World Wide Journal of Multidisciplinary Research and Development (March-2024)

WWJMRD 2024; 10(03): 87-95
www.wwjmrd.com
International Journal
Peer Reviewed Journal
Refereed Journal
Indexed Journal
Impact Factor SJIF 2017: 5.182 2018: 5.51, (ISI) 2020-2021: 1.361
E-ISSN: 2454-6615

Ismail Marouani
Department of Electronic Engineering, Applied College, University of Ha'il, Saudi Arabia.

Correspondence:
Ismail Marouani
Department of Electronic Engineering, Applied College, University of Ha'il, Saudi Arabia.

Numerical integration techniques applied to Transient Stability Analysis for Multimachine Systems

Ismail Marouani

Abstract
The transient stability of multi-machine power systems is a critical aspect in ensuring the reliable and secure operation of electrical grids. This paper presents an in-depth investigation and analysis of transient stability in multimachine systems, focusing on identifying potential stability issues and proposing effective solutions. The paper begins by introducing the concept of transient stability and its importance in maintaining system resilience during disturbances. It highlights the challenges posed by the increasing complexity and interconnectedness of modern power systems, emphasizing the need for accurate transient stability analysis techniques. Various analytical methods and tools employed for transient stability analysis are reviewed. The advantages and limitations of each approach are discussed, providing insights into their applicability in different system scenarios. The paper also addresses the key factors affecting transient stability, such as generator characteristics, system topology, and control strategies. It explores the impact of various disturbances, such as faults, load variations, and network configuration changes, on system stability by determining the critical clearing time (CCT). Furthermore, it investigates the influence of protective devices, such as circuit breakers and relays, on transient stability and proposes strategies to enhance their coordination. To improve transient stability, the paper proposes Numerical integration techniques. The effectiveness of these techniques in enhancing system stability is evaluated through The six-bus power system network of an electric utility company that used as test system.

Keywords: Numerical integration techniques, Transient Stability, multimachine systems, disturbances, critical clearing time (CCT).

1. Introduction

The reliable and secure operation of electrical power systems is of paramount importance for maintaining the stability and quality of power supply to consumers. Transient stability, which refers to the ability of a power system to maintain synchronism and recover from disturbances, plays a crucial role in ensuring the resilience of the grid. With the increasing complexity and interconnectedness of modern power systems, the analysis and assessment of transient stability in multi-machine systems have become vital for power system engineers and operators [1-3].
Transient stability analysis involves studying the dynamic behavior of power system components, such as generators, transmission lines, and loads, during transient events such as faults, sudden changes in load, or network configuration alterations. The objective is to assess the system's ability to withstand these disturbances and return to a stable operating condition within an acceptable time frame.
The analysis of transient stability in multi-machine systems is more challenging compared to single-machine systems due to the complex interaction and interdependence of multiple generators, interconnected transmission lines, and control devices. The dynamic response of each machine and the interaction among them during transient events can significantly influence the stability of the entire system. Therefore, a thorough understanding of these interactions and their impact on system stability is crucial for effective analysis and control.
Transient stability analysis techniques have evolved over the years, driven by advancements in computational capabilities and the need to tackle the increasing complexity of power systems. Various analytical methods, numerical simulations, and time-domain simulations

have been developed to assess transient stability. These techniques consider factors such as generator characteristics, system topology, control strategies, and protective devices to evaluate stability margins and identify critical issues [4,5].
The importance of transient stability analysis extends beyond ensuring the reliable operation of power systems. It also helps in system planning, design, and the development of control strategies to maintain stability under various operating conditions and contingencies. Additionally, transient stability analysis plays a crucial role in the integration of renewable energy sources, grid modernization efforts, and the enhancement of system resilience against emerging challenges, such as the increasing penetration of electric vehicles.
This paper aims to provide a comprehensive overview of transient stability analysis for multi-machine systems. It will explore the various analytical methods, simulation techniques, and control strategies employed in transient stability analysis. The paper will also highlight the challenges posed by system complexity and identify potential solutions to enhance transient stability. By consolidating existing knowledge and addressing current research gaps, this paper aims to contribute to the development of more robust and secure electrical grids capable of withstanding transient disturbances.
Transient stability analysis for multi-machine systems involves several key steps and mathematical formulations to assess the system's ability to withstand transient disturbances and maintain stability. The following formulation highlights the essential concepts and vocabulary associated with transient stability analysis [6,7]:

- Solve the Initial Load Flow: The first step in transient stability analysis is to compute the steady-state operating conditions of the power system by solving the load flow equations. This involves finding the voltage magnitudes, phase angles, and power flows in the network under normal operating conditions.
- Kron Reduction Formula: To simplify the analysis of multi-machine systems, the Kron reduction formula is often employed. This formula allows the reduction of the detailed network representation into an equivalent system with aggregated generator and load buses. The reduced system retains the essential dynamic characteristics while reducing computational complexity.
- Numerical Integration Techniques: Transient stability analysis involves solving the swing equation, which describes the dynamic behavior of synchronous generators during transient events. Numerical integration techniques, such as the Euler method, Runge-Kutta methods, or the trapezoidal rule, are applied to solve the swing equation numerically. These techniques approximate the system's state variables (rotor angles, speeds, and electrical variables) at successive time intervals.
- Swing Equation: The swing equation represents the dynamic response of synchronous generators and is a key component of transient stability analysis. It is derived from the mechanical and electrical equations of the generator. The swing equation describes the acceleration of the rotor angle with respect to time and incorporates factors such as generator inertia, damping, electrical power input, and mechanical torque.
- System Equations and State Variables: Transient stability analysis involves formulating a set of differential equations that describe the dynamic behavior of the system. These equations typically include the swing equation for each generator, differential equations for other system components (such as excitation systems and voltage regulators), and algebraic equations representing the network equations (Kirchhoff's laws). The state variables of the system include rotor angles, rotor speeds, voltages, and currents.
- Stability Assessment and Critical Clearing Time: Once the system equations are formulated and solved using numerical integration techniques, stability assessment is performed. The critical clearing time (CCT) is a key metric used to evaluate transient stability. It represents the time duration after a disturbance until the system reaches its stability limit. The CCT provides an indication of the system's ability to withstand disturbances without losing stability.

In transient stability analysis for multi-machine systems, solving the initial load flow, applying the Kron reduction formula, utilizing numerical integration techniques to solve the swing equation, and assessing stability through the critical clearing time are crucial steps in understanding and evaluating the system's transient stability characteristics.

2. Numerical integration techniques of transient stability

Numerical integration techniques, such as the Euler method and Runge-Kutta methods, offer several advantages in the context of transient stability analysis and solving differential equations [8-10]:

- Flexibility: Numerical integration techniques provide flexibility in handling complex systems with nonlinearities and time-varying parameters. They can handle a wide range of system configurations and dynamic behaviors, making them suitable for transient stability analysis in multi-machine systems.
- Accuracy: While numerical integration techniques introduce approximations, they can provide accurate solutions when appropriately applied. Higher-order methods, such as the Runge-Kutta methods, offer improved accuracy by using multiple stages and evaluating derivatives at intermediate points. This accuracy is crucial for capturing the fine details of transient behavior and accurately assessing stability.
- Efficiency: Numerical integration techniques offer computational efficiency compared to analytical and symbolic methods, especially for systems with a large number of equations and variables. They allow for efficient time-stepping by updating the system's state variables at discrete time intervals, enabling the simulation of system dynamics over extended time periods.
- Adaptability: Numerical integration techniques can adapt to different time steps based on the system's behavior. They allow for variable time stepping, where smaller time steps are used during critical transients or when the system dynamics change rapidly, and larger time steps are used during stable periods. This adaptability balances accuracy and computational efficiency.
- Robustness: Numerical integration techniques are

robust and can handle a wide range of system conditions, including abrupt changes, discontinuities, and nonlinearities. They can handle events such as faults, load variations, and control actions with stability and accuracy, making them suitable for transient stability analysis in realistic power system scenarios.
- Implementation Simplicity: Numerical integration techniques are relatively straightforward to implement compared to analytical methods. Once the system equations are formulated, numerical integration methods offer practical and accessible approaches for solving the equations and obtaining numerical solutions.
- Widely Used and Tested: Numerical integration techniques have been extensively used and tested in various scientific and engineering applications, including power system analysis. They have well-established theoretical foundations and are supported by a wealth of existing numerical methods and algorithms, making them reliable and trusted tools for transient stability analysis.

While numerical integration techniques have advantages, it is important to select an appropriate method based on the specific system characteristics, accuracy requirements, and computational resources available. The choice between methods like the Euler method, Runge-Kutta methods, or more advanced techniques depends on the desired tradeoff between accuracy and computational efficiency in a given transient stability analysis scenario.

The Euler method and Runge-Kutta methods are numerical integration techniques that can handle nonlinearities and time-varying parameters in differential equations, including those encountered in transient stability analysis. An explanation of how each method handles these factors as follow:

Euler Method:

The Euler method is a simple and straightforward numerical integration technique that approximates the solution of a differential equation by stepping forward in time with a fixed time step. Here's how it handles nonlinearities and time-varying parameters:
- Nonlinearities: In the Euler method, the nonlinearities in the system equations are considered at each time step. At each time increment, the derivatives of the state variables are evaluated based on the current values of the state variables. These derivatives capture the effect of nonlinear elements, such as power system components or control devices, in the system equations. The method then uses these derivatives to update the state variables for the next time step.
- Time-Varying Parameters: The Euler method can handle time-varying parameters by updating the values of these parameters at each time step. If the system equations involve time-varying parameters, such as varying loads or control signals, the method considers the current values of these parameters during the evaluation of the derivatives. This ensures that the time-varying nature of these parameters is properly accounted for in the integration process.

While the Euler method is relatively simple to implement, it has limitations in terms of accuracy, especially for systems with rapid changes or highly nonlinear behaviors. To address these limitations, more advanced methods like the Runge-Kutta methods are often employed.

Runge-Kutta Methods:

Runge-Kutta methods are a family of numerical integration techniques that provide higher accuracy by evaluating derivatives at multiple stages within each time step. These methods handle nonlinearities and time-varying parameters as follows:
- Nonlinearities: Runge-Kutta methods employ a series of stages to approximate the derivatives of the state variables within each time step. These stages involve evaluating the derivatives at intermediate points using a weighted combination of derivative evaluations. By considering multiple stages, Runge-Kutta methods capture the effect of nonlinearities more accurately than the Euler method.
- Time-Varying Parameters: Similar to the Euler method, Runge-Kutta methods can handle time-varying parameters by updating their values at each time step. The derivatives at each stage within the method's iterations are evaluated based on the current values of the state variables and the time-varying parameters. This ensures that the impact of time-varying parameters on the system dynamics is properly accounted for during the integration process.

Runge-Kutta methods, particularly higher-order variants like the fourth-order Runge-Kutta method (RK4), provide improved accuracy compared to the Euler method. They achieve this by employing multiple derivative evaluations and using weighted combinations of these evaluations to update the state variables. The increased accuracy of Runge-Kutta methods makes them more suitable for handling complex systems with nonlinearities and time-varying parameters encountered in transient stability analysis.

Apart from the Euler method and Runge-Kutta methods, there are other numerical integration techniques commonly used in transient stability analysis. Some of these techniques include:
- Trapezoidal Rule: The Trapezoidal Rule is a widely used numerical integration method that provides better accuracy compared to the Euler method. It approximates the solution by taking the average of the derivatives at the current and next time steps. The Trapezoidal Rule balances the contributions from the current and future states, resulting in improved accuracy and stability.
- Backward Euler Method: The Backward Euler method is an implicit numerical integration technique that approximates the solution by using the derivative evaluated at the next time step. It provides unconditional stability and can handle stiff systems, where the time constants of the system components vary significantly. The Backward Euler method is particularly useful when dealing with systems with fast dynamics or rapidly changing parameters.
- Gear's Method: Gear's method is a family of implicit numerical integration techniques that provide higher accuracy and stability compared to the Euler method. These methods use a combination of backward and forward differences to approximate the derivatives. Gear's methods are particularly effective for stiff systems and can handle systems with varying time steps.
- Adams-Bashforth-Moulton Methods: Adams-

Bashforth-Moulton methods are explicit numerical integration techniques that use a combination of backward and forward differences to approximate the derivatives. These methods estimate the derivative at the next time step using previous derivative values. Adams-Bashforth methods are useful for solving systems with moderate stiffness and are often combined with other techniques for higher accuracy.

- Predictor-Corrector Methods: Predictor-Corrector methods combine the predictions made by an explicit method with the corrections obtained from an implicit method. These methods utilize both forward and backward differences to estimate the derivatives and update the state variables. The Adams-Moulton method combined with the Adams-Bashforth method is a common predictor-corrector approach used in transient stability analysis.

The choice of numerical integration technique depends on several factors, including the system's characteristics, desired accuracy, computational efficiency, and the presence of stiff or rapidly changing dynamics. Different methods have their strengths and limitations, and selecting an appropriate method involves considering the specific requirements and constraints of the transient stability analysis scenario [11,12].

Table 1: Advantages and disadvantages of some commonly used numerical integration techniques in transient stability analysis.

Techniques	Advantages	Disadvantages
Euler Method	Simplicity: The Euler method is straightforward to implement and understand	- Accuracy: The Euler method has relatively low accuracy, especially for systems with rapid changes or highly nonlinear behaviors.
	Computational Efficiency: It requires fewer computations per time step compared to more advanced methods.	- Stability: It may exhibit stability issues, particularly for stiff systems or when the time step is not sufficiently small.
Runge-Kutta Methods	- Accuracy: Runge-Kutta methods, especially higher-order variants like RK4, offer improved accuracy compared to the Euler method.	- Computational Complexity: Runge-Kutta methods require more computations per time step than the Euler method, leading to increased computational overhead.
	- Flexibility: They can handle a wide range of nonlinearities and time-varying parameters in the system equations.	- Stiff Systems: While Runge-Kutta methods can handle a range of systems, they may struggle with stiff systems characterized by widely varying time constants.
Trapezoidal Rule	- Accuracy: The Trapezoidal Rule provides better accuracy than the Euler method and is relatively simple to implement.	- Computational Complexity: The Trapezoidal Rule involves more computations per time step compared to the Euler method, increasing computational requirements.
	- Stability: It is unconditionally stable, making it suitable for a wide range of systems.	
Backward Euler Method:	- Stability: The Backward Euler method is unconditionally stable and can handle stiff systems effectively.	- Computational Complexity: It requires solving nonlinear equations at each time step, making it computationally more demanding than explicit methods.
	- Accuracy: It offers improved accuracy compared to the Euler method.	
Gear's Method:	- Stability and Accuracy: Gear's methods provide high stability and accuracy, especially for stiff systems.	- Computational Complexity: They involve more computations and additional equation solving per time step, resulting in increased computational overhead.
Adams-Bashforth-Moulton Methods:	- Accuracy: Adams-Bashforth-Moulton methods offer higher accuracy than the Euler method and are suitable for systems with moderate stiffness.	- Stability: These methods can be conditionally stable and may require smaller time steps to maintain stability.
	- Efficiency: They are explicit methods and require fewer computations per time step compared to implicit methods.	
Predictor-Corrector	Accuracy: Predictor-Corrector methods combine the benefits of explicit and implicit methods, offering improved accuracy compared to explicit methods alone.	- Computational Complexity: Predictor-Corrector methods involve additional computations and equation solving compared to explicit methods, resulting in increased computational requirements.
	- Stability: They can handle a range of system dynamics, including moderately stiff systems.	

An explanation of the advantages and disadvantages of some commonly used numerical integration techniques in transient stability analysis presented in table 1. It's important to note that the advantages and disadvantages listed above are generalizations, and the performance of each numerical integration technique can vary depending on the specific system characteristics, accuracy requirements, and computational resources available. The choice of method should be made considering the specific needs and constraints of the transient stability analysis scenario.

3. Problem formulation

The problem formulation for transient stability analysis in multi-machine systems involves modeling the dynamic behavior of the system following disturbances and formulating the necessary equations to analyze its transient stability. A general problem formulation using the swing equation, which is a fundamental equation in transient stability analysis is as follow [13-15]:

3.1. System Modeling

Consider a multimachine power system consisting of N synchronous machines connected through transmission lines and interconnected by a network of buses. The system is subjected to disturbances, such as faults, load changes, or generator tripping, which cause transient deviations in the machine rotor angles and speeds.

3.2. Network Equations

In addition to the swing equation, the problem formulation involves incorporating the network equations that describe the power flow and electrical behavior of the system. These equations include nodal power balance equations, bus voltage equations, and line flow equations, among others.

The electrical power output can be given by:

$$S_{ei}^{*} = E_i'^{*}.I_i \quad (1)$$

or

$$P_{ei} = \Re[E_i'^{*}.I_i] \quad (2)$$

Where:

$$I_i = \sum_{j=1}^{m} E_j'.Y_{ij} \quad (3)$$

$$E_i' = V_i + jX_d'.I_i \quad (4)$$

$$I_{bus} = Y_{bus}.V_{bus} \quad (5)$$

Using voltages and admittances in polar form:

$$P_{ei} = \sum_{j=1}^{m} |E_i'|\,|E_j'|\,|Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \quad (6)$$

Prior to disturbance:

$$P_{ei} = P_{mi} = \sum_{j=1}^{m} |E_i'|\,|E_j'|\,|Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \quad (7)$$

P_{mi}: Mechanical power input to machine i.

P_{ei}: Electrical power output of machine i.

3.3. Swing Equation

The swing equation is a key equation used to describe the dynamic response of synchronous machines during transient events. It relates the rate of change of rotor angle (δ) to the mechanical power input (P_m), electrical power output (P_e), inertia constant (H), and damping coefficient (D).

For each machine i in the system, the swing equation can be formulated as:

$$\frac{d^2\delta_i}{dt^2} + D_i.\frac{d\delta_i}{dt} + ws_i^2.(P_{mi} - P_{ei}) = 0 \quad (8)$$

where:

δ_i: Rotor angle of machine i with respect to a reference.

D_i: Damping coefficient of machine i.

ωsi: Synchronous speed of machine i.

P_{mi}: Mechanical power input to machine i.

P_{ei}: Electrical power output of machine i.

The swing equation with damping neglected for machine i:

$$\frac{H_i d^2 S_i}{\pi f_0 dt^2} = P_{mi} - \sum_{j=1}^{m} |E_i'|\,|E_j'|\,|Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \quad (9)$$

Where:

$$H_i = \frac{S_{Gi}}{S_B}.H_{Gi} \quad (10)$$

H_i: inertia constant of machine i on the MVA base S_B.

H_{Gi}: inertia constant of machine i on the MVA base S_{Gi}.

$$\frac{dS_i}{dt} = \Delta w_i \ ; i = 1, \dots, m \quad (11)$$

$$\frac{d\Delta w_i}{dt} = \frac{\pi f_0}{H_i}(P_m - P_e^f) \quad (12)$$

Where:

P_e^f: postfault power.

3.4. Initial and Boundary Conditions

The problem formulation requires specifying the initial conditions, which include the initial rotor angles, speeds, and voltages of the machines. Additionally, boundary conditions, such as fault clearing times or control actions, need to be defined to capture specific transient events.

3.5. Solution Method

To solve the transient stability problem, numerical integration techniques are employed to approximate the solution of the differential equations. The choice of the numerical integration method depends on factors such as accuracy requirements, system characteristics, stability considerations, and computational efficiency.

By formulating the necessary equations, incorporating network equations, defining initial and boundary conditions, and selecting an appropriate solution method, the problem formulation for transient stability analysis in multi-machine systems provides a foundation for conducting simulations, analyzing system response to disturbances, and assessing the transient stability limits of the power system.

4. Simulation results

To study the transient stability of the 3-machine test system mentioned in reference [16], two cases are considered. In both cases, a fault occurs and is subsequently cleared, and the transient stability is analyzed. Here are the formulations for the two cases:

- A 3-phase to ground fault occurs on the line [5, 6] near bus 6 in the 3-machine test system. The fault is cleared by opening the circuit breakers at both ends of the line between buses 5 and 6. The objective is to study the transient stability, specifically focusing on the critical clearing time (CCT). The CCT represents the time it takes to clear the fault such that the system remains stable.
- A 3-phase to ground fault occurs on the line [4, 6] near bus 6 in the 3-machine test system. The fault is cleared by opening the circuit breakers at both ends of the line between buses 4 and 6. Similar to Case 1, the transient stability analysis is performed, and the focus is on determining the critical clearing time (CCT).

In both cases, the 3-machine test system consists of three synchronous machines, denoted as Machine 1, Machine 2, and Machine 3. Machine 1 is considered the reference unit due to its relatively large MW capacity compared to the other generating machines. The system includes transmission lines with their respective constants, loads represented on a 100 MVA base, and machine constants.

The structure of the studied power system model is depicted in Figure 1 in the reference [16]. By performing transient stability analysis for these two cases, the aim is to assess the system's ability to maintain stable operation after the fault is cleared, and specifically to determine the critical clearing time (CCT) for each case. The CCT represents a crucial parameter that helps evaluate the stability limits of the system and design appropriate protection schemes.

By simulating the system dynamics and analyzing the responses of machines and buses during and after fault clearance, the transient stability analysis provides insights into the system's behavior, stability margins, and potential stability issues. These findings assist in making informed decisions for system planning, protection scheme design, and control strategies to ensure the reliable and secure operation of multimachine power systems.

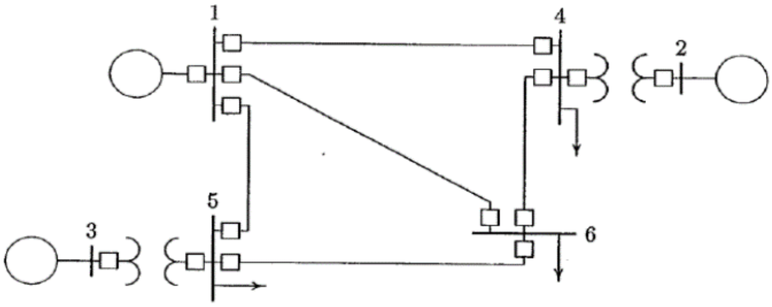

Fig 1: 3-machine test system.

Case 1. Fault occurs on the line [5, 6] near bus 6

From figure 2, After a fault occurs in the power system, the phase angle differences between machines are observed. In this case, the phase angle differences δ21 and δ31 reach maximum values of δ21 = 123.9 degrees and δ31 = 62.95 degrees, respectively. However, it is observed that the phase angle differences start to decrease, and the machines' swings synchronize. This behavior indicates stability in the system. The fault is cleared within 0.4 seconds, and the system is found to be stable during the fault clearing time.

In the figure 3, the swing curves, specifically for machine 2, are analyzed. It is observed that the phase angle of machine 2 increases without limit, indicating instability in the system. The fault is cleared within 0.5 seconds, and the system is found to be unstable during the fault clearing time.

To further analyze the system's stability, the simulation is repeated with a fault clearing time of 0.45 seconds. It is determined that the system is critically stable during this clearing time.

Table 2 presents the reduced Y matrix values for different system conditions. The matrix values are provided for the prefault, during fault, and post-fault states. These values represent the electrical characteristics of the power system, including the network impedance and admittance, which are essential in transient stability analysis.

Table 2. Reduced Y matrix.

Type of network	Node	1	2	3
Prefault	**1**	0.3517 - 2.8875i	0.2542 + 1.1491i	0.1925 + 0.9856i
	2	0.2542 + 1.1491i	0.5435 - 2.8639i	0.1847 + 0.6904i
	3	0.1925 + 0.9856i	0.1847 + 0.6904i	0.2617 - 2.2835i
Faulted	**1**	0.1913 - 3.5849i	0.0605 + 0.3644i	0.0523 + 0.4821i
	2	0.0605 + 0.3644i	0.3105 - 3.7467i	0.0173 + 0.1243i
	3	0.0523 + 0.4821i	0.0173 + 0.1243i	0.1427 - 2.6463i
Fault cleared	**1**	0.3392 - 2.8879i	0.2622 + 1.1127i	0.1637 + 1.0251i
	2	0.2622 + 1.1127i	0.6020 - 2.7813i	0.1267 + 0.5401i
	3	0.1637 + 1.0251i	0.1267 + 0.5401i	0.2859 - 2.0544i

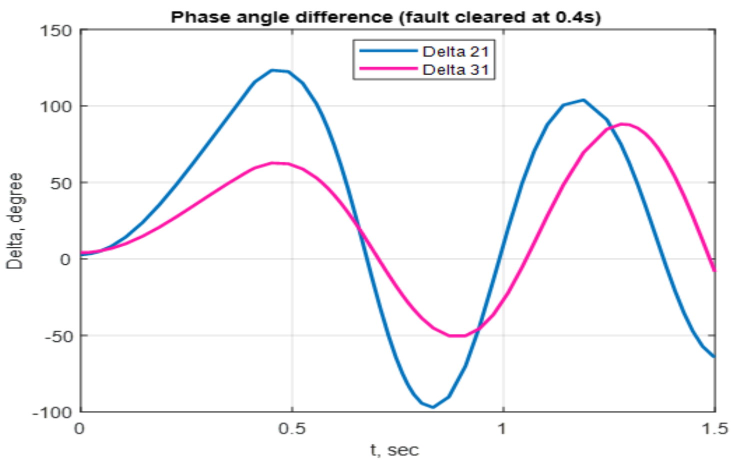

Fig 2: Plots of angle difference for machine 2 and 3, fault cleared at 0.4s.

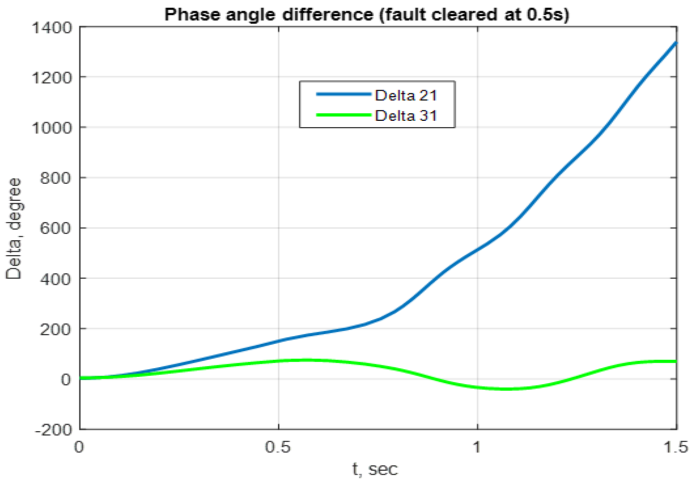

Fig 3: Plots of angle difference for machine 2 and 3, fault cleared at 0.5s.

By evaluating the behavior of phase angle differences, machine swings, and the reduced Y matrix values, the transient stability analysis provides insights into the stability status of the power system during and after the fault. This information aids in understanding the system's response to disturbances, identifying critical clearing times, and designing appropriate protection and control measures to ensure stable and secure operation of the power system.

Case 2. Fault occurs on the line [4, 6] near bus 6

After a fault occurs in the power system, the phase angle differences between machines, namely δ21 and δ31, are monitored. In the figure 3, the phase angle differences reach maximum values of δ21 = 111.8343 degrees and δ31 = 57.8619 degrees, respectively. However, it is observed that the phase angle differences start to decrease, and the machines' swings synchronize. This behavior indicates stability in the system. The fault is cleared within 0.4 seconds, and the system is found to be stable during the fault clearing time.

Figure 5 depicts the swing curves, specifically for machine 2. It is observed that the phase angle of machine 2 increases without limit, which indicates instability in the system. The fault is cleared within 0.5 seconds, and the system is found to be unstable during the fault clearing time.

To further analyze the system's stability, the simulation is repeated with a fault clearing time of 0.45 seconds. It is determined that the system is critically stable during this clearing time.

Table 3 presents the reduced Y matrix values for various system conditions, including the prefault, during fault, and post-fault states. These matrix values represent the electrical characteristics of the power system, such as impedance and admittance, which are crucial in transient stability analysis. The table provides necessary information to understand the system's behavior and evaluate the impact of fault conditions on the system's stability.

Table 3. Reduced Y matrix.

Type of network	Node	1	2	3
Prefault	1	0.3517 - 2.8875i	0.2542 + 1.1491i	0.1925 + 0.9856i
	2	0.2542 + 1.1491i	0.5435 - 2.8639i	0.1847 + 0.6904i
	3	0.1925 + 0.9856i	0.1847 + 0.6904i	0.2617 - 2.2835i
Faulted	1	0.1913 - 3.5849i	0.0605 + 0.3644i	0.0523 + 0.4821i
	2	0.0605 + 0.3644i	0.3105 - 3.7467i	0.0173 + 0.1243i
	3	0.0523 + 0.4821i	0.0173 + 0.1243i	0.1427 - 2.6463i
Fault cleared	1	0.3951 - 2.8461i	0.1811 + 0.9679i	0.2417 + 1.0391i
	2	0.1811 + 0.9679i	0.5179 - 2.2378i	0.1103 + 0.4668i
	3	0.2417 + 1.0391i	0.1103 + 0.4668i	0.3171 - 2.2153i

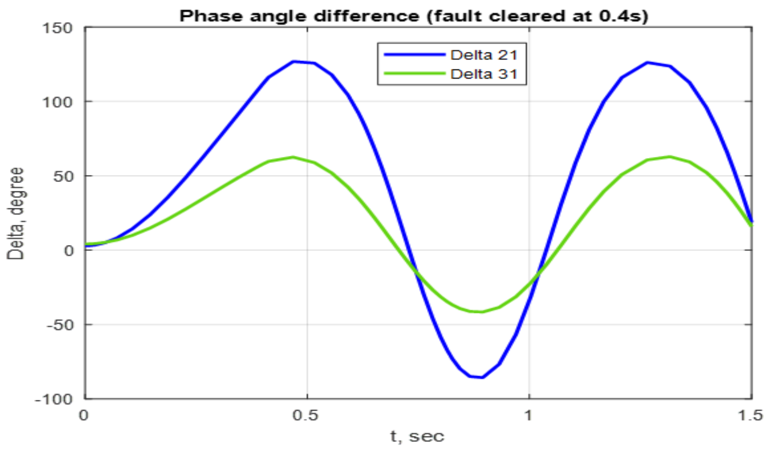

Fig 4. Plots of angle difference for machine 2 and 3, fault cleared at 0.4s

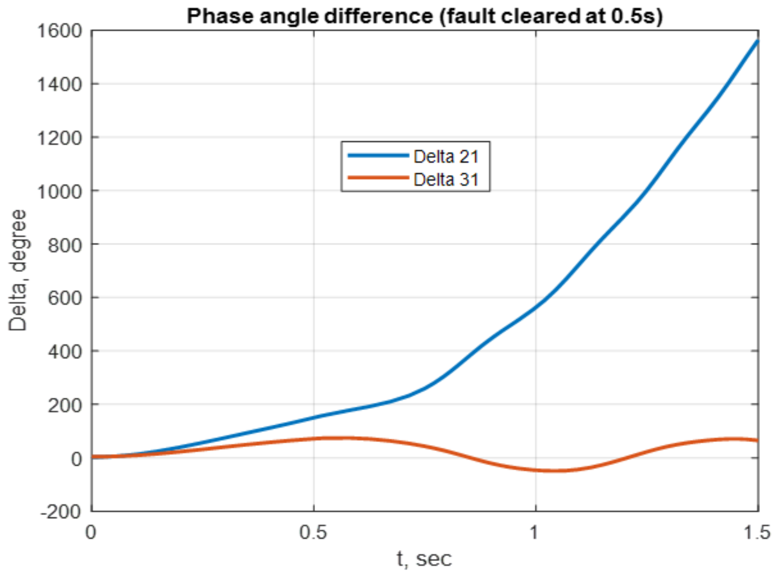

Fig 5. Plots of angle difference for machine 2 and 3, fault cleared at 0.5s

By examining the phase angle differences, machine swings, and the reduced Y matrix values, the transient stability analysis allows for an assessment of the power system's stability during and after the fault event. This knowledge assists in identifying critical clearing times, designing appropriate protection schemes, and making informed decisions to ensure the secure and reliable operation of the power system.

5. Conclusion

Transient stability analysis for multi-machine systems plays a crucial role in ensuring the reliable and secure operation of power systems. This analytical work involves studying the dynamic response of the system following disturbances to assess its ability to maintain stable operation in the presence of transient events. Numerical integration techniques, such as the Euler method, Runge-Kutta methods, Trapezoidal Rule, Backward Euler method, Gear's method, Adams-Bashforth-Moulton methods, and Predictor-Corrector methods, are commonly employed to approximate the solution of the differential equations that govern the system behavior during transient events. Each method has its advantages and disadvantages in terms of accuracy, stability, computational efficiency, and ease of implementation. The choice of numerical integration technique depends on factors such as the accuracy requirements, system characteristics, stability considerations, computational resources, implementation complexity, adaptability to time-varying parameters, and availability of tools and validation studies. By performing transient stability analysis, power system engineers and researchers can gain insights into the system's response to disturbances, identify critical stability limits, design effective control strategies, and enhance the overall system reliability. This work aids in understanding the transient behavior of multi-machine systems, assessing their stability margins, and making informed decisions to mitigate potential stability issues. Furthermore, transient stability analysis contributes to the development of advanced protection and control schemes, enabling utilities to maintain reliable and secure power supply in the face of various disturbances, including faults, contingencies, and large-scale system events. Continued research and development in transient stability analysis techniques, including the exploration of new numerical integration methods, the integration of advanced machine learning and optimization techniques, and the incorporation of real-time data and control strategies, will further enhance the accuracy, efficiency, and applicability of transient stability analysis for multi-machine systems. Ultimately, this work supports the reliable operation and planning of power systems, ensuring the delivery of electricity to consumers while maintaining system stability, security, and resilience.

References

1. Eskander. GHOLIPOUR SHAHRAK« Apport de l'UPFC à l'amélioration de la stabilité transitoire des réseaux électriques », *Thèse de doctorat*, à l'université Henry Poincaré, NANCY-I,13 OCTOBRE 2003.
2. E. Gholipour and S. Saadat, « optimal operation of a genertor connected to a network Using FACTS Devices » ELECTRIMACS, Montréal, Canada, 2002.
3. A. Nabavi-Niaki and M.R. Iravani, "Steady-State and Dynamic Model of Unified Power Flow Controller (UPFC) for Power System Studies", IEEE Trans. On Power System, vol.11,N°.4, pp.1937-1943, November 1996
4. A. Nabavi-Niaki and M.R. Iravani, "Steady-State a-+nd Dynamic Model of Unified Power Flow Controller (UPFC) for Power System Studies", IEEE Trans. On Power System, vol.11,N°.4, pp.1937-1943, November 1996
5. M. Pavella, et al, "Transient Stability of Power Systems: a Unified Approach to Assessment and Control", Kluwer, 2000.
6. Amr G. Bahbah and Adly A. Girgis, New Method for Generators' Angles and Angular Velocities Prediction for Transient Stability Assessment of Multimachine Power Systems Using Recurrent Artificial Neural Network, IEEE Transactions On Power Systems, Vol. 19, No. 2, PP. 1015-1022, May 2004.
7. Y. Kato and S Iwamoto, Transient Stability Preventive Control for Stable Operating Condition With Desired CCT, IEEE Transaction On Power Systems, Vol. 17, No. 4, PP. 1154-1161,

November 2002.
8. T. Geusmi, stabilisation et dispatching économique/ environnemental des réseaux d'énergie électrique, Thèse, National School of Engineers of Sfax, Tunisia, 2007.
9. Sarajcev, P.; Kunac, A.; Petrovic, G.; Despalatovic, M. Power System Transient Stability Assessment Using Stacked Autoencoder and Voting Ensemble. Energies 2021, 14, 3148.
10. Perilla, A.; Papadakis, S.; Rueda Torres, J.L.; van der Meijden, M.; Palensky, P.; Gonzalez-Longatt, F. Transient Stability Performance of Power Systems with High Share of Wind Generators Equipped with Power-Angle Modulation Controllers or Fast Local Voltage Controllers. Energies 2020, 13, 4205.
11. Tina, G.M.; Maione, G.; Licciardello, S. Evaluation of Technical Solutions to Improve Transient Stability in Power Systems with Wind Power Generation. Energies 2022, 15, 7055.
12. Kundur, P.; Paserba, J.; Ajjarapu, V.; Andersson, G.; Bose, A.; Canizares, C.; Vittal, V. IEEE/CIGRE Joint Task Force on Stability Terms and Definitions, Definition and Classification of Power System Stability. IEEE Trans. Power Syst. 2004, 19, 1387–1401
13. Liu, Y.; Sun, K.; Yao, R.; Wang, B. Power System Time Domain Simulation Using a Differential Transformation Method. IEEE Trans. Power Syst. 2019, 34, 3739–3748.
14. Liu, Y.; Sun, K. Power System Simulation Using a Differential Transformation Method. Ph.D. Thesis, University of Tennessee, Knoxville, TN, USA, 2022. Available online: https://trace.tennessee.edu/utk_graddiss/7073 (accessed on 28 August 2022).
15. Sanchez-Gasca, J.J.; D'aquila, R.; Price, W.W.; Paserba, J.J. Variable time step, implicit integration for extended-term power system dynamic simulation. In Proceedings of the Power Industry Computer Applications Conference, Salt Lake City, UT, USA, 7–12 May 1995.
16. Hadi saadat, Power System Analysis.Milwaukee School Engineering.TK1011.S23.1999.