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On The Ternary Cubic Diophantine Equation $2(x^2+y^2)-3xy=56z^3$

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Abstract
The sequences of integral solutions to the cubic equation with four variables $2(x^2+y^2)-3xy=56z^3$ are obtained. A few properties among the solutions are also presented.

Keywords: Non-Homogeneous Cubic, Cubic with three unknowns, integral solutions

Introduction

The Diophantine equation offers an unlimited field for research due to their variety [1-4]. In particular, one may refer [5-13] for cubic equation with three unknowns. In [14-15] cubic equations with four unknowns are studied for its non-trivial integral solutions. This communication concerns with the problem of obtaining infinitely many non-zero distinct integral solutions of ternary cubic equation given by $2(x^2+y^2)-3xy=56z^3$. A few interesting properties among the solutions and special numbers are presented.

Method of analysis

The diophantine equation to be solved for its non-zero distinct integral solutions is given by

$$2(x^2+y^2)-3xy=56z^3 \quad (1)$$

Introducing the transformations

$$x=u+v, y=u-v\ ,\ u\neq v\neq 0 \quad (2)$$

in (1), it leads to

$$u^2+7v^2=56z^3 \quad (3)$$

(3) can be solved through different methods and we obtain different sets of integer solutions to (1).

Set 1

Assume $z=a^2+7b^2$ (4)

where a and b are non-zero distinct integers.

Write 56 as $56=(7+i\sqrt{7})(7-i\sqrt{7})$ (5)

Substituting (4) & (5) in (3) and applying the method of factorization, define

$$u+i\sqrt{7}v=(7+i\sqrt{7})(a+i\sqrt{7}b)^3$$

Equating the real and imaginary parts, we have

$$u=7a^3+49b^3-21a^2b-147ab^2$$

$$v=a^3-49b^3+21a^2b-21ab^2 \quad (6)$$

From (2), the integer solutions of (1), are

$$x(a,b) = 8a^3 - 168ab^2$$
$$y(a,b) = 6a^3 + 98b^3 - 42a^2b - 126ab^2$$
$$z(a,b) = a^2 + 7b^2$$

Properties

- $x(a,1) + y(a,1) - 14(2P_a^5 - 7G_a - 4t_{4,a}) \equiv 0 \pmod{98}$
- $x(a,1) - 8CP_{6,a} \equiv 0 \pmod{168}$
- $18[y(a,a) - x(a,a)]$ is a cubical integer.
- $x(a,a) + z(a,a) - 8CP_{6,a} - t_{4,a} \equiv 7 \pmod{168}$
- $26[z(a,a) + y(a,a) - x(a,a)] - 2704\,CP_{6,a} = 0.$

Set 2

Rewrite 56 as $56 = \frac{(7 + i5\sqrt{7})(7 - i5\sqrt{7})}{4}$ (7)

Substituting (4) & (7) in (3) and applying the same procedure as mentioned in the above set, we have

$$u + i\sqrt{7}v = \frac{(7 + i5\sqrt{7})}{2}(a + i\sqrt{7}b)^3$$

Equating the real and imaginary parts, we have

$$u = \frac{1}{2}(7a^3 + 245b^3 - 105a^2b - 147ab^2)$$
$$v = \frac{1}{2}(5a^3 - 49b^3 + 21a^2b - 105ab^2)$$

As our aim is to find integer solutions, choose $a = 2A, b = 2B$ in the above equations we get then the corresponding non-zero integer solutions of (1) are given by

$$u = 28A^3 + 980B^3 - 420A^2B - 588AB^2$$
$$v = 20A^3 - 196B^3 + 84A^2B - 420AB^2 \quad (8)$$

From (2), the integer solutions of (1), are

$$x(A,B) = 48A^3 + 784B^3 - 336A^2B - 1008AB^2$$
$$y(A,B) = 8A^3 + 1176B^3 - 504A^2B - 168AB^2$$
$$z(A,B) = 4(A^2 + 7B^2)$$

Note

We can write 56 in an another way as

$$56 = \frac{(119 + i5\sqrt{7})(119 - i5\sqrt{7})}{256}$$

As proceeding in the above sets, we get the corresponding non-zero distinct integer solutions of (1) given by

$$x(A,B) = 31744A^3 - 150528B^3 + 64512A^2B - 666624AB^2$$
$$y(A,B) = 29184A^3 + 275968B^3 - 118272A^2B - 612864AB^2$$
$$z(A,B) = 256(A^2 + 7B^2)$$

Set 3

One may write (3) as

$$u^2 + 7v^2 = 56z^3 * 1 \quad (9)$$

Write 1 as

$$1 = \frac{(3 + i\sqrt{7})(3 - i\sqrt{7})}{16} \quad (10)$$

Using (4), (5) and (10) in (9) and applying the method of factorization, define

$$(u + i\sqrt{7}v) = \frac{1}{4}\left[(3 + i\sqrt{7})(7 + i\sqrt{7})(a + i\sqrt{7}b)^3\right] \quad (11)$$

Equating real and imaginary parts of (9), we have

$$u = \frac{1}{4}(14a^3 + 490b^3 - 210a^2b - 294ab^2)$$
$$v = \frac{1}{4}(10a^3 - 98b^3 + 42a^2b - 210ab^2)$$

As our aim is to find integer solutions, choosing $a = 4A, b = 4B$ in the above equations we obtain

$$u = 224A^3 + 7840B^3 - 3360A^2B - 4704AB^2$$
$$v = 160A^3 - 1568B^3 + 672A^2B - 3360AB^2 \quad (12)$$

In view of (2), the integer solutions to (1) are given by

$$x(A,B) = 384A^3 + 6272B^3 - 2688A^2B - 8064AB^2$$
$$y(A,B) = 64A^3 + 9408B^3 - 4032A^2B - 1344AB^2$$
$$z(A,B) = 16(A^2 + 7B^2)$$

Properties

- $x(1,B) - 6y(1,B) + 50176\,CP_{6,B} \equiv 0 \pmod{21504}$
- $3x(A,1) - 2y(A,1) - 6144\,P_{A-1}^3 \equiv 0 \pmod{20480}$
- $z(A,A) - x(A,A) - 128\,t_{4,A}$ is a cubical integer.
- $y(A,A) - x(A,A)$ is the product of a cubical integer and a perfect square.

Conclusion

In this paper, we have presented sets of infinitely many non-zero distinct integer solutions to the ternary cubic equation, given by $2(x^2 + y^2) - 3xy = 56z^3$. As Diophantine equations are rich in variety due to their definition, one may attempt to find integer solutions to higher degree diophantine equation with multiple variable along with suitable properties...

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