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Arotaritei Dragos
University of Medicine and Pharmacy „ Grigore T.Popa", Department of Medical Biosciences, Iasi, Romania.

Ilea Mihail-Ovidiu
University of Medicine and Pharmacy „ Grigore T.Popa", Department of Medical Biosciences, Iasi, Romania.

Correspondence:
Ilea Mihail-Ovidiu
University of Medicine and Pharmacy „ Grigore T.Popa", Department of Medical Biosciences, Iasi, Romania.

Solving linear programming problems using the simplex method

Arotaritei Dragos, Ilea Mihail-Ovidiu

Abstract
A linear programming problem is said to be in standard form when all constraints become equations and all variables are nonnegative. An inequality becomes an equality by introducing a new variable, called a compensation variable. A linear programming problem is in tabular form when all constraints are equations.
The simplex method is an iterative procedure for solving linear programming problems in tabular form. The simplex method generates new basic feasible solutions that increase the value of the objective function (or at least leave it unchanged) by generating new tabular forms for the system of equations. When no further improvement can be made, the optimal solution has been reached.

Keywords: Simplex Method, Linear Programming, Optimal Solution.

1. Introduction
The use of matrices in various fields of study, especially in economics, is very necessary. The simplex method has several advantages over other matrix methods, as it can be used to solve linear optimization problems whose constraints are inequalities while Crammer's rule and Jacoby method are used in solving systems of linear equations. Linear programming is a method or mathematical approach for determining and obtaining the best results such as maximum profit in several economic activities such as production projects, based on a certain optimal criterion(s) [1,2]. The word linear is used to describe a proportional relationship between two or more variables in modeling, and the word programming means the planning of economic activities to obtain alternative solutions to problems that use limited resources by adopting a particular strategy among different strategies to achieve the desired goal [3] .

2. Materials and methods
The simplex method consists of three steps:

1. Find the largest positive value for $z_j - c_j$. This will designate the pivot column. If there is no such value, then the optimal solution has already been found.
2. For each positive value in the pivot column, find the ratio: (right-hand side)/(corresponding element in the pivot column).
- The minimum ratio establishes the pivot line.
- At the intersection of the pivot column and the pivot line, the pivot element is found.

3. Generate the new tabular form as follows:
- Divide the pivot line by the pivot element;
- For all other lines, multiply the new line generated in point (a). By the corresponding element in the pivot column and extract it from the current line.
- Fill in the cells of the table and go to step one.

3. Results & Discussion

A problem of determining economic activity can be given by the form:

$$\min\begin{cases} f(x_1,x_2,x_3,x_4)=3x_1-2x_2+x_3-x_4 \\ \begin{cases} x_1-x_2+2x_3+x_4\le 6 \\ x_1+x_2-x_3-2x_4=4 \\ 2x_1+3x_3+x_4\le 8 \end{cases} \\ x_i\ge 0, i=\overline{1,4} \end{cases}$$

- We introduce the objective function: $f(x)=3x_1-2x_2+x_3-x_4$
- We introduce the compensation variables: $\begin{cases} x_1-x_2+2x_3+x_4+x_5=6 \\ x_1+x_2-x_3-2x_4=4 \\ 2x_1+3x_3+x_4+x_6=8 \end{cases}$
- The linear programming problem becomes:

$$\min\begin{cases} f(x_1,x_2,x_3,x_4)=3x_1-2x_2+x_3-x_4 \\ \begin{cases} x_1-x_2+2x_3+x_4+x_5=6 \\ x_1+x_2-x_3-2x_4=4 \\ 2x_1+3x_3+x_4+x_6=8 \end{cases} \\ x_i\ge 0, i=\overline{1,4} \end{cases}$$

- We will write the expanded matrix of the system:

$$A=\begin{pmatrix} 1 & -1 & 2 & 1 & 1 & 0 & 6 \\ 1 & 1 & -1 & -2 & 0 & 0 & 4 \\ 2 & 0 & 3 & 1 & 0 & 1 & 8 \end{pmatrix}$$

- From the problem statement, the variables x_2,x_5,x_6 must be main.
- It is observed that the variables x_5,x_6 are already main.
- We will multiply the second line by (1) and add it to line one like this:

$$\begin{pmatrix} 1 & -1 & 2 & 1 & 1 & 0 & 6 \\ 1 & 1 & -1 & -2 & 0 & 0 & 4 \\ 2 & 0 & 3 & 1 & 0 & 1 & 8 \end{pmatrix} \approx \begin{pmatrix} 2 & 0 & 1 & -1 & 1 & 0 & 10 \\ 1 & 1 & -1 & -2 & 0 & 0 & 4 \\ 2 & 0 & 3 & 1 & 0 & 1 & 8 \end{pmatrix}$$

- The basic admissible solution (we set the secondary variables to zero : $x_1=x_3=x_4=0$).
- It will turn out that : $x_2=4, x_5=10, x_6=8 \rightarrow \overline{x_0}=(0,4,0,0,10,8)$
- From the Simplex table it can be seen that its P_3 components are : $(1,-1,3)$
- $$\begin{cases} z_0=0\cdot 10+(-2)\cdot 4+0\cdot 8=-8, z_1=0\cdot 2+(-2)\cdot 1+0\cdot 2=--2 \\ z_2=0\cdot 0+(-2)\cdot 1+0\cdot 0=-2, z_3=0\cdot 1+(-2)\cdot(-1)+0\cdot 3=2 \\ z_4=0\cdot(-1)+(-2)\cdot(-2)+0\cdot 1=4, z_5=0\cdot 1+(-2)\cdot 0+0\cdot 0=0 \\ z_6=0\cdot 0+(-2)\cdot 0+0\cdot 1=0 \end{cases}$$

- From the table we obtain: $\begin{cases} z_1 - c_1 = -5, z_2 - c_2 = 0, z_3 - c_3 = 1 \\ z_4 - c_4 = 5, z_5 - c_5 = 0, z_6 - c_6 = 0 \end{cases}$
- We apply the entry criterion: we will consider the maximum of all positive numbers of the form ($z_j - c_j$).
- From the table it is observed that the maximum is 5, it will result that the column of element 5 will enter the base, therefore P_4.
- We apply the exit criterion.
- We will calculate: $\min(\frac{P_0}{P_4}) = \min(\frac{10}{-1}, \frac{4}{-1}, \frac{8}{1}) = 8$.
- It will result that the line of element 4 will exit the base, so it will exit the base.
- From the table it is observed that the pivot of the basic change is 1.
- Only positive numbers are considered in the exit criterion!
- From the Simplex table it can be seen that its P_1 components are: $(4,5,2)$ From the Simplex table we obtain:

$$\begin{cases} z_1 - c_1 = -15, z_2 - c_2 = 0, z_3 - c_3 = -14 \\ z_4 - c_4 = 0, z_5 - c_5 = 0, z_6 - c_6 = -5 \end{cases}$$

- From the table it can be seen that the optimality criterion is satisfied.
- The problem admits a unique optimal solution if : $z_j - c_j < 0, (\forall j = \overline{1,6})$
- From the table it can be seen that $z_j - c_j = 0$ there is , so we can say that the solution is not unique.
- From the Simplex table: $x_{optima} = (0,20,0,8)$.
- From the Simplex table: $f(x) = -48 < 0$.

Table 1: Simplex Table.

			3	-2	1	-1	0	0
B	C_B	P_0	P_1	P_2	P_3	P_4	P_5	P_6
x_5	0	10	2	0	1	-1	1	0
x_6	-2	4	1	1	-1	-2	0	0
x_4	0	8	2	0	3	**1**	0	1
z_j		-8	-2	-2	2	4	0	0
$z_j - c_j$			-5	0	1	5	0	0
x_5	0	18	4	0	4	0	1	1
x_3	-2	20	5	1	5	0	0	2
x_4	-1	8	2	0	3	1	0	1
z_j		-48	-12	-2	-13	-1	0	-5
$z_j - c_j$			-15	0	-14	0	0	-5

4. Conclusions

The scope of application of mathematical tools is very vast in microeconomics. Linear programming plays a crucial role in microeconomics. The use of mathematics in economics ensures accuracy. Therefore, in this article, we have analyzed the key role of the Simplex algorithm in microeconomics. The simplex method has several advantages over other matrix methods, as it can be used to solve linear optimization problems whose constraints are inequalities while Crammer's rule and the Jacobians method are used in solving systems of linear equations. We can say that linear programming is a method or mathematical approach for determining and obtaining the best results such as: maximum profit in several economic activities, production projects, based on a certain optimal criterion (optimal criteria).

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