
WWJMRD 2018; 4(2): 75-79
www.wwjmrd.com
*International Journal*
*Peer Reviewed Journal*
*Refereed Journal*
*Indexed Journal*
*UGC Approved Journal*
*Impact Factor MJIF: 4.25*
e-ISSN: 2454-6615

**Akhmedov M.Sh.**
Department «Higher mathematics», Bukhara Technological Institute of Engineering, Bukhara, Republic of Uzbekistan

**Ruziyev T.R.**
Department «Physics» Bukhara State Medical Institute, Bukhara, Department of Informatics and Biophysics

**Umarov A.O.**
Department «Higher mathematics», Bukhara Technological Institute of Engineering, Bukhara, Republic of Uzbekistan

**Atoyev A.B.**
Department «Higher mathematics», Bukhara Technological Institute of Engineering, Bukhara, Republic of Uzbekistan

**Correspondence**:
**Akhmedov M.Sh.**
Department «Higher mathematics», Bukhara Technological Institute of Engineering, Bukhara, Republic of Uzbekistan

# Spatial Oscillations Varied Viscoelastic Pipeline under AC Varied Internal Pressure

**Akhmedov M.Sh., Ruziyev T.R., Umarov A.O., Atoyev A.B.**

**Abstract**
In work on the basis of the bar theory, the flexural - tensional vibration of a viscous elastic curved pipeline is considered by the action of the internal with a moving ideal fluid. The problem of oscillations of a viscous elastic rod is solved.

**Keywords:** liquid, pipeline, Fourier method, relaxation nuclei.

## Introduction

Curved pipelines are often used in engineering and construction [1,2,3]. For example, the supply of fuel from the fuel tanks of an aircraft in the combustion chamber of engines is carried out by means of pipelines. Therefore, the problem of studying the oscillation of pipelines with a flowing liquid is an actual problem and of practical interest.

## Basic relations and statement of the problem

We consider the spatial oscillation of a curved pipeline and the incompressible fluid contained in it in the relative axis Oz (Fig. 1) passing through the supports. It is assumed that the pipeline is under the influence of a variable internal pressure. The velocity of the fluid is neglected. The length of the pipeline is-$l$, the thickness of its wall is -h, and the total mass of a homogeneous pipeline and liquid $m = m_1 + m_2$. In this formulation of the problem, we will neglect longitudinal forces of inertia in comparison with transverse forces. Pipeline element $dz$ and mass $dm = (m/l)dz$. Transverse distributed load $q_n$ on the pipeline is expressed by the formula

$$q_n = -\frac{m}{l}\left(\frac{\partial^2 w}{\partial t^2} - g\cos\theta\right) + p_i F_i \frac{\partial^2 w}{\partial z^2} \quad (1)$$

where $F_i = \pi R_i^2$, $p_i = p_0 + p_v \sin(\Omega t + \varphi)$, $w$- deflection of a pipeline element, $\Omega, \varphi, p_0, p_v$ - values of the circular frequency, the initial phase, the static and the amplitude of the dynamic components of the variable internal pressure $p_i$ in the pipeline, $R_i, F_i$ internal radius and cross-sectional area of the pipeline, $t$ -time.

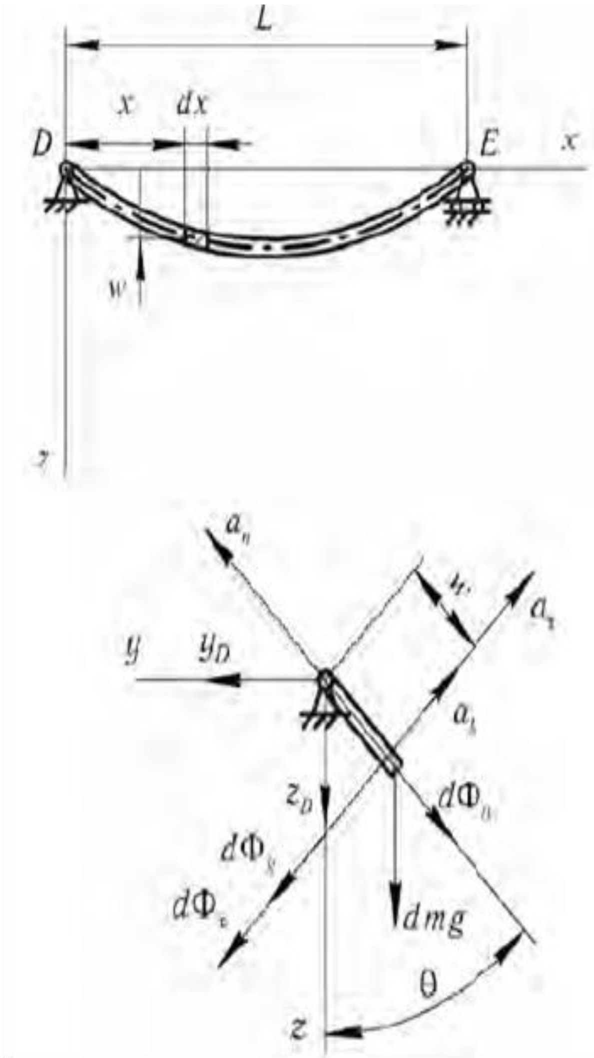



**Fig 1:** Calculation scheme

The magnitude of the buoyancy force $dF_A$ Archimedes acting on a pipe element in length $dz$, equal to $dF_A = \rho_c \pi R_k^2 g dz,\ R_k = R_i + h,$ where $\rho_c$ - fluid density, $g = 9.8 m/c^2$, force $d\vec{F}_c$ resistance of movement of a pipe element is determined by the Stokes formula [4

$$d\vec{F}_c = -\mu \vec{V}_a dz,$$

Where $\vec{V}_a$ - absolute element speed, $\mu$ - coefficients of resistance, depending on the viscosity of the liquid and the shape of the inner surface of the pipe. According to the theorem on addition of velocities $\vec{V}_a = \vec{V}_1 + \vec{V}_2$, where $\vec{V}_1$ and $\vec{V}_2$ - relative and portable velocity of the pipe element. In addition, the latter is given by

$$V_1 = \frac{\partial w}{\partial t},\ V_2 = \frac{\partial \theta}{\partial t}.$$

In this way, $d\vec{F}_c$ resistance can be represented in the form

$$dF_{c1} = \mu \frac{\partial w}{\partial t} dz,\ dF_{c2} = \mu w \frac{\partial \theta}{\partial t} dz,$$

where $\theta$ - angle of rotation of the tube as a solid with respect to Oz. Total moment $M_z$ forces of restoration (or visco-elasticity) in the supports is directly proportional to the angle $\theta$ turning the tube like a rigid body about the axis Oz:

$$M_z = c_0 \left[ \theta(t) - \int_0^t R_c(t-\tau)\theta(t) d\tau \right],$$

Where $R_c(t-\tau)$– relaxation core; $c_0$– instantaneous modulus of elasticity. The tangent $a_\tau$ to the trajectory, normal $a_n$ and Coriolis $a_k$ acceleration of the selected pipeline element are equal

$$a_\tau = w \frac{d^2\theta}{dt^2},\ a_n = w \left(\frac{d\theta}{dt}\right)^2, a_k = 2 \frac{d\theta}{dt} \frac{\partial w}{\partial t}.$$

Thus, the forces of inertia $dF_\tau, dF_n, dF_k$ of the selected pipeline element will be recorded

$$dF_\tau = dm \cdot w \cdot \frac{d^2\theta}{dt^2}, dF_n = dm \cdot w \cdot \left(\frac{d\theta}{dt}\right)^2, dF_\tau = 2dm \cdot \frac{d\theta}{dt} \frac{dw}{dt}.$$

Equation of pipeline equilibrium in the form of a sum of moments of all applied forces and inertia forces relative to the axis Oz

$$-\int_{(m)} (dmg - dF_A) w \sin\theta - \int_{(m)} w dF_\tau - \int_{(m)} w dF_k - M_z = 0, \quad (2)$$

Where $g$ - gravitational acceleration. Equation (2) after some transformations and taking into account

$$T = \frac{\tilde{E}F}{2l} \int_0^l \left(\frac{\partial w}{\partial z}\right)^2 dz,$$

It takes the form

$$\frac{\partial^2 w}{\partial t^2} + \frac{\tilde{E}Jl}{m} \frac{\partial^4 w}{\partial z^4} - \frac{T_i}{m} \frac{\partial^2 w}{\partial z^2} - g_1 \cos\theta - w\left(\frac{d\theta}{dt}\right)^2 = 0, \quad (3)$$

Where $T_i = T - p_i F_i, J = \pi R_i^3 h$ - axial moment of inertia of the cross-sectional area of the pipeline, $g_1 = g - F_k / m$. Bending movements of the pipeline satisfying the boundary conditions

$$w(0,t) = 0;\ \frac{\partial^2 w(0,t) = 0}{\partial z^2}; w(l,t) = 0;\ \frac{\partial^2 w(l,t) = 0}{\partial z^2}, \quad (4)$$

we take in the form

$$w = W_0 \sin \frac{\pi z}{l} + \sum_{k=1}^{\infty} w_k(t) \sin \frac{k\pi z}{l}, \quad (5)$$

Where $W_0$ and $w_k(t)$ -amplitude of static and dynamic components of bending movements. Substituting solution (5) into equations (3) and (4) and applying to the Bubnov-Galerkin procedure [5], after simple transformations we obtain (*κ=0*)

$$\frac{d^2\theta}{dt^2}(W_0+w_0(t))^2+\frac{2\mu}{m}\frac{d\theta}{dt}+2.0(W_0+w_0(t))\left(\frac{2g_1}{\pi}\sin\theta+\frac{d\theta}{dt}\frac{dw_0}{dt}\right)=0;$$
$$\frac{d^2w_0}{dt^2}+\frac{\mu l}{m}\frac{dw_0}{dt}+\frac{(J\pi^4)\tilde{E}}{l^3m}(W_0+w_0(t))=\frac{4g_1}{\pi}\cos\theta+(W_0+w_0(t))\left(\frac{d\theta}{dt}\right)^2- \quad (6)$$
$$-\frac{\pi^2}{ml}\left[\frac{\tilde{E}F_i\pi^2}{4l^2}(W_0+w_0(t))^2-F_i(p_0+p_v\sin(\Omega t))\right](W_0+w_0(t))^2.$$

The system of equations (6) is solved under the following initial conditions

$$t=0:\theta=\theta_0,\ \dot{\theta}=d\theta/dt=\omega_0;\quad w_0=0,\quad dw/dt=0. \quad (7)$$

Here $\theta_0$ , $\omega_0$ - the initial angle of rotation and the angular velocity of the pipeline deviation from the vertical plane. When $\theta(t)=0, w_0(t)=0, p_v=0$, then we obtain the following nonlinear integral equation for determining the quasistatic component of the deflection of the pipeline $W_0$

$$B_1W_0^3+(B_2'+B_2'')W_0-B_2'\int_0^t R_E(t-\tau)W_0(\tau)d\tau-B_1\int_0^t R_E(t-\tau)W_0^3(\tau)d\tau-B_3=0,$$

where

$$B_1=\frac{\pi^4E_0F_i}{4L^2},B_2'=\frac{\pi^4E_0J}{L^2},\ B_2''=\pi^2F_ip_0,\ B_3=\frac{4gmL}{\pi}.$$

If $R_E(t-\tau)=0$, then the results of calculations are obtained [6]. If $\theta=const$ , тогда (6) принимает следующий вид

$$\frac{d^2\overline{w}_0}{dt^2}+A\overline{w}_0(t)+B\varepsilon(\overline{w}_0(t))^2-A\varepsilon\int_0^t R(t-\tau)w(\tau)d\tau-B\varepsilon\int_0^t R(t-\tau)(w(\tau))^2d\tau=f(t), \quad (8)$$

where

$$\overline{w}_0(t)=W_0+w(t),A=\frac{(J\pi^4)E_0}{l^3m},\ B=\frac{E_0F_i\pi^4}{4l^3m},\quad f(t)=\frac{4g}{\pi}\cos\theta+\frac{\pi^2}{ml}\left[F_i(p_0+p_v\sin(\Omega t))\right].$$

system of integro-differential equations (8) is solved by the perturbation method. A general form, the system of integro-differential equations (2.16) in an elastic formulation $R(t-\tau)=0$ ) is given in the works [7,8].

**Consider the free oscillations of the pipeline**

For this purpose, it is assumed $p_v=0,\Omega=0,\varphi=0$. Linearization of the system of differential equations (6), then we obtain the following system of equation

where

$$C_1=\frac{2c_0}{W_0^2m}\ C_2=\frac{4g_1}{\pi W_0},d_1=\frac{\pi^2}{ml}\left(\frac{\pi E_0J}{l^2}+\frac{3\pi^2E_0F_i}{4l^2}W_0^2\right)\ d_2=F_i\ p_0\frac{\pi^2}{ml}.$$

This formula corresponds to the results obtained in [7]. When $R(t-\tau)=0$, then the frequencies $\omega_1$ and $\omega_2$ natural oscillations of the pipeline will be determined by formulas

$$\omega_1=\frac{2}{W_0^2}\left(\frac{c_0}{m}+\frac{2g_1}{\pi}W_0\right),\ \omega_2=\frac{\pi^2}{m_0l^2}\left(\pi^2\frac{E_0J}{l^2}+\frac{3\pi^2E_0F_i}{4l^2}W_0^2-F_ip_0\right) \quad (9)$$

When the viscoelastic properties of pipelines are taken into account, then (9) is expressed by means of the transcendental equation for the angular and bending oscillations of the pipelines

$$\omega^2-\frac{4}{W_0^2}\left\{\frac{c_0}{m}\left[1-\Gamma_{c_0}^c(\omega_R)-i\Gamma_{c_0}^s(\omega_R)\right]+\frac{2g_1}{\pi}W_0\right\}^2=0$$

$$\omega^2 - \frac{\pi^4}{m^2 l^2}\left\{ \left( \frac{\pi^2 E_0 J}{l^2} + \frac{3\pi^2 E_0 F_i}{4l^2} \right) \left[ 1 - \Gamma_E^c(\omega_R) - i\Gamma_E^s(\omega_R) \right] - F_i p_0 \right\}^2 = 0,$$

Where $\omega = \omega_R + i\omega_I$ - complex frequency,

$$\Gamma_{c_0}^c(\omega_R) = \int_0^\infty R_{c_0}(\tau)\cos\omega_R\tau\, d\tau,$$

$$\Gamma_E^c(\omega_R) = \int_0^\infty R_{c_0}(\tau)\cos\omega_R\tau\, d\tau,$$

$$\Gamma_{c_0}^s(\omega_R) = \int_0^\infty R_{c_0}(\tau)\sin\omega_R\tau\, d\tau$$

$$\Gamma_E^s(\omega_R) = \int_0^\infty R_E(\tau)\sin\omega_R\tau\, d\tau,$$

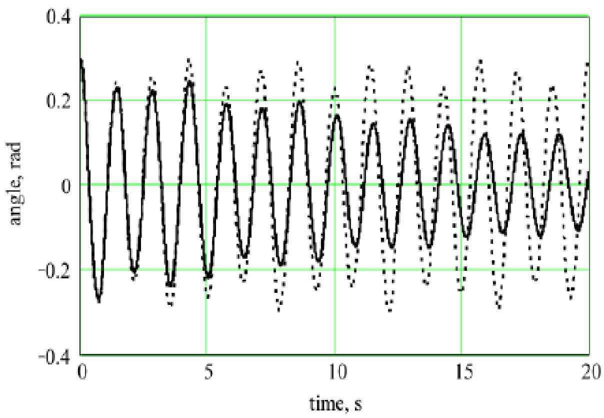
Respectively, the cosine and sine Fourier images of the relaxation core of the material. Let us investigate the influence of the buoyancy force of Archimedes, the forces of inertia of Carioles, the resistance force and the magnitude of the static component of the internal pressure in the liquid, and also the geometric and physic-mechanical parameters of the tube on its free vibrational motion. Numerical results. The numerical solution of problem (6) was determined by the Runge-Kutta method. The results of calculations for the following values basic parameters:

$l = 3m,\ c = 0,\ R_i = 0.29m,\ h = 0.006,\ \theta_0 = 0.3rad,$

$E = 2.1x10^{11}\,Па, \omega_0 = 0\,рад/сек,\ m = 6.142\,x10^3\,кг$

Fig. 2.-3 shows plots of the angle $\theta$ of rotation and the dynamic deflection $w_0(t)$ the middle point of the span of the pipe from the time t, respectively. The solid lines in the graphs show the results of the calculation, taking into account the resistance forces, and the dashed lines - without taking these forces into account. Calculations were carried out for two variants of the coefficients $\mu$ and density $\rho_0$: 1. $\mu = 25\,Pas$, $\rho_0 = 800kg/m^3$ .2. $\mu = 0.025\,Pas$, $\rho_0 = 1.25kg/m^3$. Figures 1-2 illustrate the results of calculations for two mentioned above options.

Taking into account the above, it is possible to draw the following conclusions.



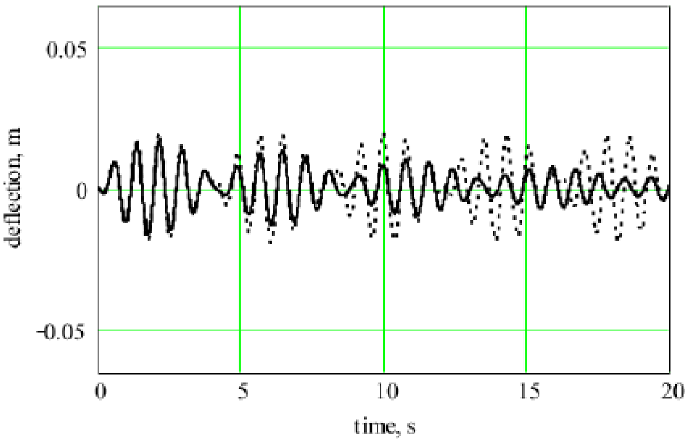
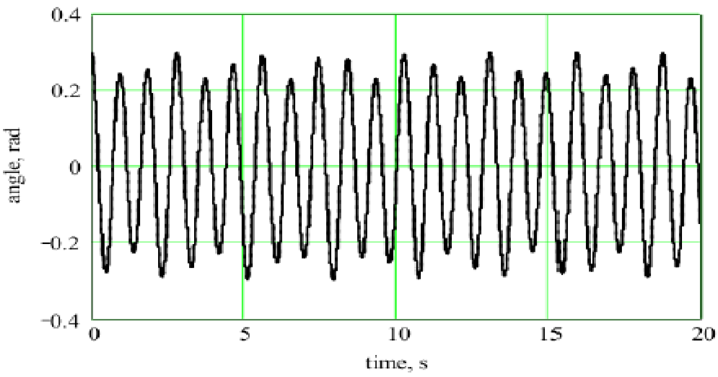




**Fig.2:** Dependencies of the angle of rotation $\theta$ and deflection $w_0$ middle point of the span of the pipe from time $t$ at $p_0 = 50bar, \mu = 25Pas, \rho_0 = 800kg/m^3$



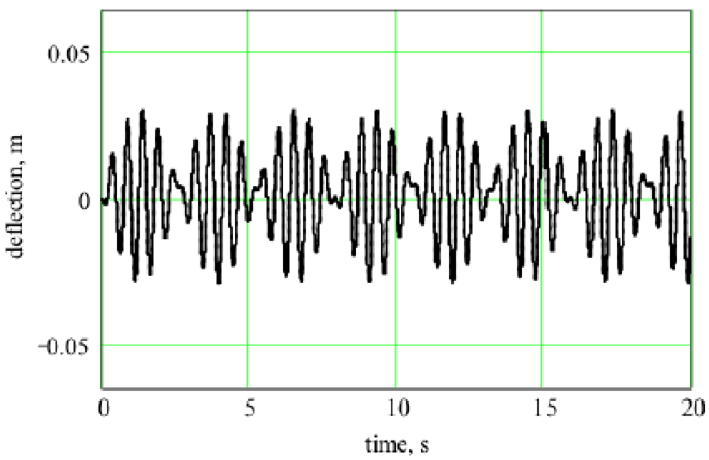




**Fig. 3:** Dependence of the angle of rotation $\theta$ and deflection $w_0$ middle point of the span of the pipe from time $t$ at $p_0 = 50bar, \mu = 0.025Pas, \rho_0 = 1.25kg/m^3$.

**Conclusions**

Based on the developed approximate mathematical model of the flexural-rotational vibrational movements of the pipeline, its free oscillations were investigated. It is established that with increasing static component of internal pressure, an increase in the amplitude of free bending vibrations and an increase in the frequency of free rotational vibrations of the tube simultaneously occur. It is shown that for relatively large values of the resistance forces, free bending, rotational vibrations of the tube decay with time.

**References**


1. Safarov I.I., Teshayev M.Kh. Vibrations of curvilinear tubes interacting with the medium. Analysis, concepts, methods. LAP, Lambert Academic Publishing (Germany). 2012. 245p.
2. Safarov I.I., Teshayev M.Kh., Boltaev Z.I. Wave processes in a mechanical waveguide. Fundamentals, concepts, methods. LAP, Lambert Academic Publishing (Germany). 2012, 275p.
3. Safarov I.I., Teshaev M.Kh., Kilichev O. Dynamic stressed states of thin-walled pipelines. Lambert Academic Publishing (Germany). 2016. 230p.
4. 4. Bozorov M.B., Safarov I.I., Shokin Yu.I.. Numerical simulation of oscillations of dissipatively homogeneous and inhomogeneous mechanical systems. - Novosibirsk, 1996. -187p.
5. Safarov I.I., Teshayev M. H., Madjidov M. Natural Oscillations of Viscoelastic Lamellar Mechanical Systems with Point Communications. Applied Mathematics, 2014, 5, PP. 3018-3025
6. Safarov I.I., Qilichov O. Sh.Mathematical Modeling of Static Stress-Strain State of Parallel Tubes Located In an Elastic Environment. Case Studies Journal 2015, 4, 3,, PP.40-51.
7. Forsberg K. Influence of boundary conditions on the modal characteristics of thin cylindrical shells // AIAA J. -1964. –Vol. 2. №2. - P. 2150- 2157.
8. Gilat Rivka Aboudi Jacob the Lyapunov exponents as a quantities criterion for the dynamic buckling of composite plates//Int. J. Solids and Struct.-2002.–39. №2. - P. 467-481.