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The control of human psycho-affective stability using differential equations

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Abstract
Love brings people together. Today we are so lucky that we no longer need to form a couple in order to survive or have children. Society no longer conditions us, but allows us to enjoy in individuality many things that in the past were only possible through and as a couple. Now, the main motivation of a relationship is the love between the partners. But is this enough for the partners to endure together? What actually keeps two people together in the long run? Relationships in which partners survive beyond the end of the passionate period turn out to be shaped by an extremely important and far more long-lasting resource than passion: compatibility. Among all the factors that contribute to marital happiness, the degree of compatibility, or matching, is the most important criterion, as shown by studies conducted on relationships that pass the test of time. We used the mathematical notion of stability and the MATLAB program to model the stability of the most famous lovers: Romeo and Juliet.

Keywords: Love, Ordinary Differential systems equations, Mathematical models, MATLAB.

1. Introduction

Fairy tales and movies (Hollywood, but not only) idealize the couple relationship and reinforce the idea of "soul mate" and "destiny". The romantic idea of love, "that love" that makes you lose your mind, is the ultimate desire of many of us. Because the dream is extremely attractive: songs, poems, paintings were composed out of love. Over time, numerous authors have tried to define and explain the phenomenon of falling in love from different perspectives, offering descriptions and, sometimes, personal examples. Thus, some associate falling in love with a "divine madness" similar to mystical experience, while others characterize it as a "self-abandonment", a strong temptation to let yourself is absorbed by the other. Hendrix [1] believes that people do not fall in love by chance and that, in this process, the unconscious plays an essential role. Thus, for a while, lovers cling to the illusion of romantic love, which implies different roles played at an unconscious level. The author claims that we choose our partners for two important reasons:

- A first reason would be because they have the positive but also the negative traits of our parents or those who took care of us when we were children.
- The second reason would be that the partners we choose have the role of compensating for our own positive traits that were eliminated in the process of childhood socialization.

What is a functional couple? What makes the two partners stay together for a long time? What are the ingredients of a stable, lasting relationship? These are just some of the questions that most of the people who have had the experience of being in a relationship or who have dreamed at least once of being with someone ask themselves. An essential role in lasting and healthy couple relations is the effective communication with the partner. This requires a good self-knowledge and knowledge of the partner and a permanent adaptation to the situation. Thus, for the partners to understand each other, it is especially necessary for them to listen to each other. The ability to listen is the ability to focus on your partner's messages, to decipher and understand the way he thinks and feels, to see the world through his eyes, detaching yourself from your own interests, needs, opinions and prejudices. Thus, clear communication, expressing emotions and needs and the ability to listen are key elements in a lasting relationship. But there is a big difference between the story and the reality. Just as the Facebook pictures of a couple rarely reflect the actual state of affairs between the two, the reality is that you run into all kinds of problems, refusals, failure to meet expectations and you don't understand what is happening, where the problem comes from.

2. Materials and methods

We will present a very simple model of romantic relationships [2], which assumes that the person's feelings are affected only by the other person's feelings. We will note through $x(t)$ Romeo's feelings, through $y(t)$ Juliet's feelings. These feelings will be measured on a scale of -1 to 1, with negative values representing a level of anxiety and positive values that will represent a level of affection. To design an appropriate system to describe how the feelings of the two lovers change over time, proposed the following system of differential equations [3]:

$$\begin{cases} \dfrac{dx}{dt} = \alpha y(t) \\ \dfrac{dy}{dt} = \beta x(t) \\ x(0) = x_0, y(0) = y_0 \end{cases} \tag{1}$$

The constants α, β denotes "terms of relationship." A positive constant indicates that there is an affection of the other person and thus the feelings between the two lovers in- crease. A negative constant, on the other hand, indicates that the other person's affection largely rejects these feelings and thus there is uncertainty in their relationship. The best known application of systems of differential equations in the study of the dynamics of the relationship between Romeo and Juliet was proposed by the mathematician Strogatz in 1994. An obvious difficulty in any model that defines love between two individuals is what is meant by love and how we can model in a meaningful way.

The Strogatz type model of a love affair between Romeo and Juliet is of the form [4]:

$$\begin{cases} \dfrac{dx}{dt} = ax(t) + by(t) \\ \dfrac{dy}{dt} = cx(t) + dy(t) \\ x(0) = x_0, y(0) = y_0 \end{cases} \tag{2}$$

Where $x(t)$ quantify Romeo's love (or hate if $x(t) < 0$) for Juliet at the time and $y(t)$ quantify Juliet's love for Romeo (or hate if $y(t) < 0$) .

Parameters a, b, c, d quantify Romeo's romantic style and Juliet's romantic style as follows [5]:

- The parameter a would describe how Romeo is encouraged by his feelings
- The parameter b would describe how Romeo is encouraged by Juliet's feelings.
- The parameter c would describe how Juliet is encouraged by her feelings
- The parameter d would describe how Juliet is encouraged by Romeo's feelings

3. Results & Discussion

Particular case one: If we consider the system: (3)

$$\begin{cases} \dfrac{dx}{dt} = 0 \\ \dfrac{dy}{dt} = cx(t) + dy(t) \\ x(0) = x_0, y(0) = y_0 \\ a = b = 0 \end{cases},$$

- We can say that Romeo's feelings towards Juliet never change, but there remains an indifference towards Juliet .
- Studying this model, we can observe Juliet's reaction to Romeo's indifference.

$$\begin{cases} \dfrac{dx}{dt} = 0 \\ \dfrac{dy}{dt} = cx(t) + dy(t) \\ x(0) = x_0, y(0) = y_0 \end{cases} \leftrightarrow \begin{cases} x(t) = C_1 \\ \dfrac{dy}{dt} = cC_1 + dy(t) \\ x(0) = x_0, y(0) = y_0 \end{cases} \leftrightarrow \begin{cases} x(t) = C_1 \\ y(t) = -\dfrac{cC_1}{d} + C_2 e^{dt} \\ x(0) = x_0, y(0) = y_0 \end{cases} \tag{4}$$

The solution is of the form : (5) $\begin{cases} x(t) = x_0 \\ y(t) = -\dfrac{cx_0}{d} + (y_0 + \dfrac{cx_0}{d})e^{dt} \end{cases}$

- $\dfrac{dy}{dt} = dy(t) + cC_1 \rightarrow \dfrac{dy}{dt}e^{-dt} - dy(t)e^{-dt} = cC_1e^{-dt} \rightarrow \dfrac{d}{dt}(y(t)e^{-dt}) = cC_1e^{-dt}$

$y(t)e^{-dt} = -\dfrac{cC_1}{d}e^{-dt} + C_2 \rightarrow y(t) = -\dfrac{cC_1}{d} + C_2e^{dt}$

Particular case two: If we consider the system: (6) $\begin{cases} \dfrac{dx}{dt} = x(t) + y(t) \\ \dfrac{dy}{dt} = x(t) + y(t) \\ x(0) = x_0, y(0) = y_0 \\ a = b = c = d = 1 \end{cases}$

- $\begin{cases} y(t) = \dfrac{dx}{dt} - x(t) \\ \dfrac{dy}{dt} = x(t) + y(t) \end{cases} \leftrightarrow \begin{cases} y(t) = x'(t) - x(t) \\ x''(t) - x'(t) - x(t) = 0 \end{cases} \leftrightarrow \begin{cases} y(t) = x'(t) - x(t) \\ x(t) = C_1e^{\frac{1+\sqrt{5}}{2}t} + C_2e^{\frac{1-\sqrt{5}}{2}t} \end{cases} \leftrightarrow$

The solution is of systems (4) is the form (7) $\begin{cases} y(t) = \dfrac{\sqrt{5}-1}{2}C_1e^{\frac{1+\sqrt{5}}{2}t} - \dfrac{\sqrt{5}+1}{2}C_2e^{\frac{1-\sqrt{5}}{2}t} \\ x(t) = C_1e^{\frac{1+\sqrt{5}}{2}t} + C_2e^{\frac{1-\sqrt{5}}{2}t} \end{cases}$

Note: Let's deal with the equation : $x''(t) - x'(t) - x(t) = 0$

- We attach the characteristic equation: $\lambda^2 - \lambda - 1 = 0 \rightarrow \lambda_{1,2} = \dfrac{1 \pm \sqrt{5}}{2}, m_1 = m_2 = 1$
- The solution is : $x(t) = C_1e^{\frac{1+\sqrt{5}}{2}t} + C_2e^{\frac{1-\sqrt{5}}{2}t}$

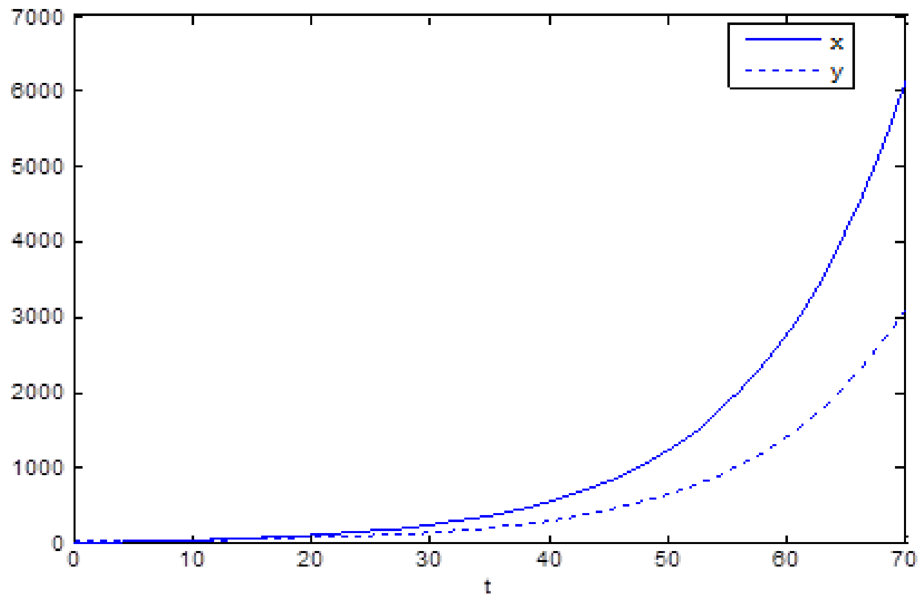

Fig. 1: The two are very eager to have a relationship.

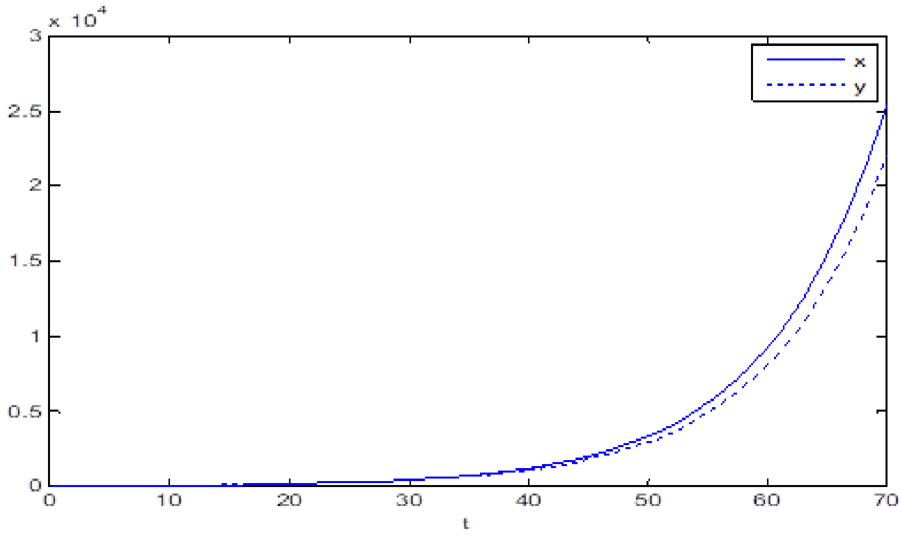

Fig. 2: We have mutual love or quarrel

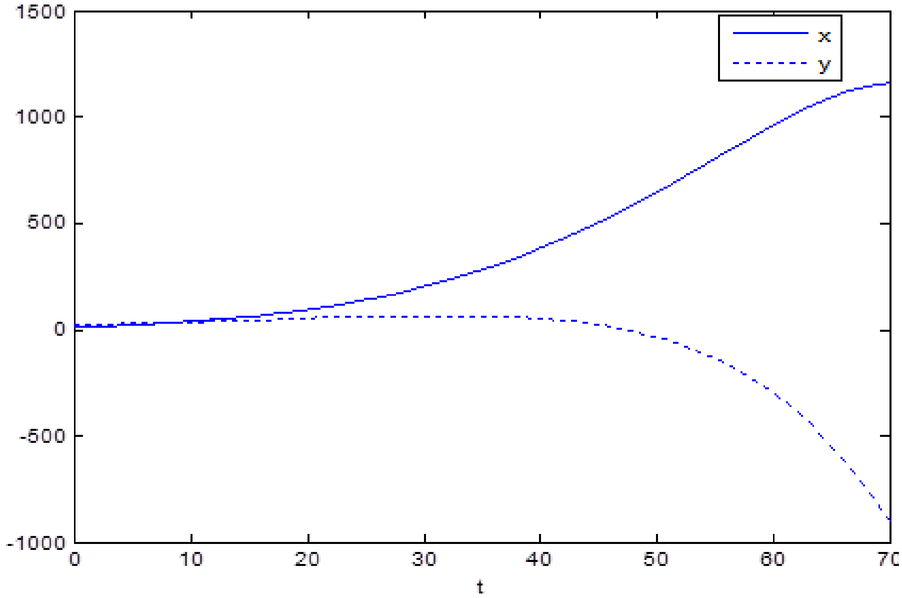

Fig. 3: We have an endless cycle of love versus strife

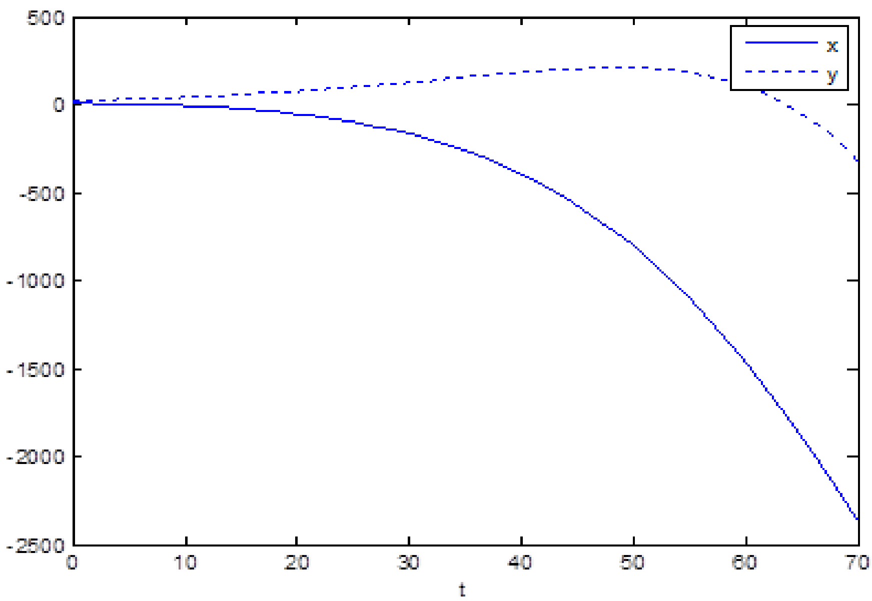

Fig. 4: Case: $b < 0, c > 0$. We have an endless cycle of love versus strife

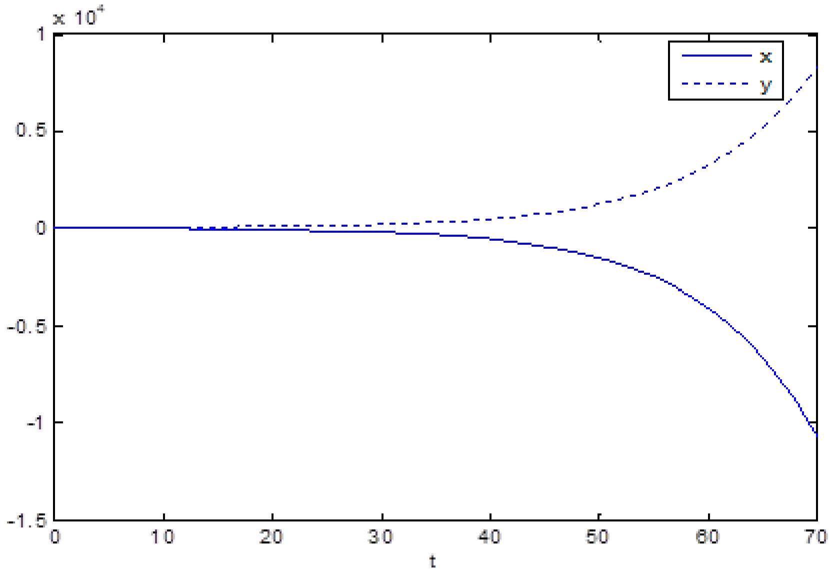

Fig. 5: Case: $b < 0, c < 0$. We have an unrequited love between the two lovers

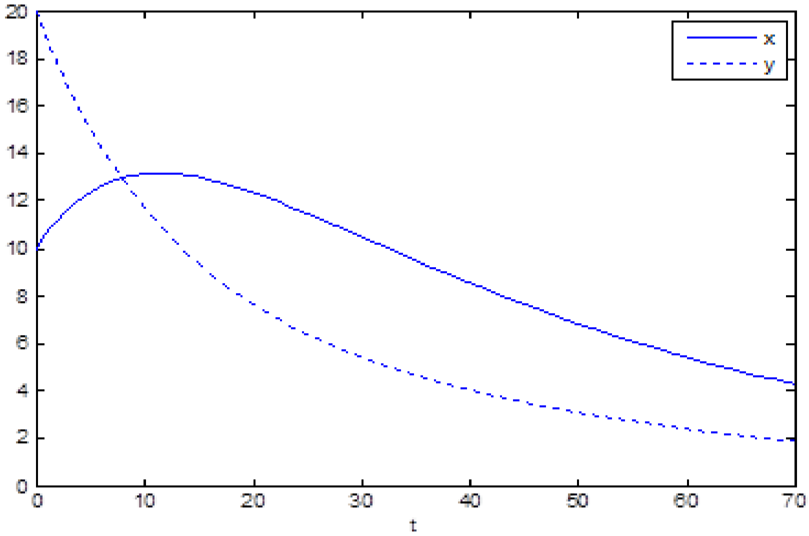

Fig. 6: Case: $a + d < 0$. We have a cold relationship between the two lovers

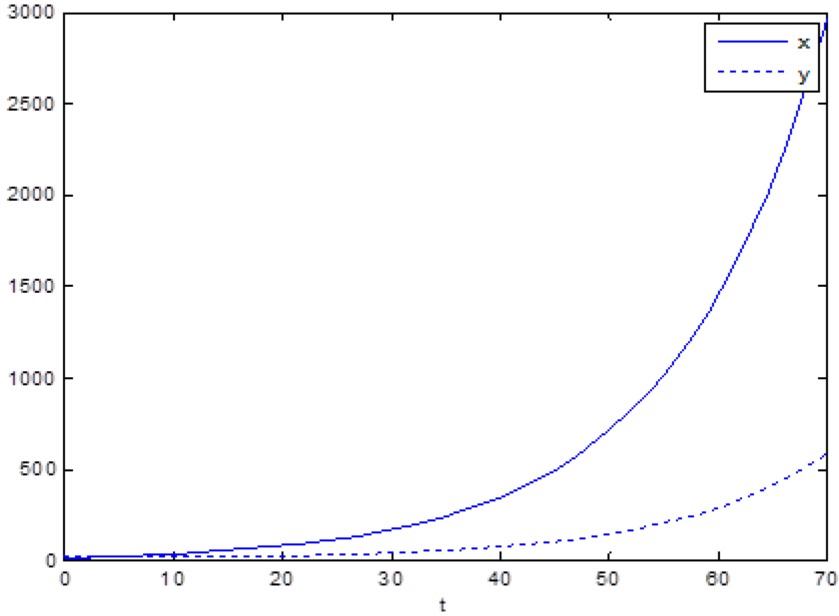

Fig.7: Case: $a + d > 0$. We have an intensity of feelings between the two lovers

Stability solution

We consider the general form of the system (8): $\begin{cases} \dfrac{dx}{dt} = f(x,y) \\ \dfrac{dy}{dt} = g(x,y) \end{cases}$

To determine the behavior of system solutions (6) we will define two notions: equilibrium and stability.

Note: The equilibrium points are determined as follows: $\begin{cases} f(x,y) = 0 \\ g(x,y) = 0 \end{cases}$ (9)

- For system (8), we obtain : $\begin{cases} cx + dy = 0 \\ ax + by = 0 \end{cases}$, $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$
- Because $\det A \neq 0 \rightarrow$ the system only admits the null solution: $x = y = 0$. We will have only one equilibrium point: $E(0,0)$

For stability [6], we will define the Jacobi matrix:

$$J(x,y) = \begin{bmatrix} \dfrac{\partial f(x,y)}{\partial x} & \dfrac{\partial f(x,y)}{\partial y} \\ \dfrac{\partial g(x,y)}{\partial x} & \dfrac{\partial g(x,y)}{\partial y} \end{bmatrix}$$

A. Equilibrium is a node if both eigenvalues of the Jacobian evaluated at the equilibrium points are real, distinct, nonzero, and of the same sign

- The node is locally stable if the eigenvalues are negative.
- The node is locally unstable if the eigenvalues are positive

B. The equilibrium is a saddle node if the two eigenvalues of the Jacobian evaluated at the equilibrium points are real, distinct, nonzero, and of different signs.

C. The equilibrium is a spiral if the two eigenvalues of the Jacobian evaluated at the equilibrium points are complex numbers with non-zero real parts.

- The spiral is locally stable, if the real parts of the eigenvalues are negative
- The spiral is locally unstable if the real parts of the eigenvalues are positive

$$\begin{cases} f(x,y) = ax + by, g(x,y) = cx + dy \\ \dfrac{\partial f}{\partial x} = a, \dfrac{\partial f}{\partial y} = b \\ \dfrac{\partial g}{\partial x} = c, \dfrac{\partial g}{\partial y} = d \end{cases} \rightarrow J = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

- We will calculate the eigenvalues:

$$\det(J - \lambda I_2) = 0 \rightarrow \begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0 \rightarrow (a-\lambda)(d-\lambda) - bc = 0$$

- We obtain the equation of the second degree:

$$\lambda^2 - \lambda(a+d) + (ad - bc) = 0,$$

$$\Delta = (a+d)^2 - 4(ad-bc) \rightarrow \lambda_{1,2} = \frac{(a+d) \pm \sqrt{(a+d)^2 - 4(ad-bc)}}{2}.$$

From the theory of the equation of the second degree we obtain

- If $\Delta > 0 \leftrightarrow (a+d)^2 > 4(ad-bc)$ the equation admits two real solutions.
- If $\Delta = 0 \leftrightarrow (a+d)^2 = 4(ad-bc)$ the equation admits only one solution
- If $\Delta < 0 \leftrightarrow (a+d)^2 < 4(ad-bc)$, the equation admits conjugate complex roots

Special case three **:** We consider the special case, where both Romeo and Juliet are not masters of their own feelings, but simply an attraction to the other ($a = d = 0$)

System (8) is of the form :
$$\begin{cases} \dfrac{dx}{dt} = by(t) \\ \dfrac{dy}{dt} = cx(t) \\ x(0) = x_0, y(0) = y_0 \end{cases} \quad (10)$$

- $$\begin{cases} \dfrac{dx}{dt} = by(t) \\ \dfrac{dy}{dt} = cx(t) \end{cases} \leftrightarrow \begin{cases} x''(t) = by'(t) \\ y'(t) = cx(t) \end{cases} \leftrightarrow \begin{cases} x''(t) = bcx(t) \\ y'(t) = cx(t) \end{cases} \leftrightarrow \begin{cases} x(t) = C_1 e^{\sqrt{bc}t} + C_2 e^{-\sqrt{bc}t} \\ y'(t) = cx(t) \end{cases}$$

The solution is of the form :
$$\begin{cases} x(t) = C_1 e^{\sqrt{bc}t} + C_2 e^{-\sqrt{bc}t} \\ y(t) = \dfrac{cC_1}{\sqrt{bc}} e^{\sqrt{bc}t} - \dfrac{cC_2}{\sqrt{bc}} e^{-\sqrt{bc}t} \end{cases} \quad (11)$$

From the initial conditions we get :

$$\begin{cases} x(t) = (\dfrac{x_0}{2} + \dfrac{y_0\sqrt{bc}}{2c})e^{\sqrt{bc}t} + (\dfrac{x_0}{2} - \dfrac{y_0\sqrt{bc}}{2c})e^{-\sqrt{bc}t} \\ y(t) = \dfrac{c_1}{\sqrt{bc}}(\dfrac{x_0}{2} + \dfrac{y_0\sqrt{bc}}{2c})e^{\sqrt{bc}t} - \dfrac{c_2}{\sqrt{bc}}(\dfrac{x_0}{2} - \dfrac{y_0\sqrt{bc}}{2c})e^{-\sqrt{bc}t} \end{cases} \quad (12)$$

In this case, the eigenvalues of the Jacobi matrix are : $\lambda_{1,2} = \pm\sqrt{bc}$.

- The equilibrium is a saddle node in this case, because the two eigenvalues of the Jacobian evaluated at the equilibrium points are real, distinct, nonzero, and of different signs.
- In all the subcases that we will analyze, we will take into account that : $b \cdot c > 0$.

The dynamics of the relationship between the two lovers will depend on the following b, c parameters**:**

- We have a mutual love (or mutual hatred): $b > 0, c > 0$
- We only have one-sided love (one loves and the other hates or is indifferent): $b < 0, c < 0$.
- We have an infinite love (hate) cycle of the two: $b \cdot c > 0$

Special case four **:** We consider the special case, where the two lovers are exactly opposite in feelings: $\begin{cases} c = -b \\ d = -a \end{cases}$

System (8) is of the form : (13)
$$\begin{cases} \dfrac{dx}{dt} = ax(t) + by(t) \\ \dfrac{dy}{dt} = -bx(t) - ay(t) \\ x(0) = x_0, y(0) = y_0 \end{cases}$$
sau:
$$\begin{cases} \dfrac{dx}{dt} = -dx(t) - cy(t) \\ \dfrac{dy}{dt} = cx(t) + dy(t) \\ x(0) = x_0, y(0) = y_0 \end{cases}$$

- In this case, the eigenvalues of the Jacobi matrix of system (13) are : $\lambda_{1,2} = \pm\sqrt{a^2 - b^2}$.
- The equilibrium is a saddle node in this case, because the two eigenvalues of the Jacobian evaluated at the equilibrium points are real, distinct, nonzero, and of different signs.
- In all the subcases that we will analyze, we will take into account that : $a^2 - b^2 > 0$

- $$\begin{cases}\dfrac{dx}{dt}=ax(t)+by(t)\\ \dfrac{dy}{dt}=-bx(t)-ay(t)\end{cases}\leftrightarrow\begin{cases}y(t)=\dfrac{x'(t)-ax(t)}{b}\to y'(t)=\dfrac{x''(t)-ax'(t)}{b}\\ \dfrac{x''(t)-ax'(t)}{b}=-bx(t)-a\dfrac{x'(t)-ax(t)}{b}\end{cases}\leftrightarrow$$
- $$\begin{cases}y(t)=\dfrac{x'(t)-ax(t)}{b}\\ x''(t)-(a^2-b^2)x(t)=0\end{cases}\leftrightarrow\begin{cases}y(t)=\dfrac{x'(t)-ax(t)}{b}\\ x(t)=C_1e^{\sqrt{a^2-b^2}t}+C_2e^{-\sqrt{a^2-b^2}t}\end{cases}\leftrightarrow$$

The solution is of the form :
$$\begin{cases}y(t)=\dfrac{(\sqrt{a^2-b^2}-a)}{b}C_1e^{\sqrt{a^2-b^2}t}-\dfrac{(\sqrt{a^2-b^2}+a)}{b}C_2e^{-\sqrt{a^2-b^2}t}\\ x(t)=C_1e^{\sqrt{a^2-b^2}t}+C_2e^{-\sqrt{a^2-b^2}t}\end{cases}$$

The dynamics of the relationship between the two lovers will depend on the following a,b parameters:

- Romeo becomes a willing and enthusiastic person who works very hard: $a\cdot b>0$, and in addition he becomes a hermit (he lives in extreme asceticism, with only one goal, that of praying incessantly for everyone).
- We only have a cautious love of Romeo towards Juliet: $a\cdot b<0$, and in addition Romeo becomes a narcissistic type (Romeo considers himself a special person, superior to ordinary people. In Romeo, self-esteem is high but fragile, being very concerned about the way which also shows how favorably he is seen by those around him. He takes great pleasure in compliments and is very vulnerable to criticism.

Special case five: We consider the special case, where the two lovers are exactly opposite in feelings: $\begin{cases}c=b\\ d=a\end{cases}$

System (8) is of the form: (14) $$\begin{cases}\dfrac{dx}{dt}=ax(t)+by(t)\\ \dfrac{dy}{dt}=bx(t)+ay(t)\\ x(0)=x_0,\,y(0)=y_0\end{cases}$$ sau: $$\begin{cases}\dfrac{dx}{dt}=dx(t)+cy(t)\\ \dfrac{dy}{dt}=cx(t)+dy(t)\\ x(0)=x_0,\,y(0)=y_0\end{cases}$$

- In this case, the eigenvalues of the Jacobi matrix of the system are: $\lambda_{1,2}=a\pm b$
- The equilibrium is a saddle node in this case, because the two eigenvalues of the Jacobian evaluated at the equilibrium points are real, distinct, nonzero, and of different signs.
- In all the subcases that we will analyze, we will take into account the following cases $|a|<|b|,|a|>|b|$

$$\begin{cases}\dfrac{dx}{dt}=ax(t)+by(t)\\ \dfrac{dy}{dt}=bx(t)+ay(t)\end{cases}\leftrightarrow\begin{cases}y(t)=\dfrac{x'(t)-ax(t)}{b}\to y'(t)=\dfrac{x''(t)-ax'(t)}{b}\\ \dfrac{x''(t)-ax'(t)}{b}=bx(t)+a\dfrac{x'(t)-ax(t)}{b}\end{cases}$$

The dynamics of the relationship between the two lovers will depend on the following a,b parameters:

- We only have a cautious love of the two lovers for $|a|<|b|$ again between the two lovers there will be either a passionate love or quarrels, depending on the initial conditions.
- We have a cautious love of the two for $|a|>|b|$, but their relationship turns into a state of mutual apathy.

Special Case six: We consider the special case where Romeo's feelings for Juliet are not affected by her feelings for him ($b=0$) as well as his feelings for her ($a=0$).

$$\begin{cases} \dfrac{dx}{dt} = 0 \\ \dfrac{dy}{dt} = cx(t) + dy(t) \\ x(0) = x_0, y(0) = y_0 \end{cases}$$

System (8) is of the form (15)

In this case, the eigenvalues of the Jacobi matrix of the system are: $\lambda_1 = d, \lambda_2 = 0$.

- $$\begin{cases} \dfrac{dx}{dt} = 0 \\ \dfrac{dy}{dt} = cx(t) + dy(t) \\ x(0) = x_0, y(0) = y_0 \end{cases} \leftrightarrow \begin{cases} x(t) = A \\ \dfrac{dy}{dt} = d \cdot y(t) + cA \end{cases} \rightarrow \begin{cases} y(t) = -\dfrac{cA}{d} + B \cdot e^{dt} \\ x(t) = A \end{cases}$$

$$\frac{dy}{dt} = dy(t) + cA \rightarrow \frac{dy}{dt} e^{-dt} - dy(t) \cdot e^{-dt} = cA \cdot e^{-dt} \rightarrow \frac{d}{dt}(y \cdot e^{-dt}) = cAe^{-dt} \rightarrow$$

$$y(t) \cdot e^{-dt} = -\frac{cA}{d} \cdot e^{-dt} + B \rightarrow y(t) = -\frac{cA}{d} + B \cdot e^{dt}$$

- If Romeo loves Juliet ($x(t) > 0$), she will only love him back if she is a prudent or narcissistic lover ($cd < 0$).
- If Juliet is narcissistic ($d > 0$), then her love will grow uncontrollably or she will end up hating him depending on the initial conditions, but in this case her feelings will never die.
- If $d < 0$, we can talk about stable equilibrium of the system**.**

8. Conclusions

A functional couple is born, most of the time, from that "divine madness" in which the person is intoxicated with euphoria and energy through the high production of endorphins, encephalin and phenyl ethylamine in the brain. But, when the effect of these natural narcotics diminishes, it takes effort, involvement, continuous adaptation to the other, open communication, active listening, mutual understanding, intellectual, emotional and sexual intimacy, for a couple to pass the durability test. We can say that using the notion of stability (as a mathematical notion) that the success of the relationship depends on the level in which it has as a foundation a solid friendship and fulfills the needs of the two partners. For this to be possible, each partner needs to communicate their needs and make an effort to find out the other's needs and to recognize them without making value judgments about whether or not they are right, reasonable, wrong or inappropriate.

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