0.9)

Table 2: Complementary Relations

$\widetilde{\Lambda}$	VB	B	US	S	G	VG	E
$\widetilde{\Lambda}^c$	E	VG	G	S	US	B	VB

Suppose a business organization wants to promote their employee. Organization have a panel having three experts Dm_k, $(k = 1,2,3)$. They evaluate the employee according to the desired criteria of promotion. They evaluate three alternatives Alt_i, where $i = 1,2,3$ according to the four parameters i.e p_1, academics, p_2, performance, p_3, experience, p_4, discipline with weight vector $\lambda = (0.25,0.4,0.2)$. λ is the weight of each alternative. And $\mathcal{W}t = (0.35,0.2,0.25,0.4)$ is weight vector of each parameter.

Step 1, We consider all parameter of benefit type. So three normalized DMt given by the DMs Dm_k, $(k = 1,2,3)$ are are provided in Tables 3, 4, and 5.

Table 3: Normalized DMt $\tilde{\mathcal{R}}^{(1)}$

Alt_1	G	B	E	E
Alt_2	VG	US	VG	VG
Alt_3	E	VB	S	VG

Table 4: Normalized decision matrix $\tilde{\mathcal{R}}^{(2)}$

Alt_1	VG	VB	VG	VG
Alt_2	G	B	E	E
Alt_3	VG	VB	G	E

Table 5: Normalized decision matrix $\tilde{\mathcal{R}}^{(3)}$

Alt_1	G	B	VG	VG
Alt_2	VG	VB	E	VG
Alt_3	G	B	G	E

Aggregate of all the normalized fuzzy DMts to find collective IT2F DMt $\tilde{\mathcal{R}} = (\widetilde{\Lambda}_{ij})_{3\times 4}$

$$\tilde{R} = \begin{pmatrix} \widetilde{\Lambda}_{11} & \widetilde{\Lambda}_{12} & \widetilde{\Lambda}_{13} & \widetilde{\Lambda}_{14} \\ \widetilde{\Lambda}_{21} & \widetilde{\Lambda}_{22} & \widetilde{\Lambda}_{23} & \widetilde{\Lambda}_{24} \\ \widetilde{\Lambda}_{31} & \widetilde{\Lambda}_{32} & \widetilde{\Lambda}_{33} & \widetilde{\Lambda}_{34} \end{pmatrix}$$

By using the formula $\widetilde{\Lambda}_{ij} = \bigoplus_{k=1}^{l} \lambda_k \widetilde{\Lambda}_{ij}^{(k)}$, we have

$$\widetilde{\Lambda}_{11} = ((0.5475,0.6325,0.6325,0.7625,1; 0.6325,0.6325,0.6975,0.7650,0.9)$$

$$\widetilde{\Lambda}_{12} = (0.0425,0.0875,0.0875,0.2175,1; 0.0875,0.0875,0.1525,0.7650,0.9)$$

$$\widetilde{\Lambda}_{13} = (0.6625,0.7475,0.7475,0.8200,1; 0.7475,0.7475,0.7775,0.7650,0.9)$$

$$\widetilde{\Lambda}_{14} = (0.6625,0.7475,0.7475,0.8200,1; 0.7475,0.7475,0.7775,0.7650,0.9)$$

$$\widetilde{\Lambda}_{14} = \widetilde{\Lambda}_{13}$$

$$\widetilde{\Lambda}_{21} = (0.5575,0.6425,0.6425,0.7675,1; 0.6425,0.6425,0.7050,0.7650,0.9)$$

$$\widetilde{\Lambda}_{22} = (0.0675,0.1575,0.1575,0.3075,1; 0.1575,0.1575,0.2325,0.7650,0.9)$$

$$\widetilde{\Lambda}_{23} = (0.6975,0.7825,0.7825,0.8375,1; 0.7825,0.7825,0.7950,0.7650,0.9)$$

$$\widetilde{\Lambda}_{24} = (0.6775,0.7625,0.7625,0.8275,1; 0.7625,0.7625,0.7850,0.7650,0.9)$$

$$\widetilde{\Lambda}_{31} = (0.6225,0.7075,0.7075,0.8000,1; 0.7075,0.7075,0.7475,0.7650,0.9)$$

$$\widetilde{\Lambda}_{32} = (0.0425,0.0625,0.0625,0.1675,1; 0.0625,0.0625,0.1150,0.7650,0.9)$$

$$\widetilde{\Lambda}_{33} = (0.4175,0.5275,0.5275,0.6975,1; 0.5275,0.5275,0.6125,0.7650,0.9)$$

$$\widetilde{\Lambda}_{34} = (0.6975,0.7825,0.7825,0.8375,1; 0.7825,0.7825,0.7950,0.7650,0.9)$$

$$\widetilde{\Lambda}_{34} = \widetilde{\Lambda}_{23}$$

Step 2, On the basis of $\tilde{R}$ we use the weighted averaging operator $\widetilde{D}m_i = \bigoplus_{j=1}^{m} (\omega_j \widetilde{\Lambda}_{ij})$ to find the value of each Dm_i of the alternative Alt_i, $(i = 1,2,3)$, i.e,

$Dm_1 =$
$(0.6308,0.7248,0.7248,0.8434,1; 0.7248,0.7248,0.7800,0.9180,0.9)$

$Dm_2 =$
$(0.6540,0.7570,0.7570,0.8705,1; 0.7570,0.7570,0.8060,0.9180,0.9$

$Dm_3 =$
$(0.6098,0.7050,0.7050,0.8229,1; 0.7050,0.7050,0.7558,0.9180,0.9$

Step 3, Now, we find $Rank(Dm_i)$ of values Dm_i, where $i = 1,2,3$.

$Rank(Dm_1) = 6.3233, Rank(Dm_2) = 6.4940$, and $Rank(Dm_3) = 6.2046$

Step 4, Here, we can see that $Rank(Dm_2) > Rank(Dm_1) > Rank(Dm_3)$. Thus, order of the alternatives Alt_1, Alt_2, Alt_3 is ,

$Alt_2 > Alt_1 > Alt_2$

Hence, the most favorite employee for promotion is Alt_2.

8 An example of MCDM through ranking value with completely known weights

Now we deal this Example by ranking value discuss in the first section of this chapter.

Step 1, Similarly to the first algorithm we calculate $\tilde{\mathcal{R}} = (\tilde{\Lambda}_{ij})_{3\times 4}$

$$\tilde{\mathcal{R}} = \begin{pmatrix} \tilde{\Lambda}_{11} & \tilde{\Lambda}_{12} & \tilde{\Lambda}_{13} & \tilde{\Lambda}_{14} \\ \tilde{\Lambda}_{21} & \tilde{\Lambda}_{22} & \tilde{\Lambda}_{23} & \tilde{\Lambda}_{24} \\ \tilde{\Lambda}_{31} & \tilde{\Lambda}_{32} & \tilde{\Lambda}_{33} & \tilde{\Lambda}_{34} \end{pmatrix}$$

By using the formula $\tilde{\Lambda}_{ij} = \bigoplus_{k=1}^{l} \lambda_k \tilde{\Lambda}_{ij}^{(k)}$ we have

$$\tilde{\Lambda}_{11} = (0.5475,0.6325,0.6325,0.7625,1; 0.6325,0.6325,0.6975,0.7650,0.9)$$

$$\tilde{\Lambda}_{12} = (0.0425,0.0875,0.0875,0.2175,1; 0.0875,0.0875,0.1525,0.7650,0.9)$$

$$\tilde{\Lambda}_{13} = (0.6625,0.7475,0.7475,0.8200,1; 0.7475,0.7475,0.7775,0.7650,0.9)$$

$$\tilde{\Lambda}_{14} = (0.6625,0.7475,0.7475,0.8200,1; 0.7475,0.7475,0.7775,0.7650,0.9)$$

$$\tilde{\Lambda}_{14} = \tilde{\Lambda}_{13}$$

$$\tilde{\Lambda}_{21} = (0.5575,0.6425,0.6425,0.7675,1; 0.6425,0.6425,0.7050,0.7650,0.9)$$

$$\tilde{\Lambda}_{22} = (0.0675,0.1575,0.1575,0.3075,1; 0.1575,0.1575,0.2325,0.7650,0.9)$$

$$\tilde{\Lambda}_{23} = (0.6975,0.7825,0.7825,0.8375,1; 0.7825,0.7825,0.7950,0.7650,0.9)$$

$$\tilde{\Lambda}_{24} = (0.6775,0.7625,0.7625,0.8275,1; 0.7625,0.7625,0.7850,0.7650,0.9)$$

$$\tilde{\Lambda}_{31} = (0.6225,0.7075,0.7075,0.8000,1; 0.7075,0.7075,0.7475,0.7650,0.9)$$

$$\tilde{\Lambda}_{32} = (0.0425,0.0625,0.0625,0.1675,1; 0.0625,0.0625,0.1150,0.7650,0.9)$$

$$\tilde{\Lambda}_{33} = (0.4175,0.5275,0.5275,0.6975,1; 0.5275,0.5275,0.6125,0.7650,0.9)$$

$$\tilde{\Lambda}_{34} = (0.6975,0.7825,0.7825,0.8375,1; 0.7825,0.7825,0.7950,0.7650,0.9)$$

$$\tilde{\Lambda}_{34} = \tilde{\Lambda}_{23}$$

Step 2, In this step, we compute $\mathcal{R} = (r_{ij})_{3\times 4}$ of $\tilde{\mathcal{R}} = (\tilde{\Lambda}_{ij})_{3\times 4}$ given as,

$$\mathcal{R} = \begin{pmatrix} 5.7724 & 2.6770 & 6.3754 & 6.3754 \\ 5.8257 & 3.0628 & 6.5458 & 6.4490 \\ 6.1661 & 2.5312 & 5.1848 & 6.5458 \end{pmatrix}$$

Step 3, We use weight vector $\mathcal{W}t = (0.35,0.2,0.25,0.4)$ and the ith row value r_{ij}, $(i = 1,2,3)$ of $\mathcal{R}$ to compute the RV r_i of the alternative o_i, $(i = 1,2,3)$.

$r_1 = \sum_{j=1}^{4} (\omega_j r_{1j}) = 8.6469, r_2 = \sum_{j=1}^{4} (\omega_j r_{2j}) = 8.8278$, and $r_3 = \sum_{j=1}^{4} (\omega_j r_{3j}) = 8.5234$

Step 4, The rank of the alternatives Alt_i based on ranking values $r_i (i = 1,2,3)$ is $Alt_2 > Alt_1 > Alt_3$. Hence, the most favorite employee for promotion is Alt_2.

The comparison of above discussed methods show that the second approach is more effective, then former approach for IT2 fuzzy GDM problems, but in aggregation process it doesn't give the complete decision information.

9 MCDM based on partial known information about parameters

Suppose the partial information about parameters is given by DMs as follows,

$Dm_1, 0.15 \leq w_1 \leq 0.45$

$Dm_2, w_3 - w_2 \geq w_4 - w_1$

$Dm_3, w_3 - w_1 \geq 0.05$

$w_4 \geq w_1$

Then, $0.05, w_4 \geq w_1$. Now we choose the best employee for the promotion with partial known information about parameter weights to adopt the method i.e.

Step 1, In first step we normalize IT2 DMts $\bar{\mathcal{R}}^k = (\bar{\Lambda}_{ij}^k)_{n\times m}$, where $i = 1,2,\ldots,l$ given by DMs Dm_k, where $(k = 1,2,\ldots,l)$ utilizing IT2 fuzzy WAAO i.e $\tilde{\Lambda}_{ij} = \bigoplus_{k=1}^{l} \lambda_k \tilde{\Lambda}_{ij}^{(k)}$. Its particulars are to sum up all individual $\bar{\mathcal{R}}^k = (\bar{\Lambda}_{ij}^k)_{n\times m}$ $(i = 1,2,\ldots,l)$ into a CNIT2DMt $\bar{\mathcal{R}} = (\bar{\Lambda}_{ij})_{n\times m}$.

Step 2, On the basis of defined rank of IT2FN we find the RVMt $\mathcal{R} = (r_{ij})_{n\times m}$ of the CNIT2DMt $\bar{\mathcal{R}} = (\bar{\Lambda}_{ij})_{n\times m}$.

Step 3, To get best weight vectors $\omega^{(i)} = (w_1^{(i)}, w_2^{(i)}, \ldots, w_m^{(i)})^t$, where $(i = 1,2,\ldots,n)$ corresponding to the alternatives o_i, where $i = 1,2,\ldots,n$ by utilizing model-1 to construct the weight matrix Ω.

Step 4, Construct the matrix $(\mathcal{R}\,\Omega)^t\,(\mathcal{R}\,\Omega)$ and compute the normalized eigenvector

$V_N = (V_{N1}, V_{N2}, V_{N3})^t$ for the matrix $(\mathcal{R}\,\Omega)^t\,(\mathcal{R}\,\Omega)$.

Step 5, Compute the weight vector $\mathcal{W}t = \Omega\, \mathcal{V}_N$.
Step 6, On the basis of $\tilde{\mathcal{R}} = (\tilde{\Lambda}_{ij})_{3\times 4}$, we use the WAO $\tilde{\Lambda}_i = \bigoplus_{j=1}^{m} (\omega_j \tilde{\Lambda}_{ij})$ to get $\tilde{D}m_i = \bigoplus_{j=1}^{m} (\omega_j \tilde{\Lambda}_{ij})$ of the alternative Alt_i, $(i = 1,2,3)$.
Step 7, Compute RV $Rank(\tilde{D}m_i)$ of Dm_i, $1 \le i \le n$. After Step 7 we shall be able to choose the best alternative and shall be able to make a decision.
Related Example Consider the above problem to find the best choice. so we can
Step 1, Similarly, we use the above $\tilde{\mathcal{R}}^k = (\tilde{\Lambda}_{ij}^k)_{n\times m}$ and aggregate all individuals normalized IT2F DMts $\tilde{\mathcal{R}}^k = (\tilde{\Lambda}_{ij}^k)_{3\times 4}$, $k = 1,2,3$ into $\tilde{\mathcal{R}} = (\tilde{\Lambda}_{ij})_{3\times 4}$.i.e

$$\tilde{\mathcal{R}} = \begin{pmatrix} \tilde{\Lambda}_{11} & \tilde{\Lambda}_{12} & \tilde{\Lambda}_{13} & \tilde{\Lambda}_{14} \\ \tilde{\Lambda}_{21} & \tilde{\Lambda}_{22} & \tilde{\Lambda}_{23} & \tilde{\Lambda}_{24} \\ \tilde{\Lambda}_{31} & \tilde{\Lambda}_{32} & \tilde{\Lambda}_{33} & \tilde{\Lambda}_{34} \end{pmatrix}$$

By using the formula $\tilde{\Lambda}_{ij} = \bigoplus_{k=1}^{l} \lambda_k \tilde{\Lambda}_{ij}^{(k)}$, we have

$$\tilde{\Lambda}_{11} = (0.5475,0.6325,0.6325,0.7625,1; 0.6325,0.6325,0.6975,0.7650,0.9)$$

$$\tilde{\Lambda}_{12} = (0.0425,0.0875,0.0875,0.2175,1; 0.0875,0.0875,0.1525,0.7650,0.9)$$

$$\tilde{\Lambda}_{13} = (0.6625,0.7475,0.7475,0.8200,1; 0.7475,0.7475,0.7775,0.7650,0.9)$$

$$\tilde{\Lambda}_{14} = (0.6625,0.7475,0.7475,0.8200,1; 0.7475,0.7475,0.7775,0.7650,0.9)$$

$$\tilde{\Lambda}_{14} = \tilde{\Lambda}_{13}$$

$$\tilde{\Lambda}_{21} = (0.5575,0.6425,0.6425,0.7675,1; 0.6425,0.6425,0.7050,0.7650,0.9)$$

$$\tilde{\Lambda}_{22} = (0.0675,0.1575,0.1575,0.3075,1; 0.1575,0.1575,0.2325,0.7650,0.9)$$

$$\tilde{\Lambda}_{23} = (0.6975,0.7825,0.7825,0.8375,1; 0.7825,0.7825,0.7950,0.7650,0.9)$$

$$\tilde{\Lambda}_{24} = (0.6775,0.7625,0.7625,0.8275,1; 0.7625,0.7625,0.7850,0.7650,0.9)$$

$$\tilde{\Lambda}_{31} = (0.6225,0.7075,0.7075,0.8000,1; 0.7075,0.7075,0.7475,0.7650,0.9)$$

$$\tilde{\Lambda}_{32} = (0.0425,0.0625,0.0625,0.1675,1; 0.0625,0.0625,0.1150,0.7650,0.9)$$

$$\tilde{\Lambda}_{33} = (0.4175,0.5275,0.5275,0.6975,1; 0.5275,0.5275,0.6125,0.7650,0.9)$$

$$\tilde{\Lambda}_{34} = (0.6975,0.7825,0.7825,0.8375,1; 0.7825,0.7825,0.7950,0.7650,0.9)$$

$$\tilde{\Lambda}_{34} = \tilde{\Lambda}_{23}$$

Step 2, To find $R = (r_{ij})_{3\times 4}$ we use the ranking formula define in start of this chapter. Therefore, we get

$$\mathcal{R} = \begin{pmatrix} 5.7724 & 2.6770 & 6.3754 & 6.3754 \\ 5.8257 & 3.0628 & 6.5458 & 6.4490 \\ 6.1661 & 2.5312 & 5.1848 & 6.5458 \end{pmatrix}$$

Step 3, Given that the information about weights of parameter is given by DMs $Dm_i, i = 1,2,3$ as follows, respectively,

$$Dm_1, 0.15 \le w_1 \le 0.45$$

$$Dm_2, w_3 - w_2 \ge w_4 - w_1$$

$$Dm_3, w_3 - w_1 \ge 0.05$$

$$w_4 \ge w_1$$

Then, $H = \{0.15 \le w_1 \le 0.45, w_3 - w_2 \ge w_4 - w_1, w_3 - w_1 \ge 0.05, w_4 \ge w_1\}$
Now by using the model-1 we find the weight vector of each alternative corresponding to each DM. For finding these weight vectors we adopt two techniques of optimization that are Big M-method and Two phase-method. After solving this optimization problem, we get

$$\omega^{(1)} = (\omega_1^{(1)}, \omega_2^{(1)}, \omega_3^{(1)}, \omega_4^{(1)})^t = (0.05,0,0.45,0.5)^t$$

$$\omega^{(2)} = (\omega_1^{(2)}, \omega_2^{(2)}, \omega_3^{(2)}, \omega_4^{(2)})^t = (0.05,0,0.45,0.5)^t$$

$$\omega^{(3)} = (\omega_1^{(3)}, \omega_2^{(3)}, \omega_3^{(3)}, \omega_4^{(3)})^t = (0.225,0,0.275,0.5)^t$$

Moreover, we can verify this result by Tora software.

$$\Omega = \begin{pmatrix} 0.05 & 0.05 & 0.225 \\ 0 & 0 & 0 \\ 0.45 & 0.45 & 0.275 \\ 0.5 & 0.5 & 0.5 \end{pmatrix}$$

Step 4, Compute the matrix $(\mathcal{R}\,\Omega)^t\,(\mathcal{R}\,\Omega)$, i.e,

$$(\mathcal{R}\,\Omega)^t\,(\mathcal{R}\,\Omega) = \begin{pmatrix} 116.9909 & 116.9909 & 116.5229 \\ 116.9909 & 116.9909 & 116.5229 \\ 116.5229 & 116.5229 & 116.1113 \end{pmatrix}$$

find the eigenvalues of the matrix $(\mathcal{R}\,\Omega)^t\,(\mathcal{R}\,\Omega)$ that are $E = (350.057, 0.036, 1.31 \times 10^{-14})$ Maximum eigenvalue of the matrix $(\mathcal{R}\,\Omega)^t\,(\mathcal{R}\,\Omega)$ is 350.057. Therefore, we find the eigenvector V corresponding to 350.057 that is

$$V = (-0.57809, -0.57809, -0.575867)^t$$

Now, we normalize this eigenvector V and we get

$$V_N = \| V \| = \left(\frac{-0.57809}{-1.732047}, \frac{-0.57809}{-1.732047}, \frac{-0.575867}{-1.732047}\right)$$

$$V_N = (0.3338, 0.3338, 0.3325)$$

Step 5, Weight vector

$$\omega = \Omega\, V_N =$$

$$\begin{pmatrix} 0.05 & 0.05 & 0.225 \\ 0 & 0 & 0 \\ 0.45 & 0.45 & 0.275 \\ 0.5 & 0.5 & 0.5 \end{pmatrix}\begin{pmatrix} 0.3338 \\ 0.3338 \\ 0.3325 \end{pmatrix} = (0.108, 0, 0.392, 0.500)$$

$$Dm_1 = \omega_1\widetilde{\Lambda}_{11} + \omega_2\tilde{A}_{12} + \omega_3\widetilde{\Lambda}_{13} + \omega_4\widetilde{\Lambda}_{14}$$

$$Dm_1 = \omega_1\widetilde{\Lambda}_{21} + \omega_2\widetilde{\Lambda}_{22} + \omega_3\widetilde{\Lambda}_{23} + \omega_4\widetilde{\Lambda}_{24}$$

$$Dm_1 = \omega_1\widetilde{\Lambda}_{31} + \omega_2\widetilde{\Lambda}_{32} + \omega_3\widetilde{\Lambda}_{33} + \omega_4\widetilde{\Lambda}_{34}$$

Step 6, Use $\widetilde{D}m_i = \bigoplus_{j=1}^{m} (\omega_j\widetilde{\Lambda}_{ij})$ to get $Dm_i, (i = 1,2,3)$

$$Dm_1 = (0.6501, 0.7351, 0.7351, 0.8139, 1; 0.7351, 0.7351, 0.7689, 0.7651, 0.9)$$

$$Dm_2 = (0.6724, 0.7574, 0.7574, 0.8250, 1; 0.7574, 0.7574, 0.7803, 0.7651, 0.9)$$

$$Dm_3 = (0.5797, 0.6746, 0.7787, 0.7786, 1; 0.6745, 0.6745, 0.7184, 0.7651, 0.9)$$

Step 7, Now we Compute $Rank(\widetilde{D}m_i)$ of Dm_i, $1 \leq i \leq n$. By using the ranking value formula that we already define in start of the chapter, we get

$Rank(Dm_1) = 6.3128, Rank(Dm_2) = 6.4229$, and $Rank(Dm_3) = 5.9779$

Step 8, Here, we can see that $Rank(Dm_2) > Rank(Dm_1) > Rank(Dm_3)$. The preference order of the alternatives Alt_1, Alt_2, Alt_3 is

$Alt_2 > Alt_1 > Alt_3$

Thus, we can say that the best choice is the alternative Alt_2. In this Example we see that the procedure adopt here we determine the weights of parameters from the CIT2FDMt with the partial known information and also in the process of aggregation it avoid losing and distorting the original decision information.

Conclusion

We examine MAGDM problems in IT2 FS, and proposed a method to deal with the situations where values of attributes are considered as IT2 FSs. Information related to weight of attributes is incomplete. In this method first convert all individual "normalized interval type-2 fuzzy decision matrix (NIT2FDMts)" into the "collective interval type-2 fuzzy decision matrix (CIT2FDMt)" by using the "interval type-2 fuzzy weighted arithmetic average operator (IT2 fuzzy WAAO)". In the form where we have information related to weight of attributes is partial known, we construct the RVMt of the "collective normalized interval type-2 fuzzy decision matrix (CNIT2FDMt)", and establish some optimization models to find weight of attributes. Then, we utilize obtained weight of attributes and the IT2 fuzzy WAAO to obtain ranking of the objects/alternatives and then choose the best choice. All this process illustrated by Example concerning with a business organization, searching for the best employee for the promotion on the behalf of desired criteria.

Author Contributions

All the authors have equally contributed towards the formation of this paper.

Conflicts of Interest

The authors declare no conflict of interest.

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Institutional security for special education as conceived in this discourse broadly means protection, safe guard, regulatory policy, directives and care initiated, pursued to develop, improve special needs education and prevent persons with disabilities from